TWO BOOKS ABOUT AIRPLANES.

Aeronautics: A class text. By Edwin B. Wilson. New York. John Wiley and Sons. vi + 265 pp.

Grundlagen der Flugtechnik: Entwerfen und Berechnung von Flugzeugen. Von Dr.-Ing. H. G. Bader. Berlin, B. G. Teubner, 1920. Mit 47 Figuren im Text. vi + 194 pp.

The mathematical treatment of the motion of an airplane requires a knowledge of two subjects, the dynamics of a rigid body, and the theory of fluid motion. The particular application is that of a rigid body moving in any way through a gas and supported by the reaction of the gas. In order to get this support, it is necessary that the body should have such a shape that the pressure of the gas may have full play, and that there be attached a power system which, by driving the body through the air, shall furnish a support equal to the weight of the body. If the body is required to rise, additional power is necessary to furnish an upward force to create the vertical acceleration. The first problem is, therefore, a consideration of the pressures to be produced when the body is to have uniform motion in a horizontal straight line, and next, the additional pressures when it is required to rise. Moreover, the best shape of the body for the purpose in view must be considered.

It is a familiar observation in watching the flight through the air of any object, such as a thin plate or a sheet of cardboard, that it rarely keeps in a fixed direction, or remains parallel to its first position, unless some device is used for the purpose. Even then there will be considerable deviations. In fact, one may almost say that, in general, the motion of a body through a fluid is unstable, and that, in order to keep it steady on a straight track, it is necessary to check the slight deviations as they arise. In other words, human agency is required for steering such an object. On the other hand, careful research, both mathematical and practical, have shown that we can often find devices which will maintain stability within certain limits. In the case of the airplane, investigation of the stability of any particular type is of fundamental importance, and this should extend not only to small deviations, but also to any that are likely to arise.

As far as the motion of the body is concerned when the various forces are given, the well known methods and results of ordinary dynamics can be applied immediately with full confidence. There are no fundamental difficulties to be overcome, except that of reducing complicated formulas to arithmetic. This is chiefly a matter of calculation, which is often laborious. The forces are of three kinds, those due to gravitation which can always be calculated, those due to the thrust of the power system for which we usually have the necessary data, and finally, those due to the air pressures. Theoretically, the latter should be deducible from the equations of hydrodynamics, but practically so little has been achieved in this effort, that in most cases recourse has been had to experiment both for the general laws and the detailed results.

It is really rather astonishing how little has been deduced for even the simplest case, say that of a sphere moving with constant velocity through a liquid. Solutions are known and are fully worked out for the case of a fluid without viscosity. But if the velocity exceeds a certain rather low limit, and the fluid has only a slight viscosity, the motion of the fluid and its pressures on the surface of the sphere bear little or no resemblance to those given by the solution for a non-viscous fluid. The motion is said to beome *turbulent,* which is merely a word to express our ignorance of its essential properties and our inability to calculate it. The mathematical solution for a perfect fluid cannot be treated as a first approximation to that in a slightly viscous fluid. The same fact vitiates nearly all the cases hitherto treated. This is made particularly obvious when we open the chapter on viscosity in a standard treatise, such as Lamb's *Hydrodynamics*, and find that in the very simplest problems the coefficient of viscosity appears in the denominator so that its vanishing makes the expressions infinite. Of course, there is an explanation of this result, but the explanation is after all really an apology for the inherent weakness of all the known methods of treatment for the motion of a solid through a liquid.

Thus, when we come to learn about the distribution of air pressures on a moving plane, we find that hydrodynamics helps but little. This is partly due to circumstances. During the war, when the greatest advance was made in the development of flying machines, it was necessary to obtain results as quickly as possible, and as there was little hope of getting them from theory, experimental data were gathered by a numerous body of workers. It is from them that most of our present knowledge comes. But with calmer times, it is to be hoped that theory may take its proper share in the investigations and not only gather up into general formulas the known* numerical data, but also deduce further consequences and make advances on its own account.

