Cell	Value of t	ξ	η	ζ	$\theta$	
1234	0	0	0	0	0	
5123	bz	0	0	$\boldsymbol{b}$	0	
5134	bx	b	0	0	0	
5124	$oldsymbol{by}$	0	b	0	0	
6235	c(x+y)+(b-c)z-c	$\boldsymbol{c}$	$\boldsymbol{c}$	b-c	$\boldsymbol{c}$	
6345	(b-c)x+c(y+z-1)	b-c	$\boldsymbol{c}$	$\boldsymbol{c}$	$\boldsymbol{c}$	
6234	(x+y+z-1)(3c-b)	x+y+z-1)(3c-b) 3c-b3c-b3c-b				
6245	(b-c)(x+z-1)+cy	b-c	c	b-c	b c	

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## MATHEMATICAL PHYSICS FOR ENGINEERS.

Einführung in die Theorie des Magnetismus. Von R. Gans. Leipzig, Teubner, 1908. vi + 110 pp.

Einführung in die Maxwellsche Theorie der Elektrizität und des Von Cl. Schaefer. Leipzig, Teubner, Magnetismus. 1908. viii + 174 pp.

Funktionentafeln mit Formeln und Kurven. Von E. Jahnke und F. Emde. Leipzig, Teubner, 1909. xii + 176 pp.

Professor Jahnke is editing a series of texts for engineers. The exact title of the series is Mathematisch-physikalische Schriften für Ingenieure und Studierende. It is published by the firm of B. G. Teubner with the usual typographical excellence of all such works issued by this house. The titles of the volumes which have already appeared, in addition to the three at the head of this review, are Elektromagnetische Ausgleichsvorgänge in Freileitungen und Kabeln, Die Theorie der Besselschen Funktionen,\* Die Vektoranalysis und ihre Anwendungen in der theoretischen Physik (2 volumes),† Theorie der Kräftepläne. There are further announced some twentyfive volumes to which titles and authors have already been assigned, and we are assured that a longer continuation of the series is in contemplation. The texts, or should they be called tracts, are all of moderate size, varying from a bit over 100 to somewhat less than 200 pages. From those which have hitherto appeared it is already evident that each has a definite aim and mission to fulfil, which indeed it does fulfil, and that the series as a whole, if not carried so far as to lose its vitality, will form a

<sup>\*</sup> Reviewed in this BULLETIN, volume 16 (1910), p. 385. † Reviewed in this BULLETIN, volume 17 (1910), p. 100. ‡ Still others have appeared since the date of this review.

valuable collection for students, engineers, and (let us add) mathematicians.

As it is chiefly mathematicians to whom this review is addressed, it will not be amiss to state why this series of tracts seems to us particularly useful to them. In the first place mathematicians, as a general rule in this country at least, are certainly not over-familiar with mathematical physics. doctorate is given at our universities to many a student of mathematics who has hardly a respectable knowledge of mechanics — to say nothing of the mathematical theories of optics, electromagnetism, thermodynamics, and other physical sciences. The specialist thus turned out may be very highly trained, indeed somewhat fine from over-training, but he is essentially uneducated, essentially narrow. He is unable to appreciate where a great deal of his mathematics arose and what were the physical problems which necessarily called it into being. Moreover, he is really not infrequently as yet unfit to teach with a broad unprejudiced view the elementary calculus, elementary mechanics, and advanced calculus which he may be called upon to teach. Under these circumstances it is but small wonder that many leaders in the various departments of our scientific and engineering schools take an indifferent if not antagonistic attitude toward mathematical instruction as it is. In the second place it is very difficult for the young mathematician who feels the inadequacy of his training upon the practical side to make up his deficiency. The massive works on mathematical physics are written for those accustomed to such subjects and are as difficult and perplexing reading to the mathematician as the modern mathematics is to the physicist. What is needed is a series of tracts, written in a simple explanatory style and each dealing with some well connected and restricted field. Toward mathematical physics the mathematician is really in the position of a Studierender. Jahnke's series seems admirably fitted not only to serve the class for whom it is intended but to render great service in broadening such of our young pure mathematicians as would desire to extend their knowledge in the direction which is most likely to add life and value to their teaching and sympathy toward their colleagues in technical departments.

