SHORTER NOTICES.

Oeuvres de G. H. Halphen. Publiées par les soins de C. Jordan, H. Poincaré, E. Picard, avec la collaboration de E. Vessiot. Vol. I, 1916, xliv + 570 pp. Vol. II, 1918, vii + 560 pp. Paris, Gauthier-Villars.

Picard in his *Notice sur la vie et les travaux de Georges-Henri Halphen* and Poincaré after him in his *Notice sur Halphen* urged that one could distinguish among the mathematicians of thirty years ago two well-marked and opposing tendencies of thought separating mathematical labors into two distinct categories and the mathematicians themselves, perhaps less definitely, into two classes. Those of the one class are preoccupied principally with enlarging the field of known notions; the others prefer to devote their energies primarily to penetrating more deeply into notions which already have been analyzed and elaborated. This distinction of classes is just as valid today as it was when Picard and Poincaré insisted upon it in 1890; and it is likely to remain so as long as mathematics develops along the lines already marked out.

Those who are most concerned with extending the frontiers of science often find it necessary to leave their new ideas without much elaboration and to proceed to a general account of the lay of the field, so to speak, in order to obtain at once a comprehensive view. In such investigations many questions will be raised and not answered, it is very difficult to stand always clear of errors in detail, lines of investigation which deserve to be followed up must at most be only indicated, and many promising thoughts must be dismissed altogether.

The mathematicians of the second class, of whom Halphen is one of the greater, are more intimately concerned with a desire to give to their work a character of absolute perfection. An error with respect to even the smallest detail becomes a matter of acute pain. Whatever these mathematicians touch they wish to achieve to the point of leaving unanswered no question which their investigation raises. They seek nothing less than to put their thought into a form of absolute perfection and beauty.

These two directions of mathematical thought are observed also in the different branches of the science. One can probably

still say (as Picard said in 1890) that in a general way the tendency to comprehensive investigation and broad characterization is found more often in matters relating to the theory of functions, and the tendency to deep penetration in the works on modern algebra and analytic geometry.

The work of Halphen is marked principally by the tendency to deep penetration. He devoted himself for the most part to algebra and geometry, fields apparently best suited to such a temperament. He analyzed the difficult problems attacked to the point of complete and definitive results. Everywhere his work is marked by the effort to leave not a single thing unachieved.

After the researches of de Jonquières and Chasles there was a lively investigation of algebraic systems of conics depending on a single parameter. Chasles had found, by a sort of induction, a general law giving the number of conics satisfying a given condition. This number was composed of a sum of two terms, each of them being a product of two factors one of which depended on the system and the other on the condition. Halphen, simultaneously with several other mathematicians, sought a proof of the law of Chasles; and he believed that he had found one. Later he perceived an error in his argument and consequently took up again a study of the question. After a long investigation his labors were rewarded through the discovery of a complete solution of the problem by means of a method of great originality. The result is distinguished by a characteristic deep penetration.

Halphen's masterpiece, according to Poincaré, is his memoir on twisted algebraic curves, crowned in 1881 by the Berlin Academy. This theory is contrasted in a remarkable manner with that of plane curves. For the latter a single number, the degree, suffices for a complete classification. Twisted algebraic curves do not possess a like property. To give a single number is not enough to classify them. Moreover, one cannot find a system of integers enjoying with respect to twisted curves a rôle analogous to that of the degree for plane curves. Halphen obtains a satisfactory solution of the problem; but, since the solution is not susceptible of an analytical expression, its character is likely to escape the superficial reader. The results bring to light certain curious and unnoticed properties which are brought out clearly by Poincaré (vol. I, p. xxxi).

Among the other larger subjects investigated by Halphen

may be mentioned the following: singular points of algebraic curves, differential equations and invariants, elliptic functions, theory of numbers, and theory of series.

For a brief sketch of the life of Halphen, with some remarks on the character of his work, the reader may consult the *Notice* by Picard (vol. I, pp. vii–xvi). In the *Notice* by Poincaré (vol. I, pp. xvii–xliii) we have an excellent systematic analysis of his mathematical contributions. Besides this there is the *Notice* by Halphen himself (vol. I, pp. 1–47) in which his own contributions were analyzed on the occasion of his candidacy before the Paris Academy of Sciences in 1885, about four years before his death. These excellent brief accounts of his work relieve the reviewer of the duty of making an analysis of the separate memoirs. The entire works (with the exception, apparently, of the *Traité des Fonctions Elliptiques*) are to be included in four volumes, of which the third is announced as in press and the fourth in preparation.

R. D. CARMICHAEL.

A History of the Conceptions of Limits and Fluxions in Great Britain from Newton to Woodhouse. By Florian Cajori. Chicago, The Open Court Publishing Company, 1919, pp. viii + 299.

This work appears as number five in the Open Court Series of Classics of Science and Philosophy, a series which should meet with all the encouragement and support that American scholars can give in these times, when the question of the publication of such works is so critical. That such encouragement and support is justified may be seen from an examination of this latest production of Professor Cajori's pen, for he has here given to scholars one of the best of his various studies in the history of mathematics.

The work consists of twelve chapters under substantially the following titles: I. Newton; II. Printed books and articles on fluxions before 1734; III. Berkeley's *Analyst;* IV. Jurin's controversy with Robins and Pemberton; V. Textbooks immediately following Berkeley; VI. Maclaurin's *Fluxions* (1742); VII. Textbooks of the middle of the eighteenth century; VIII. Robert Heath and the controversy in his time; IX. Abortive attempts at arithmetization; X. Later works on fluxions; XI. Criticisms under the influence of