Das Wissen der Gegenwart in Mathematik und Naturwissenschaft. By EMILE PICARD. Authorized German edition von F. und L. LINDEMANN. Leipzig und Berlin, B. G. Teubner, 1913. 8vo. iv+292 pp.

This volume has for its prime object a sweeping survey of the state of the various sciences at the beginning of the twentieth century, with the intent of making very prominent the capital importance of what the scientist calls "theories." Their significance for the advance of science and what they may rationally be expected to yield is the subject for discussion. In addition to these, the rôle that the "concept" has to play, its yielding character under the influence of new facts and of the progress of new thought, is to be kept prominently in the front as one of the main features of the discussion. is viewed as an asymptotic approximation towards knowledge, the convergence of which is an undemonstrable postulate and may become as slow as that of certain series. The value of science is due to the fact that it is the child of the indissoluble Just as Carnot in marriage of the beautiful and the useful. studying heat-engines founded the science of thermodynamics, and Sainte-Claire-Deville in studying platinum founded the science of chemical mechanics; or on the other hand, as Newton in writing the Principia made it possible for navigators to sail unknown seas in greater security, or Ampère and Faraday with their interest in the action of currents upon currents made the modern electric power station a possibility; so in all ages science has proceeded from two parents: the love of order and harmony in thought, and the desire for those useful inventions that would make life more comfortable

The book is practically a translation of Picard's La Science moderne et son Etat actuel, but the Introduction has been condensed and modified, and the first chapter has had some changes. In our judgment the Introduction to the original could very profitably have been kept, inasmuch as it states in a more definite way the exact point of view from which the subject is to be examined. We will therefore extract somewhat from this original source in order to show the capital importance of the book itself for all thinkers, and particularly for those who are inclined to yield to the seductions of a certain will-o'-the-wisp, which lures them to hunt for an ultimate and completely finished philosophy of the universe.

After stating that it is not possible to go deep into the questions that arise with regard to science without analysing the manner in which we acquire knowledge, the author points out that the only source of knowledge is the flowing stream of The phenomena separate into two classes, consciousness. those which come into our consciousness through our senses and which our will is not sufficient to give birth to; and those which arise without the intermediation of the senses, as thought and memory. Thus we have the vague notions of the exterior and of the interior worlds. Through experience and the use of many organs under the action of the will, the illusions of sense are eliminated, and the residue of the exterior world we call the real. This involves also the appreciation of certain invariable relations between a first sensation and those we can This real must also be considered as it is cause to follow it. viewed by society, in particular by that part of society which is made up of those called *sane*. These invariant relations depend upon successive experiments of growing precision, and become the more valuable in proportion as they permit us to predict with greater certainty. Among these invariants arising from the exterior real we find certain ones that are independent of ourselves, others that depend upon us: the objective elements and the subjective elements. Among those that are objective some are constant in such a way as to give us the idea of thing, others are variable, like motion or position. Among the subjective elements there are some from which we abstract certain parts, forming concepts. These concepts are not absolutely rigid, but are subject to modification from new facts or from a sharper distinction of objective and subjective elements. The important point is that concepts are fundamental in the genesis of science, and scientific knowledge tends to development by their means, but their arbitrariness shows the presence of the human mind working upon the data of experience. In this connection logic is seen to be an admirable instrument, powerless to create, but powerful to transform concepts and bring out their unexpected consequences, thus leading us frequently to more extended concepts or to more general hypotheses. We come also to believe in laws of nature under the impulsion of our esthetic feeling and from our observation of the practical utility of such a belief. It is hopeless to attempt to demonstrate the principle of causality, but nevertheless we act as if it were true, and come to the

expected result. A system of concepts associated with laws or particular facts and transformed by convenient deductions, laid upon the framework of certain hypotheses, so as to produce a stable structure, is a scientific theory. The arbitrary character of the concepts is accentuated in the more arbitrary character of the hypotheses, and it often happens that the complicated network of concepts, facts, hypotheses, and reasonings of two very different theories will account equally well for a given group of phenomena. We can always make a theory agree with a set of phenomena by modifying the concepts and altering the hypotheses in the right manner. But we are under the guidance of a desire for simplicity and harmony, in all modifications.

It is thus evident that the human mind in large part creates science, and that the concepts we use and the hypotheses we connect them together with, are our own invention. And although these are conditioned in a serious way by the objective world, yet the hope of an ultimate purely objective science is a chimera, for science is measured by our own capabilities and will always be dependent upon our relations to the external world.

In consequence of the nature of science the different fields in which investigators are working must continually tend towards union with each other and the consequent revision of concepts, hypotheses, and principles. Physics and chemistry have produced physical chemistry, and the greatest suitor for the hand of the latter today is biology. One becomes a little frightened when he contemplates the breadth of learning that the investigator of the future in any one field must possess. for he may have to be acquainted with all fields of scientific learning. Even technique is becoming a complex faculty and years of practice are inevitable for the successful experimenter. Further the need of cooperation is increasing daily, so that we find mathematicians and physicists, chemists and physiologists, biologists and psychologists, all combining The schools of applied science furnish not only problems but facilities to the pure scientist. In the near future it is evident that national and international cooperation and support for the advance of science not only will be seen to be desirable, but will be necessary.

It is obvious therefore that a book like this is one that no student of science should fail to read, and he ought to have it on his shelf. We would be glad to see the day when every candidate for the doctorate would be subject to examination on such a text. We will welcome the day when one course at least is given in every university, and required of all graduate students, in this survey of all the sciences, their history, progress, and present state as to methods, principles, and correlations.

A word as to the contents of the book is all that is necessary, as no briefer account of it can be given than the text itself. The opening chapter is on the development of mathematics and its relation to the other sciences. This is followed by chapters on Mathematics and astronomy, Mechanics and energetics, The physics of the ether, The physics of matter and chemistry, Mineralogy and geology, Physiology and biological chemistry, Botany and zoology, Medicine and bacteriology. Explanatory notes by the translator close the book, and these will be welcomed by the careful student.

Anything from the pen of Professor Picard will bear the mark of that perspicuous thinker, and this difficult piece of work is no exception. Moreover the style is so charming that one finishes reading it with satisfaction that scientific exposition can still be done so perfectly.

JAMES BYRNIE SHAW.

Maxima und Minima in der elementaren Geometrie. Von RUDOLF STURM. Leipzig and Berlin, B. G. Teubner, 1910. v+138 pp.

In this book, the author makes use of nothing but elementary geometry and trigonometry, the latter being used in proving a few of the theorems. The first two theorems in the book are arithmetical, namely:

(1) Of n positive numbers whose product is given, the sum is smallest when the numbers are equal.

(2) If the sum of n positive numbers is given, the product is greatest when the numbers are equal.

These two theorems are the basis of quite a number of proofs on areas and perimeters of polygons. When the problem under consideration can be reduced to a sum or product with the necessary restrictions, the maximum or minimum values follow readily from (1) or (2). With all the possible combinations of restrictions on the sides and angles of a triangle, the author has dealt in detail. Polygons both regular and