In proceeding to the review of the three particular volumes which are now to be considered we shall adopt the chiastic order

and begin with the last. Jahnke and Emde's table of functions and graphs is a book which is likely to find an active working place in the library of any one who has often to reduce problems to numerical answers. The chief tables are essentially four-Four-figure accuracy is generally sufficient and place tables. should be adopted as the standard of the engineer, mathematician, and physicist. There are some problems which require a closer solution than one which allows an error of one or two hundredths of one per cent; but these are few and should not be put in the foreground. A volume of five-place tables of the functions which Jahnke and Emde tabulate with graphs in 176 pages would be of unwieldy size and serve few purposes for which the present work is not available. The type here selected is large, heavy, and easily read; but a paper not quite so shining would have materially lessened the eye-strain attendant upon long continued use of the table. The cuts and rulings upon which they appear are clear and can easily be read graphically for rough work. Moreover, even when the cuts are not to be used for graphical work, they are very useful in showing the general shape of the function tabulated; in some instances, however, we believe that the graphs could just as well have been omitted.

The first table is of x tan x and  $x^{-1}$  tan x and is very brief; the values of x are at each tenth (in radians) from 0.0 to 1.5. The authors remark below the graphs that: "In order to obtain intermediate values of these functions it is best to plot off the numbers here given on millimeter paper and to read the desired values from the curves thus obtained. A similar remark applies to all later tables in which the interval is taken too large." We do not understand this; it would seem as though if the interval is too large, a replot from the tables would be far worse than reading the curve as it is; what really is needed is a replot with a large number of intermediate values which are not here given in the table. In fact this particular table seems useless except for very rough work; arithmetic interpolation either forward or backward is of course out of the question. Table II contains the roots of some transcendental equations,

$$\tan x = x$$
,  $\tan x = \frac{2x}{2 - x^2}$ ,  $\cos x \cosh x = \pm 1$ ,

and others. Not only are the roots tabulated, but the formulas from which they may be computed are often given. Table III

is for the interchange of a + bi and  $re^{i}$ . The transformation

$$b/a = \tan \psi$$
,  $a + bi = a \sec \psi e^{i\psi}$ 

is made and the values of sec  $\psi$  and of  $\psi$  are tabulated according to the values of  $\tan \psi$ . If b and a are small numbers so that division and multiplication are easy, this is the best way to tabulate; but if a, b are any four-figure numbers, it would probably be better to tabulate  $\log \sec \psi$  and  $\psi$ . Table IV gives

$$\frac{\pi}{2}x$$
,  $e^x$ ,  $e^{-x}$ ,  $\frac{e^x}{\sqrt{\frac{1}{2}\pi x}}$ ,  $\frac{e^{-x}}{\sqrt{\frac{1}{2}\pi x}}$ 

The values of x run by alternate tenths from 0.0 to 6.0. The interval between the entries for x is so great that here again arithmetic interpolation is out of the question. It will therefore be seen that although the functions are given to four figures, the argument is given to only two and this table is not in any true sense a four-place table. A number of the tables are constructed in this manner; the authors are apparently relying on a replot of the tabulated values and a graphical interpolation from the curves thus obtained. For many purposes this is perhaps sufficient.

Table V is on the hyperbolic functions. Here there are eight pages of formulas giving the definition of the functions and the relation between the different functions, the addition theorems, the functions of multiple angles, the powers of sinh x and cosh x, the relation to the exponential, logarithmic, and circular functions, the derivatives of the functions and integrals which contain them either in the integrand or in the primitive, and also approximations to the functions. The only numerical tabulation is of the gudermannian angle  $\gamma$ , which is to seconds while the argument advances by hundredths from 0.00 to 5.00 and by 0.05 from 5.00 to 8.00. It is therefore apparent that to obtain  $\sinh x$  and  $\cosh x$  it is necessary to take  $\tan \gamma$  and sec y out of a trigonometric table. In view of the large amount of room given to the formulas and of the convenience and importance of the functions in numerous practical problems, we believe it would have been well also to tabulate sinh x and  $\cosh x$  or their logarithms directly to four figures as is done in B. O. Peirce's Short Table of Integrals. At this point let us remark that we believe all students of engineering should be familiar with the use of tables of hyperbolic functions; in some schools these functions are a regular part of the curriculum, and in no case would it require more than a lesson or two to introduce them.

