

## A BRIEF ACCOUNT OF THE LIFE AND WORK OF THE LATE PROFESSOR ULISSE DINI.

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THE death of Professor Ulisse Dini in his native city of Pisa, Italy, on the 28th of last October marked the passing of one whose name has long been familiar to the mathematical fraternity of the entire world and it therefore seems fitting, despite the wide separation geographically between the scene of his labors and our own, that a brief account of his life and work be given at this time in America and a small measure at least of tribute be rendered to his genius.

Dini was born on the 14th of November, 1845, of parents highly respected but of very moderate circumstances. He early manifested an unusual activity of both mind and body, thus commanding the attention and admiration of his instructors and masters who foresaw for him a future of extraordinary promise. Upon entering the neighboring university of Pisa his marked capabilities in mathematics were soon recognized and he shortly became the favorite pupil of Professor Betti, well known as one of the leaders in mathematical instruction and research at this period in Italy. Under such auspices and at the uncommonly early age of nineteen years, Dini attained the laureate and received directly afterward a government scholarship enabling him to continue his studies for a year at Paris. During this brief foreign sojourn he came chiefly under the influences of Bertrand and Hermite and formed an intimate friendship with them which extended long into later years. Returning forthwith to Pisa, his active career as teacher and investigator began in the year 1866 while he was yet scarcely of age and continued almost without interruption for over fifty years, being characterized throughout by the utmost zeal and devotion, not only to mathematical science, but to his native country, city and university, from all of which he received in turn the highest honors they could bestow.

Such in brief was Dini's career. If we inquire in a more detailed sense what his relations to mathematics actually

were and what his special achievements, it may be noted in the first place that his earliest researches lay in the field of infinitesimal geometry; more specifically, in the determination of the form and properties of certain partial differential equations which arise in the theory of applicable surfaces. His work in this connection, though extending over no more than six years (1864-1870), gave rise to some eighteen memoirs dealing chiefly with general problems in the theory of curvature and geodesics, some of which had been proposed earlier by Beltrami. At this early period of his life, however, Dini had not yet begun the researches for which he is to be regarded as famous nor had he in fact even entered seriously into that broad field, namely pure analysis, in whose development he was destined soon to play an active part. The transition of his interest and labors to this latter field took place about 1870. At this comparatively early date it will be recalled that the newer and more rigorous analysis (the so-called "modern analysis" of to-day) was but little known to the world at large, the spirit of its methods being virtually confined to the limited school of pupils immediately surrounding Weierstrass in Germany. Nevertheless, once Dini had turned his efforts in this direction, he appears to have reached within a remarkably short time a full appreciation of these newer ideas and methods. In fact, he straightway acquired such a critical insight into their significance and developed such ability and confidence in their use that he was soon independently at work carrying out for himself their manifold consequences, especially their bearing upon those concepts which lie at the foundation of analysis. Thus, by the year 1877, or seven years from the time he began, he published the treatise, since famous, entitled *Foundations for the Theory of Functions of Real Variables* (*Fondamenti per la teoria delle funzioni di variabili reali*). Much of what Dini here sets forth concerning such topics as continuous and discontinuous functions, the derivative and the conditions for its existence, series, definite integrals, the properties of the incremental ratio, etc., was entirely original with himself and has since come to be regarded everywhere as basal in the real variable theory. The book has, in fact, served as a model the world over and even at this date, which is more than forty years since its publication, it still affords one of the best available expositions of the basal concepts of analysis

as regarded from the standpoint of modern rigor, evidence of which may be found, for example, in the fact that as late as 1902 an authorized translation of the work was published in German.

The fact that Dini, though situated at the time far from any natural sources of information, was able to arrive so early at a thoroughgoing appreciation of the newer methods in analysis and to produce straightway so unique and valuable a work as the "Foundations" leads one naturally to inquire somewhat farther than we have thus far indicated into the attending circumstances. One may well ask, for example, what particular problem or incident served as his starting point and thus became his first inspiration. Dini's own explanation of this, as given in after years to inquiring friends, was somewhat as follows: While still a pupil at the university he came to feel serious doubts as to the actual meaning of some of the most fundamental concepts commonly employed in the calculus. They did not seem to be capable of the sort of definite, working formulation which one has a right to expect of every concept employed in mathematical science. While thus concerned in his own mind, he learned some years later from certain memoirs of Hankel, Schwartz, and Heine that similar doubts had been experienced by others and had already been resolved, at least in part, by Weierstrass and his school in Germany. Thereupon he obtained from this distant source such further information as was then available and, though meager and confined to special problems only, it was sufficient to impart the spirit of the newer methods and to convince him of their far reaching possibilities and consequences. He then began, single handed and alone, that remarkable series of studies and researches which culminated but a few years later in his treatise above mentioned on the Foundations.

