Auslese aus meiner Unterrichts- und Vorlesungspraxis. By HERMANN SCHUBERT. 3 volumes. G. J. Göschen, Leipzig, 1905, 1906.

IN these little books the veteran teacher and editor treats a great variety of topics more or less closely connected with the mathematical subjects studied in the German gymnasium. Originality and elegance of presentation, even in the case of the most hackneyed topics, make the lectures of interest and value. The keynote is struck in the following sentence from the author's preface:

"Denn die mathematische Didaktik sollte sich nicht auf oft erfolglose Verbesserungsvorschläge bezüglich der Verteilung und der Ausdehnung des Lernstoffs beschränken, sondern sollte umgekehrt die Pflicht fühlen und erfüllen, den zu bewältigenden Lernstoff so einfach und zugänglich zu gestalten, dass auch der minder begabte Schüler in der nun einmal von oben herab vorgeschriebenen Zeit mehr lernt und begreift, als es bisher der Fall war."

Two of the chapters deal with the calculation of logarithms, the first intended for students of the Untersekunda, the second for the Prima. The first treatment presupposes merely a knowledge of the expansions of $(a-b)(a+b)$, $(a-b)^2$, $(a-b)^3$, $(a-b)^4$, and the theorems concerning the logarithms of products and quotients, and is based on the inequality

$$2 \log x - \log (x-1) - \log (x+1) > 0.$$

The second treatment presupposes the general binomial theorem and is based on the so-called "Tripelformel" originally presented in the author's pamphlet "Elementare Berechnung der Logarithmen" (Göschen, 1903). With these elementary means, without any reference to infinite processes, the logarithms of the prime numbers less than a hundred are worked out to eight decimal places.

Many of the lectures are devoted to topics from the theory of numbers. A section of almost a hundred pages shows how the leading theorems on congruences, including quadratic remainders and the Pell equation, may be easily and rapidly obtained by starting out with the discussion of continued fractions.

The applications of number theory in elementary geometry are treated very fully. Here many of the results, not merely the presentation, are due to the author. The discussion of heronian triangles is a model of elegance. Such a triangle is one in which the area and the sides are expressible in integers. The author's method is based on the fact that the tangent of half each angle in such a triangle must be rational. Angles with this property are termed heronian. The theorem on page 13 of volume 2 should evidently be corrected so as to read: Every linear homogeneous function, with integral coefficients, of any number of heronian angles is itself an heronian angle.

The art of manufacturing problems in such a way that the solution as well as the data shall be integral is not cultivated seriously in this country.* The author takes up questions of this sort in many complicated cases: quadrilaterals with rational sides, diagonals, and area; tetrahedrons with rational edges, faces, and volume; and finally, in the last chapter of the third volume, spherical triangles with both sides and angles heronian (it is here shown that if four of the parts are heronian, the remaining two must also be heronian).

The proof of Euler's theorem on polyhedrons given in chapter 8 of the first volume is quite novel: the result is obtained by counting, in two different ways, the number of conditions which determine a polyhedron, and equating the results.

A number of the topics are taken from physics. We mention only the somewhat polemical discussion of the absolute system of units; and an elementary treatment of the pendulum, in which the period of oscillation is determined with great accuracy. The formula

$$\pi\sqrt{\frac{l}{g}}\left(1+\frac{1}{4}\sin^2\frac{\alpha}{2}\right)<t<\pi\sqrt{\frac{l}{g}}\sec\frac{\alpha}{2},$$

where α denotes the angle of oscillation, was originally published in the *Naturwissenschaftliche Wochenschrift* for 1896, but has not attracted the attention it merits.

E. KASNER.

*Students of elementary geometry usually get the impression that if two solids agree in both surface and volume they are congruent or at least symmetric. It is of course easy to give examples showing that this is not true, even in the simple case of rectangular parallelepipeds. An example in which the dimensions are all integral would be of interest. The problem is to find integral solutions (the smaller the better) of the pair of equations

$$xyz=x'y'z', \quad \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{x'}+\frac{1}{y'}+\frac{1}{z'}.$$