$\mathfrak{G}_3$. The vertex and any other three points (in general position) of a characteristic surface uniquely determine the surface. A characteristic curve through the vertex and two other points of the surface lies completely on the surface.

F. N. COLE,
Secretary.

THE TWENTY-THIRD REGULAR MEETING OF THE SAN FRANCISCO SECTION.

THE twenty-third regular meeting of the San Francisco Section of the Society was held at the University of California, on Saturday, April 12, 1913. The following members of the Society were present:

Mr. B. A. Bernstein, Professor H. F. Blichfeldt, Dr. Thomas Buck, Professor G. C. Edwards, Mr. W. F. Ewing, Professor M. W. Haskell, Professor L. M. Hoskins, Dr. Frank Irwin, Professor D. N. Lehmer, Professor W. A. Manning, Professor H. C. Moreno, Dr. L. I. Neikirk, Professor E. W. Ponzer, Professor T. M. Putnam.

Professor G. C. Edwards presided at both morning and afternoon sessions. It was decided to hold the next regular meeting of the Section at Stanford University, on October 25, 1913.

The following program was presented:

(1) Mr. B. A. BERNSTEIN: "A set of postulates for the algebra of positive rational numbers with zero."

(2) Professor H. F. BLICHFELDT: "On the arithmetic value of quadratic forms."

(3) Dr. L. I. NEIKIRK: "The analytical geometry of functional space."

(4) Professor T. M. PUTNAM: "Concerning the residues of certain sums of powers of integers to a prime modulus."

(5) Dr. H. W. STAGER: "A geometrical transformation, with some applications to certain systems of spheres."

Abstracts of the papers follow below.

1. In volume 3 of the *Transactions*, Professor Huntington gives a set of postulates for the positive rational numbers. By modifying Professor Huntington's set, Mr. Bernstein obtains a complete set of postulates for the algebra of positive rational numbers with zero.

2. Let f be a positive definite quadratic form in n variables and of determinant D. The following theorem has been proved by Hermite: such a set of integers, not all zero, may be substituted for the variables that the value of f is less than or equal to $\gamma D^{1/n}$, where γ is a number which depends upon n only. Various approximations to γ have been found, that given by Minkowski being the smallest for large values of n. Professor Blichfeldt proves that Minkowski's value for γ may be divided by a number which approaches $\sqrt{2}$ for large values of n.

3. In a paper presented to the International Congress of Mathematicians for 1906 M. Padoa defines all euclidean geometrical concepts in terms of two, the point and the distance, which were left undefined, but subjected to certain postulates. In two papers in the *Nouvelles Annales* M. Fréchet defines the point by an infinite set of numbers $x_1, x_2, x_3, \cdots$ (or for short x) and the distance between the two points $(x, y) = \sqrt{\Sigma(x_j - y_j)^2}$. These satisfy the postulates of M. Padoa and give a generalization of n-dimensional euclidean geometry.

Dr. Neikirk with Kowalewski defines the point by a functional coordinate, $x(j)$, $a \leqq j \leqq b$ (or $0 \leqq j \leqq 1$), and the distance $(x, y)^2 = \int_a^b [x(j) - y(j)]^2 dj$, and by using generalizations of M. Padoa's definitions, gets a generalization of M. Fréchet's geometry.

The n-dimensional plane is defined and shown to be abstractly identical with n-dimensional euclidean space, and the ∞-dimensional plane is also defined and shown to be abstractly identical with Fréchet's space.

The functional space has still higher forms analogous to the plane. The integral equation becomes interpretable as the generalization of the theorem that, in space of n dimensions, n planes of $n - 1$ dimensions intersect in a single point, provided the determinant formed from the coefficients of the coordinates in their equations is not zero.

The paper also considers some other properties of this space. The important results in the paper are the integral relations which are pointed out and the method of their proof suggested by analogy to euclidean geometry.

4. The question of whether or not the congruence $\frac{2^p - 1}{p} \equiv 0 \pmod{p}$ is ever possible, may be made to depend on the

congruence $1^{p-2} + 2^{p-2} + \cdots + \left(\frac{p-1}{2}\right)^{p-2} \equiv 0 \pmod{p}$. Professor Putnam discusses the general congruence of this type, $1^r + 2^r + \cdots + \left(\frac{p-1}{2}\right)^r \equiv a \pmod{p}$, and shows that by expressing a as a fraction it may be given a value for any fixed r (less than p) that is independent of p.

5. In a paper in the *Proceedings of the Edinburgh Mathematical Society,* Professor Allardice considered a geometrical transformation in the plane, $\tan \frac{1}{2}\varphi = k \tan \frac{1}{2}\vartheta$, where ϑ is the angle formed by the enveloping tangents of a curve with a given straight line l, the axis of transformation, and φ is the angle formed by l and t, a system of lines through the intersection of l and t, which envelop the transformed curve of c. In the present paper, Dr. Stager considers analytically a similar transformation in space and applies it to certain systems of spheres. The method of transformation applied to space is as follows: Let α be a given plane and P be any solid. Further, let a plane β be tangent to P and intersect α in i, making with α an angle ϑ. If through i we draw a plane β', making with α an angle φ, such that $\tan \frac{1}{2}\varphi = k \tan \frac{1}{2}\vartheta$, the envelop of β' is defined as the "transform of P." The paper concludes with a number of applications of the method.

W. A. Manning,
Secretary of the Section.

THE TOTAL VARIATION IN THE ISOPERIMETRIC PROBLEM WITH VARIABLE END POINTS.

BY DR. A. R. CRATHORNE.

(Read before the Chicago Section of the American Mathematical Society, March 22, 1913.)

In the simple problem of the calculus of variations,

$$J = \int_{x_1}^{x_2} F(x, y, x', y')dt = \text{minimum},$$

the total variation can be expressed as an integral of which the integrand is the Weierstrassian E-function. It is the object of this note to express in a similar way the total vari-