With the assurance once gained that he was working upon well-grounded principles and definitions, Dini next proceeded to apply them in an extended and detailed sense. Thus, during 1877-78 he reworked and treated in his lectures a wide variety of topics taken from the usual course in higher analysis. In particular, he here gave for the first time a rigorous treatment of the general theory of implicit functions. His numerous researches at this period were left unpublished, however, being preserved only in lithograph form. Not

until the later years of his life did he undertake the considerable task of arranging the whole for publication, but it may now be found, together with much supplementary material, in his four large volumes published as late as 1915 entitled *Lessons in Infinitesimal Analysis* (*Lezioni di analisi infinitesimale*).

Following the studies of a very general nature above referred to, Dini turned his attention in 1878, or thereabouts, to a more specific field; namely, to a critical investigation of the convergence and other important properties of Fourier series. The result appeared two years later in the form of another treatise, this being entitled *Fourier Series and other Analytical Representations of the Functions of a Real Variable* (*Serie di Fourier e altre rappresentazioni analitiche delle funzioni di una variabile reale*). Here we meet with an entire book of 328 pages representing a single piece of sustained, original research and embracing results of a remarkably high order. Not only is the convergence of Fourier series studied from many points of view, but a complete and satisfactory discussion is here given for the first time of the convergence of various other allied developments well known in mathematical physics, such as the developments of an arbitrary function in terms of Bessel and Legendre functions. Moreover, a general method is finally set up for the examination of all series of the Fourier type (series in terms of normal functions). Dini's wonderful powers, both critical and inventive, doubtless reach their highest realization in this work, though, unlike the *Foundations*, the style of presentation is not well calculated to bring out the merit of the whole. This feature is largely due, as noted in the preface, to the fact that the publication proceeded at intervals in such a way that no opportunity was afforded to put the earlier parts of the work in proper adjustment with the later ones.\*

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\* As the development of an arbitrary function in terms of normal functions has been extensively considered in recent years, especially from the standpoint of the modern theory of integral equations, a word should perhaps be said as to the relation between these more recent studies and those of Dini above mentioned. Taking the question of convergence alone, the results of the more recent studies may be said to reach their culmination in the researches of Haar, *Math. Ann.*, vol. 69 (1910), p. 331 and vol. 71 (1911), p. 38. These, while exceedingly general, do not, however, include such important special cases as the Bessel and Legendre developments mentioned above. For a summary statement of Haar's results see Bôcher's address at the international congress of mathematicians at Cambridge (England), 1912, pp. 29-30, and for the underlying reason to account for the limitations which such results present, see the same address, p. 1,

Thus far we have mentioned only Dini's larger treatises. Space would hardly permit of a detailed consideration of his shorter memoirs, and fortunately this is not necessary, since the list, which comprises in all about sixty papers, has recently been prepared by his colleague, Professor Luigi Bianchi, and published along with a more extended account of his entire scientific achievements in the proceedings of the Accademia dei Lincei of last February. Suffice it to say that these memoirs pertain largely to infinitesimal geometry, to the theory of functions of a complex variable and to the study of differential equations, total and partial.

We might close our brief account at this point were it not for the fact that an adequate conception of Dini's activities can scarcely be gained from an examination of his scientific career alone. Dating from about the year 1880, he was chosen time and again to occupy positions of honor and trust in the affairs of his city, province and nation. In particular, he was at the time of his death vice-president of the national council of public instruction, a senator of the kingdom and the director of the normal college (Scuola Normale Superiore) of Pisa, in addition to his position as professor at the university. His death, therefore, was deeply mourned by a wide circle of people and institutions all of which have since united in an effort to erect a permanent monument within the city of Pisa that shall symbolize not only his genius in science but the love and esteem in which he was universally held by his countrymen.

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### SHORTER NOTICES.

*Commercial Algebra.* By WENTWORTH, SMITH and SCHLAUCH.  
Boston, Ginn and Company, 1917-1918.

THIS work appears in two volumes, Book I designed for the first year course of a commercial high school, and Book II a text for advanced classes in the same lines.

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statement (4) at the bottom of the page. The excluded cases of importance have usually been considered (though not by Dini) only through special methods adapted to the case in hand. Thus, see Hobson, *Proc. London Math. Soc.*, vol. 7 (1908), pp. 359-388; also C. N. Moore in various articles in the *Trans. American Math. Soc.* dating from 1907.