Tables VI and VII give the important functions

Si 
$$x = \int_0^x \frac{\sin x}{x} dx$$
, Ci  $x = -\int_x^\infty \frac{\cos x}{x} dx$   $(x > 0)$ ,  
Ei  $x = \int_{\infty}^{-x} \frac{e^{-x}}{x} dx = \text{li } e^x$   $(x > 0 \text{ and } x < 0)$ ,  

$$\int_0^x \cos \frac{1}{2} \pi x^2 dx = \frac{1}{\sqrt{2\pi}} \int_0^z \frac{\cos z}{\sqrt{z}} dz,$$

$$\int_0^x \sin \frac{1}{2} \pi x^2 dx = \frac{1}{\sqrt{2\pi}} \int_0^z \frac{\sin z}{\sqrt{z}} dz.$$

Not only are the functions tabulated; the direct and asymptotic series and several approximate formulas which may be used for the purpose of computing the functions are given. In like manner Table VIII, which gives the  $\Gamma$ -function, its logarithmic derivative and some related functions, contains a large number of the important formulas involving the  $\Gamma$ -function. Table IX is a very complete table of the error function and its first six derivatives. In Table X is found Pearson's function

$$F(r, \nu) = e^{-\frac{1}{2}\pi\nu} \int_0^{\pi} \sin^r x e^{\nu x} dx,$$

and the logarithms of the first 31 Bernoullian numbers. Apart from their practical services, these tables, VI–X, will be found useful to the teacher for various exercises in advanced calculus when dealing with functions defined by integrals. One of the best things a student can learn is how to compute the numerical value of a given function which occurs in his work; he is not so liable to get lost in his subject nor to worry so much over the rigorous end of it.

The elliptic functions and integrals are treated in Table XI which extends to 24 pages. Numerous introductory formulas are given connecting the functions  $F(k,\phi)$ ,  $E(k,\phi)$ , sn u, en u, dn u,  $\vartheta$ ,  $\vartheta_1$ ,  $\vartheta_2$ ,  $\vartheta_3$ , p,  $\zeta$ ,  $\sigma$ . The elliptic integrals  $F(k,\phi)$  and  $E(k,\phi)$  are tabulated at intervals of 5° for  $\alpha=\sin^{-1}k$  and of 1° for  $\phi$ .

The complete integrals are tabulated separately and, as  $\alpha$  nears 90°, the interval is gradually cut down to 6′. For computation with the aid of the  $\vartheta$ -functions and especially for all formulas involving q, the values of  $\log q^{-1}$  are given at intervals of 5′ for  $\alpha$ . Brief tables of the  $\vartheta$ -functions are also constructed. As for the Weierstrassian functions,  $\mathscr{V}$ ,  $\mathscr{V}$ ,  $\zeta$ ,  $\sigma$  are tabulated in the equianharmonic case

$$g_2 = 0$$
,  $g_3 = 1$ ,  $u = \frac{r\omega_2}{180}$ ,  $\omega_2 = 1.53995$ 

for every unit from r=0 to r=240. Tables for the mutual induction and attraction of two coaxial circular currents complete the set. The only comment we would make is to commend the table giving  $\log q^{-1}$ . Despite the trigonometri canalogies of the functions sn, cn, dn and despite the beauties of  $\mathcal{P}$ ,  $\zeta$ ,  $\sigma$ , we believe that the functions which should be emphasized in discussing elliptic functions and integrals are the  $\vartheta$ -functions. In most practical problems the convergence of the series involving q is so rapid that only a very few terms are required. Moreover, for theoretical purposes it is well to familiarize the student early in his work with integral transcendental functions, which play such an important rôle in analysis and of which the  $\vartheta$ -functions are among the simplest and most useful examples.

