tion to the advanced part of the theory of numbers is already recognized to be of lasting importance. A more elementary introduction to the ideas there presented, as well as applications to algebraic numbers, are given in Minkowski's Diophantische Approximationen, which appeared in 1907 and was reviewed for the BULLETIN by the present writer, February, 1909 (volume 15, pages 251–252).

L. E. DICKSON.

Algebra of Quantics. By E. B. ELLIOTT. Second edition. Clarendon Press, Oxford, England, 1913. xvi + 416 pp.

THIS new edition of Professor Elliott's well-known work on algebraic theory presents more changes in typography than in content or method. Although the material has been considerably increased (by one eighth is the author's liberal estimate), the number of pages has been actually decreased. This has been accomplished by more compact printing, the adoption of a new method of writing fractions, and the insertion of many important equations in the body of the text instead of printing them in separate lines. These changes do not add to the appearance of the text, and surely not to the delight of the reader, to whom the outstanding equations were of great assistance in reference. The change from d to δ in writing partial derivatives strikes the eye at once. Paragraph numbers are practically unchanged.

Conservatism marks this new edition. Professor Elliott has not abandoned the English methods for German symbolism. In a majority of paragraphs there is no change, and most of the others have only slight verbal changes or an added sentence to clear up obscurities or to suggest further deductions. Chapters V and XV alone present important modifications. In Chapter V, on binary quantics, there is considerable rearrangement, and material has been added on "Invariants as functions of the differences of roots." A half dozen added pages are devoted largely to establishing the conditions under which $F(a_1, a_2, \cdots, a_p)$ can be expressed as a function of the differences of the arguments and to proving some properties of this function. The author notes that this should have been in the first edition. Chapter XV, on restricted substitutions, has undergone the most changes and contains the most new material. The half dozen paragraphs on Boolean systems for the linear form, the quadratic, the linear and quadratic, etc., have been replaced by one short chapter which suggests the method by which the Boolean systems for quantics of low order may be calculated. Ten pages have been added on the deserving topic of orthogonal concomitants. Both the absolute and non-absolute concomitants are treated for direct and skew orthogonal transformations. Complete systems of orthogonal invariants and covariants are given for the quadratic, cubic, and quartic. Much of this theory has been developed since the first edition, although some of the theorems date back to Sylvester.

Some other changes may be worth noting. In Chapter IV a few pages on German symbolism are added and the relation between it and hyperdeterminants is shown. In Chapter VII, dealing with the operators O and Ω, a long paragraph is inserted on "Separation of gradients into seminvariants and other parts," with examples. In Chapter VIII, on generating functions, an important article is added on a method of extracting generating functions for *invariants* only, and is illustrated by the sextic case. The real generating function for the sextic has been added to the table. In Chapter IX, we find a new paragraph on Hilbert's second proof of Gordan's theorem and a short one on syzygies and syzygants. Chapter XI contains a new proof of the theorem that every G is an ΩF, where G and F are both gradients, covering the case where F contains more advanced letters than G. The possible nature of F is brought out clearly in the discussion. Chapter XII contains a new article on apolarity and another on the canonical form $(1, 0, C, D, E, 0, 1)(X, Y)^6$ of the sextic. In Chapter XIV, the concomitants of the linear form and a p-ic are given and the most important of several additions in Chapter XVI is on "A generator of all covariant ternary sources."

The above with various minor additions contain scarcely enough of novelty to warrant the entry of this second edition upon a shelf already containing the first, although the added examples are of great value. In the preface the author expresses the hope that the work will continue to appeal to the beginner and further "trusts that the book will retain for some time to come a certain value to those . . .who will need to look back upon and utilize the labors of those pioneers of modern algebraic theory, who adorned the latter half of the nineteenth century, and have now passed away." The demand for a second edition makes it reasonable to expect that this hope will be realized.

D. D. LEIB.