SHORTER NOTICES.

Die Elemente der Mathematik, Band II: *Geometrie.* Von E. BOREL. Von Verfasser genehmigte deutsche Ausgabe, besorgt von P. STÄCKEL. Leipzig, Teubner, 1909. 8vo. xii + 324 pp.

THE work here considered is the second volume of the German edition of Borel's Elements of Mathematics. It is intended as a text-book of geometry for use in the secondary schools of Germany. While the language in which it is written precludes its use as a text-book in this country, it contains so many modifications, both in subject matter and in method of presentation, of the customary course in elementary geometry, that it will be read with interest and the results obtained from the use of it will be watched carefully by all who are interested in the reform of the secondary school course in geometry in the United States.

The spirit which animates the book is stated in the preface in the following way: "The conviction that the instruction in elementary geometry ought to be revised is daily gaining ground, although certain persons strongly oppose it. These people fear that the logical structure which the Elements of Euclid has for its foundation will be torn down. Would it not be better, they say, to improve and extend this structure, as in the past, rather than to tear it down and attempt the dubious experiment of building a new one in its place? I can not participate in this view of the matter and believe, on the contrary, that, in a few decades at the latest, instruction in geometry will be based on a new principle. This new principle was arrived at only in the course of the nineteenth century by the efforts of eminent mathematicians. It consists in the realization that *elementary geometry is equivalent to the investigation of the group of motions.*"

As an illustration of the differences of this text from the customary books based on Euclid's Elements may be cited the discussion of incommensurable quantities and the proofs by limits to which they give rise. The existence of incommensurable lengths, for example, appears as a difficulty which would naturally be encountered in measurement. The difficulties in the demonstrations to which these quantities give rise are surmounted by a device which again is suggested by measurement and which exhibits all the rigor that the student is likely to be able to appreciate.

It will hardly be questioned that this text will appeal more strongly to the students' interest than Euclid, nor that the material is better selected with reference to the students' capacity to receive it, nor that the student can, by the expenditure of a given amount of energy, obtain a greater amount of mathematical information from this text than from Euclid's Elements. It still remains in doubt, however, whether the student will obtain the same thorough training in rigorous, careful reasoning in this course as under the present discipline.

C. H. SISAM.

The Foundations of Mathematics. A Contribution to the Philosophy of Geometry. By Dr. PAUL CARUS. Chicago, The Open Court Publishing Co., 1908. 141 pp.

THIS book is, mathematically speaking, a more or less popular treatise, which would appear to have for its primary object an effort to show that geometry can be obtained a priori, by abstraction, from the notion of motility, and can be constructed from this alone by making use of the principles of reasoning, *all axioms being unnecessary.*

The book opens with a historical sketch, which is fairly accurate, mentioning particularly the work of Euclid, Gauss, Riemann, Lobachevsky, Bolyai, Cayley, Klein, and Grassmann. The author then introduces chapters on "The philosophical basis of mathematics" and "mathematics and metageometry" in which his philosophical theories are presented. Briefly expressed, his doctrine seems to be about as follows: "Space is the possibility of motion, and by ideally moving about in all possible directions, the number of which is inexhaustible, we construct our notion of pure space. If we speak of space we mean this construction of our mobility. It is an a priori construction and is as unique as logic or arithmetic. There is but one space, and all spaces are but portions of this construction." Mathematical space is a priori, in the Kantian sense, not however ready made in the mind, but the product of much toil and careful thought. Mathematical space is an ideal construction, hence all mathematical problems must be settled by a priori operations of pure thought, and can not be decided by external