After Table XII on zonal harmonics, the volume closes with Table XIII on the Bessel functions. It is a long drawn-out close of 85 pages — nearly half the volume. About all the formulas that one could want for the practical use of the Bessel functions, including a long array of differential equations solvable in terms of them, are tabulated. The values of  $J_n(x)$  for  $n=\pm\frac{1}{4}$  and  $\pm\frac{1}{2}$ ,  $\pm\frac{3}{2}$ ,  $\cdots$ ,  $\pm\frac{1}{2}$  are given; for  $n=\pm\frac{1}{4}$  the interval in x is 0.2, for the others it is 1.0 from 0 to 50. Just what is the value of a table of  $J_{p+\frac{1}{2}}(x)$  to four significant figures when the interval in x is so great as to give only six points on each complete oscillation of the curve? And are we supposed to plot off these values, make a graph, and interpolate graphically? We do not know. Perhaps it is only for integral values of x that  $J_{p+\frac{1}{2}}(x)$  is needed. Tables of  $J_0(x)$  and  $-J_1(x)$  at intervals of 0.01 from 0.00 to 15.50 are offered; these are Values of  $Y_0(x)$ ,  $-Y_1(x)$ ,  $K_0(x)$ , real four-place tables.  $\begin{array}{lll} K_{1}(x), & \dot{N_{0}}(x), & -N_{1}(x), & J_{0}(ix), & -i\dot{J_{1}}(ix), & \frac{1}{2}i\pi\dot{H}_{0}^{(1)}(ix), \\ -\frac{1}{2}\pi H_{1}^{(1)}(ix), & J_{0}(x\sqrt{i}), & \sqrt{z}J_{1}(x\sqrt{i}), & N_{0}(x\sqrt{i}), & H_{0}^{(1)}(x\sqrt{i}), \end{array}$  $H_i^{(1)}(x\sqrt{i})$  are set down at less frequent intervals and over a more restricted total range; also ber x, bei x, and tables for the self-induction and resistance in a straight circular wire carrying an alternating current. Numerous sets of roots of  $J_n(x) = 0$  are also to be found. We note further some tables of  $J_n(x)$  for integral values of n from 1 to 24 and for such integral values of x as make  $J_{\nu}(x) > 10^{-18}$ ; It will still other tables. are seen that the subject of Bessel functions is very thoroughly covered, perhaps disproportionately so. We should remember, however, that the Bessel functions are very important in many applications of mathematics; apparently they are more important than the elliptic functions. Indeed if one had to choose between some mention of the Bessel functions and equal mention of elliptic functions in a second course in calculus, one should unhesitatingly choose the former, especially if the time were somewhat restricted. It has always seemed somewhat peculiar and unfortunate that B. O. Peirce in extending his Short Table of Integrals in its excellent revised edition should have added some 85 formulas on elliptic functions while having the Bessel function represented only by the definition of  $J_{\nu}(x)$ when n is integral.

From these extended comments it should appear that we have in Jahnke and Emde's Funktionentafeln an extremely useful collection of formulas and values relating to the most important functions other than elementary, that the chief tables are suited for four-place accuracy, and that there are numerous other tables sufficiently good for graphical or slide-rule work. There can be little doubt that the use of the book will be very general among all workers with mathematics.

To review Schaefer's tract on the introduction to Maxwell's theory without appearing so complete and enthusiastic an admirer of the work as to lessen the value of the review is an impossibility; the work seems to have nothing but excellencies. In fact as soon as it came to our notice we immediately introduced it to our classes and followed it closely from start to finish. We shall merely try here to explain the reason for our enthusiasm. Every one is probably familiar with at least the covers of a number of works on Maxwell's theory; those who have sought familiarity with the interior know full well the difficulties encountered; the works err either toward giving a very gross physical and equally restricted mathematical explanation

of the actions in the ether or toward amassing great arrays of mathematical formulas with too little attention to the physics So far as we know, Maxwell in his original treatise strikes a much happier mean between the extremes than his numerous followers and 'expounders.' Schaefer's text is a sort of minor Maxwell, an exposition eminently suitable for students and engineers — and mathematicians. Like Maxwell, Schaefer appreciates the great advantages derivable from using a vector analysis; but like him, he does not wish to assume the necessary knowledge on the part of his readers. Further, the author decides not to use Green's and Stokes's theorems on divergence and curl, but to develop their equivalents directly on the fields he is considering whenever he finds it necessary to obtain results which the theorems would immediately yield. This course of procedure will undoubtedly be the more satisfactory to the greater number of readers of the work.

