group for every prime divisor of its order every subgroup of G has the same property. In particular, if the symmetric group of degree n involves Sylow subgroups for every prime which divides its order, then every substitution group of degree n (and hence every group of finite order) must involve at least one Sylow subgroup for every prime which divides its order. It is very easy to prove, as Cauchy observed, that every symmetric group of degree n has the given property, and hence the theorem which was proved above as a slight extension of one due to Cauchy implies that every group of finite order involves at least one Sylow subgroup for every prime divisor of its order. As this is the main element in Sylow's theorem it is clear that Cauchy used a method which required only slight changes to yield an easy proof of the fundamental theorem known as Sylow's theorem. It would evidently be necessary only to prove that every symmetric group whose degree is a power of p involves Sylow subgroups of order p^m in order to establish the existence of Sylow subgroups in every group of finite order by means of the theorem proved above.

The preceding remarks may also serve to exhibit additional reasons for regarding Sylow's theorem as merely an extension of Cauchy's fundamental theorem, which established the fact that every group whose order is divisible by the prime p involves operators of order p. In fact, if Cauchy had used a general value of ρ instead of $\rho = 1$ in the theorem proved above, he would have arrived at Sylow's theorem by the same steps as those which led him to his fundamental theorem. The oversight of this slight increase in generality retarded Sylow's theorem nearly thirty years and made Jordan's Traité des Substitutions much more difficult reading.

EXISTENCE THEOREMS FOR CERTAIN UNSYMMETRIC KERNELS.

BY MRS. ANNA J. PELL.

In this paper is given a brief account of the existence and expansion theorems for certain integral equations with unsymmetric kernels. Full details of the method involved and a discussion of a less general integral equation are contained in an article, "Biorthogonal systems of functions with applications to the theory of integral equations," which is at present in the hands of editors. Marty * has recently treated by another method an integral equation corresponding to a special case of the functional transformation $T\,[(1), (2), (3)]$, viz.,

$$Tf(s) = \int_a^b K(s, t) f(t) dt,$$

where $K(s, t)$ is a definite symmetric kernel.

We denote by $Tf(s)$ a linear functional transformation, which transforms every continuous function into a continuous function and which has the three following properties:

$$\int_a^b [Tf(s)]^2 ds \leqq M \int_a^b [f(s)]^2 ds, \tag{1}$$

where M is a given positive quantity;

$$\int_a^b f(s) Tf(s) ds \geqq 0, \tag{2}$$

the equality sign holding only for $f(s) = 0$, or for $f(s) = p(s)$, where $p(s)$ is a continuous function such that $Tp(s) = 0$;

$$\int_a^b f_1(s) Tf_2(s) ds = \int_a^b f_2(s) Tf_1(s) ds. \tag{3}$$

The transformed function of a continuous kernel $K(s, t)$ with respect to the variable s we designate by $T_s K(s, t)$, and assume that it is continuous in s and t.

Let $L(s, t)$ be an unsymmetric kernel satisfying the condition that

$$M(s, t) = T_s L(s, t) \tag{4}$$

is a *symmetric* kernel. By means of a biorthogonal system $(u_i(s), v_i(s))$ complete as to u and such that

$$v_i(s) = Tu_i(s)$$

the integral equations

$$u(s) + \mu p(s) = \lambda \int_a^b L(s, t) u(t) dt,$$

* *Comptes Rendus*, Feb. 28 and April 25, 1910.

$$(5) \qquad \int_a^b u(s)p(s)ds = c, \qquad v(s) = \lambda \int_a^b L(t, s)v(t)dt$$

are reduced to a system of linear equations in infinitely many variables

$$x_i = \lambda \sum_{k=1}^{\infty} x_k \int_a^b \int_a^b M(s, t)u_i(s)u_k(t)dsdt,$$

where the system of coefficients is symmetric and continuous. We obtain the following theorems:

THEOREM 1. If an unsymmetric kernel $L(s, t)$ satisfies the condition (4) and if $M(s, t) \not\equiv 0$, there exists at least one characteristic number λ, which is real and of finite multiplicity, and the characteristic functions $u_i(s)$, $v_i(s)$ form a biorthogonal system belonging to the type T.

THEOREM 2. Any continuous function $f(s)$ expressible in the form

$$f(s) = \int_a^b L(t, s) Tf_1(t)dt,$$

where $f_1(s)$ is an arbitrary continuous function, can be developed into the uniformly convergent series

$$f(s) = \sum_i v_i(s) \int_a^b u_i(s)f(s)ds.$$

THEOREM 3. If an unsymmetric kernel $L(s, t)$ has an infinite number of real characteristic numbers, and if the system of corresponding characteristic functions $(u_i(s), v_i(s))$ is complete as to u, then the kernel

$$M(s, t) = T_s L(s, t)$$

is symmetric, and T is the functional transformation belonging to the biorthogonal system (u_i, v_i) and therefore has the properties (1), (2), and (3).

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