Hessian curves, least squares, line coordinates, potential, and the like, all of which when used are accompanied by explanations of their nature sufficient for the purpose of the problems.

While the author in the preface would give the impression that the fields of geometry, physics, and engineering are equally represented in the collection, yet even a casual glance reveals that the geometric side of the calculus is emphasized more than the other two, and an abundance of interesting geometric properties of curves and surfaces is brought out. Problems involving numerical calculations and the use of tables are brought in frequently, a procedure calling for definite results, and well worth while.

Instructors of the calculus will find in the collection much material to supplement any course given where new problems are welcome; while authors of text-books on the subject might easily receive many valuable suggestions from a perusal of its pages.

The volume is in every way typographically, both in print and figures, up to the high standard of the Sammlung Teubner; in its specific field it fills a practical need, and because of the excellence of the first part the volume on the integral calculus is eagerly awaited.

ERNEST W. PONZER.

Differential and Integral Calculus. By PROFESSOR DANIEL A. MURRAY. New York, Longmans, Green and Company, 1908. xviii + 491 pp.

THE aim of the author of this text-book has been (to quote from the preface) "to describe and emphasize the fundamental principles of the subject in such a way that, as much as may reasonably be expected, they may be clearly understood, firmly grasped, and intelligently applied by young students"; and again, "the aim has been to write a book that will be found helpful by those who begin the study of calculus without the guidance and aid of a teacher." This is by no means a simple undertaking in view of the inherent and essential difficulties of the subject. The notion of a limit is fundamental; and while students readily acquire more or less vague ideas on this subject, it seems to be difficult for most of them to get a clear and accurate conception of it. This difficulty must be squarely faced; fundamental definitions and principles must be set forth in language that is accurate and therefore necessarily technical.

Any attempt to state, restate, or explain these principles in language that is less technical, and therefore inaccurate or even meaningless, is not conducive to real clearness. The beginner in calculus is not helped by having the inherent difficulties of the fundamental concepts explained away. And into this sort of error, the author of the book under consideration would seem, in the judgment of the reviewer, to have fallen to some extent.

A few specific cases may be cited in support of this opinion. In the very beginning, the limit idea cannot be avoided in defining the derivative, for the reason that the fraction $\Delta y/\Delta x$ is *meaningless* when $\Delta x = 0$. When one writes, for example,

$$y = x^2, \quad \Delta y = 2x\Delta x + \Delta x^2, \quad \frac{\Delta y}{\Delta x} = 2x + \Delta x,$$

the third equation is true *except when* $\Delta x = 0$. The author seems to make no distinction between the two members of this equation. Thus, on page 6, we read, "$\Delta y/\Delta x$ will reach the value 4 when Δx decreases to zero," but $\Delta y/\Delta x$ has not been defined to mean anything other than Δy divided by Δx. On page 9 one finds, "one can confidently and accurately state what these ratios [$\Delta s_1/\Delta t_1$ and $\Delta y_1/\Delta x_1$] *will become* [the italics are from the book] when Δt_1 but Δx_1 actually reach zero." This latter case is hardly excused by a note immediately following which says, "Moreover, it should be carefully noted that *at the final stages* in the solutions of the problems in Arts. 3 and 4, $\Delta s_1/\Delta t_1$ is *not* regarded as a fraction composed of *two* quantities, Δs_1 and Δt_1, but as a *single* quantity, namely the *speed* after t_1 seconds." The symbol was certainly introduced into the problem as a fraction. Just when does it cease to be a fraction?

On page 29 we read, "It is evident that the reciprocal of an infinitesimal approaches a number which is greater than any number that can be named, namely, an infinite number. Accordingly, an infinite number may be defined as the reciprocal of an infinitesmal." Does this mean anything? It may be allowable to use the notation $x = \infty$ *after one has explained exactly what such notation is to mean;* but it is doubtful whether there is ever justification for writing (as the author does, page 185) $F(a) = \infty$, or for speaking of "the equation, $\phi'(x) = \infty$" (page 117). An excellent definition of an asymptote as the limiting position of a tangent is given on page 200; but later, on page 202, we read that a certain line "is tangent to the curve at the infinitely distant point."

Other examples of rather loose reasoning are noticeable. On page 47, having proven that

$$\frac{d}{dx}[f(x)+c]=\frac{d}{dx}[f(x)],$$

the author says, "It follows from (1) [the equation just cited] that the derivative of a constant is zero"; but the distributive law has not yet been proven. In a note at the bottom of page 117 we read, "It follows . . . that if $\phi(a)$ is a maximum (or minimum) value of $\phi(x)$, then . . . $\sqrt{\phi(a)}$ is a maximum (or minimum) value of $\sqrt{\phi(x)}$," which is obviously not true for $\phi(x)=x^2$. The argument on page 275 holds only for a continuously increasing function, which restriction is not stated in the text. The statements (page 200) that "an ellipse cannot have an asymptote" and "the parabola $y^2=4px$ has no asymptote" are flat contradictions of the statement (page 203) that "a curve whose equation is of the nth degree has n asymptotes, real or imaginary." But the reader is still more likely to be confused when he finds (page 163) an unqualified "proof" of a property of the evolute followed by a note saying, "Property (b) should not be applied thoughtlessly; for in certain circumstances, for either the curve or its evolute, the property does not hold." An instance is then cited where the property "does not hold"; but no explanation is given of this paradox of having proven something which is not true. The note is concluded by the phrase, "which is obviously absurd"; and this is certainly true in a sense probably not intended.

One's estimate of the text-book will depend entirely on whether he regards such errors as are cited above as serious and fundamental or merely trivial—if, indeed, he regards them as errors at all. The book certainly has many commendable features.

There are numerous references throughout, both to other texts of the same grade and to books where the subjects are treated more fully and rigorously. These will undoubtedly prove helpful to student and teacher alike.

Taylor's theorem is derived by extending the mean value theorem; thus giving emphasis to the remainder from the beginning, which is most desirable. Newton's application of the mean value theorem to the finding of approximate roots of equations is given, and this feature should prove to be of practical value, especially when applied to transcendental equations. The examples given are unfortunately largely rational algebraic equations, for which Horner's method is probably simpler.

The chapters relating to space geometry and kinematics are interesting and attractive. A nice distinction is made between speed and velocity, leading to the notion of normal as well as tangential components of acceleration.

The part devoted to integral calculus includes a chapter on center of mass and moment of inertia. This is followed by a chapter containing a brief treatment of ordinary differential equations of the first and second orders.

An appendix contains notes on hyperbolic functions, the intrinsic equations of a curve (in the plane), and the length of a curve in space; also a number of exercises and problems, a brief table of integrals, a useful collection of figures of curves, and a very complete index.

The mechanical features of the book are attractive, and it is conspicuously free from typographical errors.

WALTER B. CARVER.

An Elementary Treatment of the Theory of Spinning Tops and Gyroscopic Motion. By HAROLD CRABTREE. London, Longmans, Green and Company, 1909. 8vo, xii + 140 pp.

"THE object of this book is to bring within the range of the abler Mathematicians at our Public Schools and of First Year undergraduates at the Universities [in England], a subject which has hitherto been considered too difficult for any but the more advanced students in Mathematics, while even they have in many cases failed to derive more pleasure from the study of spinning tops than is contained in submitting the problem to the action of a complicated piece of Mathematical machinery which automatically, though unintelligently, turns out the correct result."

This extract from the preface shows well the character of the greater part of the book. At the present time, the interest of the subject, in view of recent developments, renders such an exposition of value to an engineer. But it may be read with profit by those who wish to obtain some idea of the grade reached by