Chapter I is on electrostatics. After a few words in the way of describing fundamental facts and giving necessary definitions, the author mentions the fluid hypotheses and Coulomb's law for point charges. He then starts in upon the question of action at a distance versus action in a medium. He shows that the field **E** which arises from any distribution of charged points satisfies the equation div  $\mathbf{E} = 0$  except at the points; this he calls the first law of action in the medium. By introducing Gauss's theorem that the induction of E through a surface is  $4\pi$  times the total charge within, the further result div  $\mathbf{E} = 4\pi\rho$ is obtained. He next shows that the field due to any set of point charges has a potential so that  $\mathbf{E} = -\nabla \phi$  or  $\nabla \times \mathbf{E} = 0$ or the line integral of E about a circuit is zero. This he calls the second law of action in the medium and he derives the relation  $\nabla \cdot \nabla \phi = -4\pi \rho$ . The peculiar beauty of this development is the clear and artful way in which mathematics founded on the conception of point charges is combined with physics founded on the conception of action in a medium. The idea of a medium is given further emphasis by the construction of a hydrodynamic analogy. After a few applications to condensers the author takes up homogeneous and non-homogeneous dielectrics, discusses the difference between free and true electricity, and gives additional applications to condensers. A short section on the energy of the field closes the chapter; but the section is not so short that the author cannot show how in the simplest case the field theory of energy leads to the same value as the

theory of action at a distance. That he cannot handle the general theorem is due merely to the unavailabilty of Green's theorem. The reader of this first chapter ought easily to obtain the idea that action at a distance and action through a medium are two different points of view in accounting for the same phenomena, that they lead to the same mathematical formulation of the problem, that through their mathematics they are interchangeable, and that, as they are thus far equivalent, the reader may feel free to adopt as definitive either hypothesis that future investigations may establish as preferable.

A brief chapter on magnetostatics constructed on the model of the previous one leads to the discussion of the electric current and its magnetic field in Chapter III. The same general method of development is also used here. The field H due to a straight current is shown to fulfil the field equation  $\nabla \cdot \mathbf{H} = 0$ , or better  $\nabla \cdot \mathbf{B} = 0$  when  $\mu$  varies. The law that the circuit integral of H is proportional to the current through the circuit is discussed for the same simple case. It is then stated that these laws hold for any field. What amounts nearly to a demonstration of Stokes's theorem is then given to establish the equations  $4\pi c^{-1}\mathbf{C} = \nabla \times \mathbf{H}$ . The equation  $\nabla \cdot \mathbf{C}$  which follows as a consequence is explained. The vector potential is introduced and the Biot-Savart law for the mutual action of current elements is deduced. Then follow the Maxwell equations  $\epsilon \mathbf{E} + 4\pi \mathbf{C} = c_{\nabla} \times \mathbf{H}$ , the discussion of Ohm's and Joule's laws, and a careful treatment of the electrostatic and electromagnetic systems of units. It may be observed that Schaefer uses the mixed system. Chapter IV on induction carries on the work to find the equations  $\mathbf{B} = -c\nabla \times \mathbf{E}$ , to integrate the equations by potentials, to present Poynting's theorem and explain its significance, to deduce the self and mutual induction of circuits, and to treat in considerable detail the phenomena connected with electric oscillations in simple and coupled circuits. Chapter V on electric waves deals with plane waves in isotropic dielectrics, reflection and refraction, (for which the laws are obtained by using the previously established boundary conditions for continuity or discontinuity of the components of D, E, H, B,) and finally the question of electromagnetic waves in metals. There is a great mass of vital electromagnetic physics in this chapter and it is all presented with the author's usual clearness and care in exposition. reader can hardly get the wrong or fail to get the right point of view.

If this brief summary of the contents of Schaefer's book can serve to justify so unreserved enthusiasm for the work, we shall be happy; we shall be happier if it can induce others to read the book with interest and study it with care. There can now be no sufficient excuse for lack of familiarity with the elements of Maxwell's great theory; this presentation is too simple, too perfect to offer any difficulties; it neither offers too much nor affords too little for a proper introduction to the subject. Jahnke is to be felicitated on securing such a tract for his series; we only wish it had appeared in English.

Gans's tract on magnetism is also written from the point of view of Maxwell's theory, which as is stated in the preface is the point of view more and more adopted in technical work. There is consequently some duplication between this and the previous volume reviewed. Another point of the preface which is noteworthy is that Gans like Schaefer would be very glad to use vector analysis if he quite dared. The cry for some sort of vector analysis is growing as the waste of space and energy and the danger of an incorrect and insufficiently physical view. which arise when cartesian analysis is used, become more promi-It is probable that mathematicians at large would do well to familiarize themselves with the elements of vector analysis for the purpose of seeing how the little that is needed so badly can best be worked into the courses in mathematics. The plan of giving vectors as a separate and somewhat elaborate course in a few institutions or even in many institutions is not a success: it does not meet the need. Not long ago one of our colleagues, not himself a mathematician, expressed the conviction that all of our engineering students who carry their mathematics into the third year should probably have some vector analysis and that the electrical engineers should certainly have In this we entirely concur. And we surely believe that the vectors should come in little by little throughout the student's course in mathematics; the time could readily be saved from analytic geometry, especially from solid analytic geometry. The mere notion and notation of the scalar product of two vectors will of itself work wonders in clarifying a large number of formulas of plane and solid geometry; without the scalar product the ideas of line and surface integrals and of Stokes's and Green's theorems are far from clear. The matter needs serious consideration and needs it now. We, as mathematicians, must not continue indefinitely to hamper the instruction (alas, too rare) in theoretical physics by our negligence toward a very fruitful and suggestive field of our own science.

