for the extremal QP at its intersection with k_2. Let $J_{QP} = S(v)$; then

$$J_{64} = S(v_4), \quad J_{c_0} = J_{12} = S(v_2),$$

$$J_{64} - J_{c_0} = -[S(v_2) - S(v_4)] = -\int_{v_4}^{v_2} \frac{dS}{dv} dv$$
$$= -\int_{v_4}^{v_2} [x_2' H_{x'} + y_2' H_{y'}] dv,$$

and we have for the total variation

$$\Delta J = J_{\bar{c}} - J_{c_0} = \int_{\tau_3}^{\tau_4} E d\tau - \int_{v_4}^{v_2} [x_2' H_{x'} + y_2' H_{y'}] dv.$$

At the point 2, C_0 cuts k_2 transversally, that is,

$$x_2'(v) H_{x'} + y_2'(v) H_{y'}|^2 = 0,$$

and in this problem, as in the simple problem of the calculus of variations, we are led to a study of the sign of E along the extremal C_0 when considering sufficient conditions.

UNIVERSITY OF ILLINOIS,
March, 1913.

A NOTE ON GRAPHICAL INTEGRATION OF A FUNCTION OF A COMPLEX VARIABLE.

BY DR. S. D. KILLAM.

(Read before the American Mathematical Society, April 26, 1913.)

THE object of this paper is to give a shorter and purely graphical method for graphical integration than that of the author in his thesis* on graphical integration of functions of a complex variable.

We can represent a function $f(z)$ of the complex variable $z = re^{i\theta}$ in the $f(z)$-plane by a system of orthogonal curves $r = r_n$ $(n = 0, 1, \cdots, n)$ and $\theta = \theta_n$ $(n = 0, 1, \cdots, n)$. We choose the values r_n and θ_n so that the $f(z)$ plane is covered by a net of small squares. We seek now a graphical representation in the $Z = X + iY$-plane of the function $Z = \int_0^{z_n} f(z)dz$,

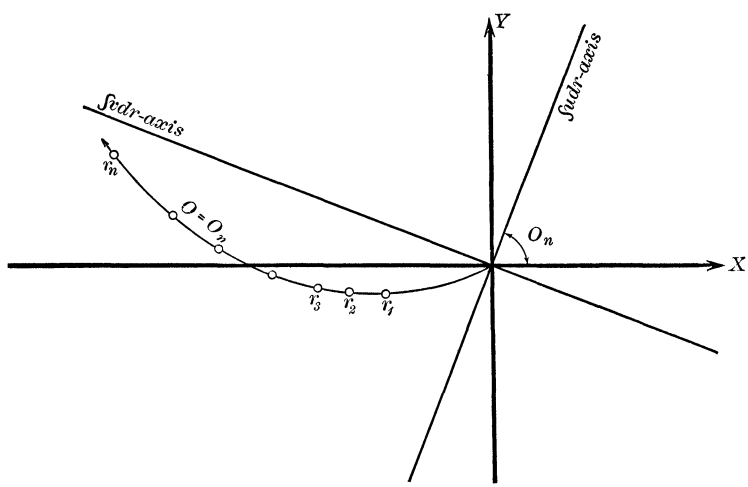
* "Über graphische Integration von Funktionen einer complexen Variabeln mit speziellen Anwendungen," Dissertation, Göttingen, 1912. Referred to in this paper as "thesis."

i. e., we seek the curves $r = r_n$ and $\theta = \theta_n$ in our Z-plane. If we integrate $f(z)$ from $z = 0$ to $z = r_n e^{i\theta_n}$ along the curve or path $\theta = \theta_n$ we have

$$X + iY = Z = \int_0^{z_n} f(z)dz = \int_0^{r_n} \{u(r, \theta_n) + iv(r, \theta_n)\} dre^{i\theta_n}$$
$$= e^{i\theta_n}\left[\int_0^{r_n} u(r, \theta_n)dr + i\int_0^{r_n} v(r, \theta_n)dr\right],$$

where $f(z) = u(r, \theta) + iv(r, \theta)$.

Now $u(r, \theta_n)$ and $v(r, \theta_n)$ are functions of the real variable r, which we can represent graphically in a $u - r$ and a $v - r$ plane;* and then integrate graphically in order to find the values of $\int_0^{r_n} udr$ and $\int_0^{r_n} vdr$ $(n = 0, 1, 2, \cdots, n)$.† In our $Z = X + iY$-plane we draw the $\int udr$ and $\int vdr$ axes so that the angle between the X axis and the $\int udr$ axis is θ_n. (See figure.)



From equation (1) we see that the factor $e^{i\theta_n}$ means that the X axis must rotate through an angle of θ_n in order to coincide with the $\int udr$ axis.

In the $\int udr - \int vdr$ plane we mark the points $R_0, R_1, \cdots, R_n$ with the coordinates $\int_0^{r_n} udr$ and $\int_0^{r_n} vdr$ $(n = 0, 1, 2, \cdots, n)$.

* See Thesis, p. 15.
† See Thesis, figs. 6 and 7.

These values we get from our graphical integration of the functions u and v, and they can be measured and carried over to the $\int udr - \int vdr$ plane; or by having our $\int udr - \int vdr$ plane on transparent paper, we can mark off the coordinates of the points $R_0, R_1, \cdots, R_n$ without the work of measuring these values. This second method also eliminates a small probable error in measurement.

Through the points $R_0, R_1, \cdots, R_n$ we draw a smooth curve, and this is our required curve $\theta = \theta_n$ in the $\int_0^{z_n} f(z)dz$ plane. In the same way we get the curves $\theta = \theta_0, \cdots, \theta_{n-1}$. Through the points r_n on each curve $\theta = \theta_n$ $(n = 0, 1, \cdots, n)$ we draw a smooth curve, and have a net of small squares covering the Z-plane which is the graphical representation of the function $Z = \int_0^{z_n} f(z)dz$.

UNIVERSITY OF ROCHESTER.

THE UNIFICATION OF VECTORIAL NOTATIONS*

BY PROFESSOR EDWIN BIDWELL WILSON.

THE unification of vectorial notations has taken several steps during the past year, but whether the steps be backward or forward, sideways or up in the air, would be difficult to say.

1. One step was forced. A report from the international committee on vector notations, appointed at Rome in 1908 with instructions to lay its recommendations before the congress at Cambridge in 1912, fell due. A member of that committee, though not in attendance at the congress, I am unable to state whether or not any report was made; but I believe that an extension of time until 1916 was asked and granted. So far as I am aware the committee apparently did not organize prior to the meeting in Cambridge last summer, and except for desultory publication on vectors by a few members of the committee, there had been no inside activity which could lead to a report. It does not appear therefore that much of a step in any direction during the past year or

* This essay may be considered as a continuation of one by the same title in this BULLETIN, May, 1910, p. 415.