The Mathematical Analysis of Electrical and Optical Wave-Motion on the Basis of Maxwell's Equations. By H. BATEMAN, M.A., Ph.D. Cambridge University Press, 1915. vi +159 pp.

THIS book is intended as an introduction to certain recent developments of Maxwell's electromagnetic theory which are directly connected with the solution of the partial differential equation of wave motion. The higher parts of the theory which are based on the dynamical equations of motion are not considered.

In Chapter I (pages 1–24) the author starts from the fundamental equations for free ether in Maxwell's electromagnetic theory and shows in the first place that solutions of these equations may be obtained by means of solutions of the fundamental wave equation

$$(1) \qquad \Omega(u) \equiv \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0,$$

where c is the velocity of light (and is taken to be a constant). A function u of x, y, z, t, satisfying equation (1), is called a wave function. In the second place the author exhibits a class of solutions of the fundamental equations of Maxwell by means of functions α and β (of x, y, z, t) of such a nature that if $F(\alpha, \beta)$ is an arbitrary function of α and β, F satisfies the partial differential equation

$$(2) \qquad \left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2 + \left(\frac{\partial F}{\partial z}\right)^2 = \frac{1}{c^2}\left(\frac{\partial F}{\partial t}\right)^2.$$

Continuing in Chapter I it is shown that the fundamental Maxwell equations for a material medium may be solved by means of functions u (of x, y, z) satisfying the equation

$$(3) \qquad \Delta u + k^2 u \equiv \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + k^2 u = 0,$$

the quantity k being a constant with respect to x, y, z, t. This equation is obviously satisfied by wave functions of the form

$$u = e^{\pm ikct} f(x, y, z).$$

In connection with a wave boundary whose equation may be expressed in the form $F(x, y, z, t) = 0$ equation (2) comes again into play and here expresses the fact that the moving wave boundary moves normally to itself with the velocity of light.

The work under review is devoted principally to deriving the properties of functions satisfying equations (1) and (3) and to the applications of the resulting theory to problems of electrical and optical wave motion. It is desirable that all types of functions satisfying equations (1) and (3) should be studied and not merely those which admit readily of application to physical problems; for in this way a clearer light is thrown on the physical problems themselves. But there is a much more important reason why the scope of the inquiry should not be restricted. The theory of wave functions forms a natural extension of the theory of functions of a complex variable and may consequently lead to results of great value for the general theory of functions. Moreover, the theory of functions of two complex variables is closely connected with the theory of wave functions. Our author recognizes this breadth of range of interest in the theory of equations (1) and (3) and takes account of it in his exposition.

In Chapter II (pages 25–34) is to be found a general survey of the different methods of solving the wave equation, including the Bernoulli method of reduction to ordinary differential equations, the generalization of wave functions through the property of linearity of the wave equation and the method of transformations (developed by Bateman) by means of which other solutions may be obtained from a single given solution.

By means of a transformation to polar coordinates and a use of the Bernoulli method of reduction to ordinary differential equations various solutions of equations (1) and (3) are obtained in Chapter III (pages 35–68), and these are employed in the investigation of several important problems connected with spherical obstacles, including the following: scattering of electromagnetic waves by spherical obstacles; damped vibrations for the space outside of a sphere; polarization and intensity of the scattered light; absorption of light by a spherical obstacle; pressure of radiation on a spherical obstacle.

In Chapter IV (pages 69–81) a similar treatment is made of equations (1) and (3) by means of a transformation to cylindrical coordinates and the results are applied in a treatment of the propagation of waves on a semi-infinite solid bounded by a plane surface and also along a straight wire of circular cross section.

A partial solution of the problem of diffraction is obtained in Chapter V (pages 82–94) by means of multiple-valued solutions of the wave equation.

Chapter VI (pages 95–109) contains a brief account of each of several transformations of coordinates which lead to solutions of the wave equation suitable for the treatment of problems connected with surfaces of revolution. These give rise to several important ordinary differential equations of which it is desirable to have a more complete theory than we possess at present.

In Chapter VII (pages 110–114) methods are exhibited for finding homogeneous solutions of the wave equation.

Chapter VIII (pages 115–140) is given to a treatment of electromagnetic fields with moving singularities. It consists principally of the author's own contributions to the theory in consideration. From the investigations of this chapter it is seen that the mathematical analysis connected with equations (1) and (3) is suitable for the discussion of three distinct theories of the universe which may be briefly described as follows: (*a*) the ether is a continuous medium and matter consists of aggregates of discrete particles; (*b*) the ether is a discontinuous medium consisting of a collection of tubes or filaments; and matter is an aggregate of discrete particles attached to the tubes; (*c*) the ether is a continuous medium and matter is an aggregate of discrete particles to which tubes are attached. In Chapters I to VII the analysis is adapted almost entirely to the first of these theories, the high development of which we owe to the pioneer work of Maxwell, Fitzgerald, Hertz, Rayleigh, Heaviside, J. J. Thomson, Lorentz and Larmor. The other theories have heretofore not received much attention; and hence the contributions of Bateman (in Chapter VIII) form a welcome addition to our previous knowledge. It is to be hoped that they will lead to further developments so that a comparison can be made between the different theories. It is likely that each of the theories will be enriched by the development of the other two.

Finally, some miscellaneous theories are briefly treated in the concluding Chapter IX (pages 141–154).

This book will be found valuable for its new contributions to the theory of equations (1) and (3), for its account of the present state of knowledge and its numerous references to the literature of these equations and for its indication of several points at which further development of the theory is desirable. Its greatest value probably lies in the way in which it makes clear that it is desirable to have a further and deeper investigation of the properties of wave functions and the wave equation. R. D. CARMICHAEL.

NOTES.

THE Swiss mathematical society held its annual meeting at Geneva, September 12–15, in affiliation with the centenary celebration of the Helvetian society of natural scientists. The following papers were read: By Professor L. G. DU PASQUIER, "On systems of complex numbers"; by Dr. G. PÓLYA, "Is the non-continuation of a power series the general case?"; by Professor M. PLANCHEREL, "On the convergence of a remarkable class of definite integrals containing an arbitrary function"; by Dr. W. H. YOUNG and Dr. G. C. YOUNG, "Integration with regard to a function of limited variation"; by Dr. G. C. YOUNG, "On curves without tangents"; by Dr. K. MIRIMANOFF and Dr. G. C. YOUNG, "On the theorem of 'tuiles'"; by Professor L. J. CRELIER, "On a particular theorem of the geometry of motion"; by Professor R. DE SAUSSURE, "Geometry of leaflets"; by Professor CAILLER, "On the analytic theory of directed bodies"; by Dr. H. BERLINER, "A new projective analytic geometry"; by Professor L. KOLLROSS, "Concerning a duality"; by Dr. F. GONSETH, "Generalization of a theorem of Poncelet"; by E. GUILLAUME, "On the impossibility of reducing the law of divergence in several variables to a composite probability."

Professor M. GROSSMANN, of the technical school at Zurich, was elected president for the following year.

AT a meeting of the Edinburgh mathematical society on November 12 the following papers were read: "The solution of difference equations by continued fractions," by J. A. STRANG; "A suggested measurement of relationship" and "The equation $x^3 + y^3 + z^3 + u^3 = 0$ where x, y, z, u are rational," by J. E. A. STEGGALL; "Notes on a triangle," by G. E. CRAWFORD; "Easy geometrical proof of a theorem of Chasles," by E. PRESS.