The original name for the unproved propositions of a mathematical science was "axiom," — a truth so simple that everyone must assent to it whenever the statement is fully comprehended. In this respect the point of view has changed completely. If a, b, $\oplus$, $\odot$ are purely abstract symbols, then no proposition whatever is evident about them. Hence the word "axiom" with its old connotation is being discarded. The paper under review uses "postulate." Other writers, as Veblen and Young, are using "assumption."

This paper should serve two distinct and very useful purposes. The writer of elementary algebras for college use will have at hand a set of postulates which will serve directly as a basis for much of his work and as a model which we hope may guide his way in making the extensions necessary to characterize the complete algebra. This should render less prevalent in the future the numerous logical incongruities that so often have marred otherwise excellent texts.

The subject with which the paper deals is confessedly abstract but the style is so lucid and the mode of treatment so simple that it should be within the reach of students even in the first years in college. It is the feeling of the reviewer that the reader who is to take his first dip into abstract mathematics cannot very well do better than to read this elegant introduction to the logical foundations of algebra.

N. J. LENNES.

Leçons sur la Théorie de la Croissance. By ÉMILE BOREL. Paris, Gauthier-Villars, 1910. vi + 168 pp.

THIS book is one of the excellent series of monographs on the theory of functions appearing under the general editorial direction of M. Borel. It has grown out of two independent courses of lectures on the theory of increase (croissance) delivered by the author at the University of Paris during the winter semesters of 1907–1908 and 1908–1909 respectively. These two courses have been coordinated and unified by M. Arnaud Denjoy.

A function $f(x)$ is said to be increasing if $f(x') - f(x'')$ is positive when $x' - x''$ is positive. The theory of increase is devoted to the investigation of the rate of change of an increasing function $f(x)$ with respect to x as x approaches infinity.

The author begins his preface by giving expression to his growing conviction that the theory of increase is the essential basis of the theory of functions. But it seems to him preferable to wait for some years before undertaking a systematic theory of functions in which the theory of increase would serve as an introduction the essential results of which would be constantly called into use. This mode of exposition will bring about a considerable simplification; but it requires the employment of new terms and new notations, and these should be introduced with a great deal of care.

Besides the introduction (13 pages) the book contains five chapters, of which the first three are devoted to the general theory of increase and the last two to its applications.

In Chapter I (19 pages) certain fundamental types of increasing functions are considered and with each is associated a sign which is used to denote its order of increase; for instance, the orders of x^n and e^x are said to be n and ω respectively. Operations of addition and multiplication of orders of increase are defined and their laws are developed.

In Chapter II (9 pages) the consideration of approximate orders of increase is taken up. The function

$$f(x)=x^n (\log x)^{2+\sin x},$$

for instance, is said to be of order (n) (read "n parenthesis"). The use of (n) in this case to denote the approximate order of $f(x)$ is justified by the fact that the order of magnitude of $f(x)$ is less than that of $x^{n+\epsilon}$ and greater than that of $x^{n-\epsilon}$, where ϵ is any positive quantity. The laws of addition and multiplication of approximate orders are investigated.

The third chapter (32 pages) is devoted to a study of the changes produced in the order of increase of a function by the integration and differentiation of the function.

Chapter IV (44 pages) on analytical applications falls into three parts. In the first part series with constant positive terms are considered. If such a series diverges, the sum of its first n terms, which increases indefinitely with n, possesses, with respect to n, an order of increase which it is useful to know. If the series is convergent one can propose the problem of determining the order of decrease of the remainder after n terms. Both of these questions are treated. Preparatory to a corresponding investigation for infinite products and as contributing to it, the author in the second part of the chapter develops the fundamental properties of the gamma function.

They are deduced in a very simple way from the preceding general theory. Finally, the third part of the chapter is given over to the study of infinite products and especially to a comparison of the increase of functions with that of their zeros.

The last chapter (51 pages) is devoted to arithmetical applications. Its object is to utilize the theory of increase in the classification of incommensurable numbers. An indication of the contents of the chapter is given in the following list of topics: Relations between incommensurable numbers and increasing functions; continued fractions; approximation by means of rational numbers to certain classes of numbers; numbers of the second order and periodic continued fractions; approximation by means of rational numbers to any numbers whatever; approximation by means of algebraic numbers; case of the number e.

In this little book M. Borel has rendered valuable service by emphasizing in an effective way the fact that the theory of increase has a place of growing importance in the development of the theory of functions.

R. D. CARMICHAEL.

Elemente der Mathematik. By JULES TANNERY. Autorisierte deutsche Ausgabe von P. KLAESS. Leipzig, Teubner, 1909. x + 339 pp. 8 marks.

Two objects dominate the teaching of freshman mathematics: the one, which may not too inaccurately be called the classical, endeavors to give the student a certain proficiency in algebraic analysis or in geometrical logic; the other, which may be called the psychological, endeavors to give him a wide view over the realm of mathematics. The classical has for its educative end skill; the psychological, vision. The classical endeavors to build the structure by finishing one room at a time. The psychological endeavors to build first the framework and then to fill in the details. Whether it is better for the great mass of freshmen, who will have little or no mathematics after that year, to learn well how to work Horner's process or to solve trigonometric equations, or whether there should be preferred a knowledge of "what is a function, its variation, and the representation of its values," or a knowledge of how we determine "tangents, areas, volumes, and the new functions, curves, or surfaces thus arising" — is a question now in