Gans starts Chapter I off with the statement that, in the neighborhood of an electric current, space has a certain condition called magnetic! He then proceeds systematically to develop this idea into the Maxwell equations curl  $\mathbf{H} = 4\pi \mathbf{C}$ . The second chapter treats induction in para- and diamagnetic bodies and results in the Maxwell equations curl  $\mathbf{E} = -\mathbf{B}$ . It may be noted that Gans is using a different set of units from Schaefer. The magnetic potential is introduced and applied to several simple systems of currents. The boundary conditions in B and H are established and applied to numerous problems of a practical technical nature. The sphere, shell, prolate ellipsoid in a constant field are also treated. As this requires the introduction of curvilinear coordinates and Laplace's equation, it is clear that Gans is not quite so chary of his mathematics as Schaefer. But is it not a curious state of affairs and quite derogatory to the acumen and common sense and honesty of teachers of mathematics to think that curvilinear coordinates can find their place where vectors are afraid to enter? go on teaching the same old things in the same old way, except that we may pay relatively more attention to rigor and relatively less to what is useful than in the old days. instance, take a look at the last edition of Serret-Harnack-Scheffers's Differential- und Integralrechnung and see how it compares with Serret's original. See whether the new material and the amplifications have not to do with limits, convergence. and uniformity rather than with line and surface integrals, Gauss's, Green's, Stokes's theorems, and potential integrals.

Chapter III takes up ferromagnetic bodies. The start is made from the experimental curves of B to H and  $\mu$  to H. The hysteresis diagram is discussed. Finally the equations

$$\nabla \cdot \mathbf{H} = 4\pi \rho, \quad \mathbf{H} = -\nabla \phi, \quad \nabla \cdot \nabla \phi = -4\pi \rho_0, \quad \phi = \int \int \int \frac{\rho_0}{r} dv$$

lead to the conception of the inverse-square law as applied to magnetism. If one looks real sharply he will find as a subheading in § 31 the mention of a magnetic pole! It is quite obvious that Gans is following out the Maxwell theory as indicated in the first sentences of the first chapter and that he is

not giving way in the slightest to the old point of view of magnetic poles and the law of the inverse square. This consistent attitude of the author is highly to be praised. In Chapter IV magnetic energy and force are treated with the aid of the Poynting vector. It is along toward the middle of this the last chapter in the tract that Gauss's methods of determining the magnetic moment of a magnet and the horizontal component of the earth's field are given. An especially commendable point is the analysis which leads up to the refutation of the common but false definition of para- and diamagnetic bodies by the position they assume relative to the lines of a magnetic field. Some mention of self- and mutual induction, Lenz's rule, and the ballistic galvanometer forms a close to the work.

It is indeed good to read a book like this of Gans where magnetism is treated systematically from the point of view of electromagnetism. The ordinary treatises on electromagnetism slip over the special field of magnetostatics with very little attention, the ordinary treatises on magnetism scarcely mention electromagnetism as other than a means and often they found electromagnetic effects on the study of magnetic shells. Now it may seem somewhat inconsistent in us to praise so heartily Schaefer's neat interrelating of action at a distance with action through a medium and still to support with almost equal ardor Gans's complete capitulation to the theory of a medium. But there is a vital difference between electrostatics and magnetostatics in that negative and positive electricity can be isolated, that is, div  $\mathbf{D} \neq 0$ , whereas a true magnetic pole is a fiction, that is, We recommend Gans to our readers as heartily  $\operatorname{div} \mathbf{B} \equiv 0.$ as we recommended Schaefer; but don't begin with Gans. certainly true that Jahnke has made an auspicious start with his series of tracts; let us hope that he can keep up on the present high level; short, snappy, scientific, modern tracts are just what we need.

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