Two or three volumes on the mathematics of the airplane have appeared in England. So far as we are aware, Professor Wilson's is the first to appear in this country. He is unusually well qualified for the task. Besides his eminence as a mathematician and a physicist, he has taught the subject for some years at the Massachusetts Institute of Technology, and has himself contributed a valuable memoir. Thus his views as to the proper approach for the student merit careful consideration. Like most of those who write introductory textbooks (and his is frankly of this character), he is naturally influenced by the character and degree of knowledge of the students who attend his own classes. This may cause difference of opinion as to what portions of previous subjects should be included. Aside from this fact, however, one can see well what is needed for the student approaching the subject of aeronautics, and the teacher can easily omit or add such matter as he may find necessary.

The first three chapters of Professor Wilson's text, constituting an introduction, give the principal facts which the student needs to know about the motion of a plane through the air. In the second chapter, the accepted law of pressure, —that it varies as the square of the velocity, the area of the surface, and the sine of the angle of attack (the angle between the plane and its direction of motion relative to the air),— is discussed, and the deviations from this statement for different velocities, shapes, and angles, as shown by experiment, are given and are illustrated by diagrams. With this as a basis, the equations of relative equilibrium of the skeleton plane are obtained in the third chapter, for steady horizontal, inclined, and circular flight. The effect of a second plane attached to the first as a *stabiliser* is also treated.

The next five chapters under the sub-title *Rigid Mechanics*, deal with the motion of the airplane under various conditions.

* The word *known* is perhaps inapplicable, for much of the information gathered by the various governmental stations is not yet available.

In the first of them, entitled *Motion in a resisting medium,* the landing problem, a free vertical fall and some other trajectories are discussed. In connection with the free fall, Professor Wilson uses an illustration showing how to construct an analytic expression which shall fit a table of numerical results derived from experiment. It may interest some readers to know that there *is* a maximum velocity of descent. Next follows a discussion of harmonic motion, which is not only sufficient for the purpose in view, but can be recommended as an excellent introduction to the subject apart from its present applications. Then follow three chapters which constitute the main treatment of the motion of an airplane from the point of view of a rigid body. The method is to form the general equations of motion, assume some form of steady motion which satisfies them, and then to determine the stability by finding the character of the small deviations from this steady motion, that is whether they are oscillatory, damped, or increase with the time. Professor Wilson introduces the student to the nature of the problem by solving some cases in two dimensions first and treating the general case in a later chapter. Those who have attempted to present Bryan's excellent but somewhat involved treatment of this subject to a class will welcome these chapters, which give the main portion of the work in sufficient detail for a good grasp of the methods to be obtained.

The third section, entitled *Fluid Mechanics,* deals partly with theoretical hydrodynamics, and partly with experimental results of observations in wind-tunnels and elsewhere. The author has tried to make the best of a bad job, but we fear that the student will recognize rather easily how little the theory assists in giving information about the air forces on an airplane. It is none the less necessary that he should learn at least the elements of fluid motion from the theoretical side, for one cannot doubt that the time will come when they will get fuller application.

A chapter on dimensions, besides having great importance for aerodynamics, on account of the necessary study of the behavior of models, fills a gap. Its use in determining the form of a functional relation between physical quantities when we know what variables are present, is occasionally hinted at but is not often adequately treated in a text-book.

Throughout the whole volume, numerical examples are used as illustrations, most of them being taken from existing machines. In addition, Professor Wilson has gathered together at the end of each chapter numerous problems for the student to solve. If we regret anything, it is the lack of references to further work for the interested student or at least to show the rank and file they have not reached the conclusion of all knowledge on the subject. But this is a mere detail which will be filled in by the instructor who uses the book.

As its title denotes, the volume by Dr. Bader is devoted to the technique of the construction of airplanes, and as such it has no very special interest to the mathematician. But it is worth noting that the author is obviously well acquainted with the applications of mathematics to dynamical problems, both from the evidence of the book itself and from the titles of his own papers given in the list of the forty-three references at the end. He is full of his subject and he realizes that, in spite of basing all his results on theory, after all the final test must come from experiment. The maze of symbols used and the references to other work in the shape of formulas taken bodily with little explanation, repels the reader to some extent. If one searches for information as to the progress made in Germany in airplane construction, he will not find very much disclosed. Nevertheless, the aerodynamic expert need not consider the volume a useless addition to his library, for there are numerous hints here and there which will well repay some thought and examination. The copy received by the reviewer in paper covers fell to pieces on opening and is now a bundle of loose sheets. The price quoted on a slip enclosed with it is $1.30, which at the present rate of exchange, is equivalent to about 80 marks. This is probably about five times the price charged in the home market.

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