point not contained in the simplex $S(i+1)$ that determines it. But that one point is a point of the simplex $S(m+1)$ by the very definition of simplex. Therefore, if one begins to count with the first set and counts through the sets in order, the number of points in the set numbered $i+1$ that have not been counted in any previous set is ${}_{m+1}C_{i+1}$. It follows that the number of points in the simplex $S(m+1)$ is

$$\sum_{j=1}^{m} {}_{m+1}C_j = 2^{m+1} - 2,$$

which is one less than the number of points in the m-space determined by the $m+1$ vertices of the simplex.

From this theorem it follows that the $l+1$ vertices of a simplex of order l determine uniquely another point, namely, the one point of the l-space determined by the simplex that is not also a point of the simplex. It is convenient to call this point the point complementary to the simplex. The triads, tetrads, pentads, etc. of the Steiner problem are found as follows: Every simplex $S(2)$ determines a triad consisting of its two vertices and the complementary point; every simplex $S(3)$ determines a tetrad consisting of its three vertices and the complementary point; and, in general, every simplex $S(l-1)$, $l \leqq k+2$, determines an *l-ad* consisting of the $l-1$ vertices and the complementary point. There are no *l-ads* for $l > k+2$.

When $n = 2^6 - 1 = 63$, it is possible to arrange the n elements in triads, tetrads, pentads, hexads, and heptads. There is no arrangement of the 63 elements in l-ads for $l > 7$. This special case was involved in Steiner's investigation of the configuration of the 28 double tangents of a quartic curve * and led him to propose for solution the "Combinatorische Aufgabe" which I have called "The tactical problem of Steiner."

ON THE SO-CALLED GYROSTATIC EFFECT.

BY PROFESSOR ALEXANDER S. CHESSIN.

(Read before the American Mathematical Society, April 24, 1909.)

IN computing the resisting couple of gyrostats or the so-called "gyrostatic effect" it is customary to assume that it is equal to $C\lambda\omega \sin\theta$, where C, λ, ω and θ denote respectively the moment of inertia of the gyrostat about its geometrical axis, the angular

* *Journal für die reine und angewandte Mathematik*, vol. 49, pp. 265–272.

velocity of spin, the angular velocity of precession and the angle of the geometrical axis with the axis of precession. It is proposed here to give the exact value of this couple.

The gyrostatic effect is due to the action of what I have called the *convective* and the *turning* forces in the motion of a body relatively to a moving system (XYZ).* Let ω be the angular velocity of this system; p, q, r its components along X, Y, Z. To simplify the results we will assume that the center of gravity of the body is at the origin of these axes. Then the principal moments M and M' of the convective and the turning forces are given by their components

$$M_x = \sum_i (y_i F_{iz} - z_i F_{iy}), \quad M'_x = \sum_i (y_i F'_{iz} - z_i F'_{iy}),$$

$$M_y = \sum_i (z_i F_{ix} - x_i F_{iz}), \quad M'_y = \sum_i (z_i F'_{ix} - x_i F'_{iz}),$$

$$M_z = \sum_i (x_i F_{iy} - y_i F_{ix}), \quad M'_z = \sum_i (x_i F'_{iy} - y_i F'_{ix}),$$

where

$$\frac{1}{m_i} F_{ix} = z_i \frac{dq}{dt} - y_i \frac{dr}{dt} + p(px_i + qy_i + rz_i) - \omega^2 x_i,$$

$$\frac{1}{m_i} F_{iy} = x_i \frac{dr}{dt} - z_i \frac{dp}{dt} + q(px_i + qy_i + rz_i) - \omega^2 y_i,$$

$$\frac{1}{m_i} F_{iz} = y_i \frac{dp}{dt} - x_i \frac{dq}{dt} + r(px_i + qy_i + rz_i) - \omega^2 z_i,$$

$$\frac{1}{m_i} F'_{ix} = -2(q\dot{z}_i - r\dot{y}_i), \quad \frac{1}{m_i} F'_{iy} = -2(r\dot{x}_i - p\dot{z}_i),$$

$$\frac{1}{m_i} F'_{iz} = -2(p\dot{y}_i - q\dot{x}_i).$$

Let now $(\Xi H Z)$ be a system of axes coinciding with the principal axes of inertia of the body.

By a series of transformations which it does not seem worth

* "On relative motion," *Transactions American Mathematical Society*, vol. 1.

while to reproduce here we obtain the following expressions for M and M':

$$M_\xi = A\dot{\omega}_\xi - (B - C)\omega_\eta\omega_\zeta, \quad M_\eta = B\dot{\omega}_\eta - (C - A)\omega_\zeta\omega_\xi,$$

$$M_\zeta = C\dot{\omega}_\zeta - (A - B)\omega_\xi\omega_\eta,$$

$$M'_\xi = (A + B - C)Q\omega_\zeta - (A - B + C)R\omega_\eta,$$

$$M'_\eta = (B + C - A)R\omega_\xi - (B - C + A)P\omega_\zeta,$$

$$M'_\zeta = (C + A - B)P\omega_\eta - (C - A + B)Q\omega_\xi,$$

where A, B, C and P, Q, R are the principal moments of inertia of the body and the components of its angular velocity Ω in the relative motion.

When two of the moments of inertia, as in the case of gyrostats, are equal, *i. e.*, $A = B$, the expressions given above will be simplified by the following selection of axes: axis P, trace of plane ΞH on plane XY; axis Q at right angle to P in the plane ΞH (to the right of P relatively to R); and R coincident with Z. Introducing Euler's angles θ, ϕ, ψ we shall now have

$$M_1 = A\dot{\omega}_1 - (A - C)\omega_2\omega_3, \quad M'_1 = (2A - C)\omega_3\phi' \sin\theta - CR\omega_2,$$

$$M_2 = A\dot{\omega}_2 - (C - A)\omega_3\omega_1, \quad M'_2 = -(2A - C)\omega_3\theta' + CR\omega_1,$$

$$M_3 = C\dot{\omega}_3, \qquad M'_3 = C(\omega_2\theta' - \omega_1\phi' \sin\theta),$$

using the symbols 1, 2, 3 in lieu of P, Q, R to indicate the corresponding components.

Were we to select the axes (XYZ) so that the Z axis coincide with the axis (ω), we would have $\omega_1 = 0$, $\omega_2 = \omega \sin\theta$, $\omega_3 = \omega \cos\theta$, and therefore

$$M'_1 = (2A - C)\omega\phi' \sin\theta\cos\theta - C\omega R \sin\theta,$$

$$M'_2 = -(2A - C)\omega\theta' \cos\theta, \quad M'_3 = C\omega\theta' \sin\theta.$$

These results show that even assuming that the system (XYZ) revolves with a constant angular velocity and that this velocity (ω) is very small (so that we may neglect terms of the order of ω^2), the commonly accepted value of the gyrostatic couple is incorrect. The last formulas reduce to $|M'| = C\omega\lambda \sin\theta$ if we assume that $\theta' = \phi' = 0$, *i. e.*, that the gyrostat axis is invariably fixed in the system (XYZ).