THE SIXTIETH MEETING OF THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE.

THE sixtieth meeting of the American Association for the Advancement of Science was held at Johns Hopkins University during the convocation week, December 28, 1908, to January 2, 1909. The president of the meeting was Professor T. C. Chamberlin, University of Chicago. The address of the retiring president, Professor E. L. Nichols, entitled "Science and the practical problems of the future," was given at the Peabody Institute on the evening of the opening day.

Comparatively few papers on pure mathematics appeared on the program of Section A because of the fact that the AMERICAN MATHEMATICAL SOCIETY held its annual meeting in affiliation with the Association. The address of the retiring vice-president, President E. O. Lovett, Rice Institute, Houston, Texas, was read by the secretary of the section. It was entitled "The problem of several bodies, recent progress in its solution," and an abstract of it has appeared in a recent number of *Science* (January 15, 1909).

The officers of the section were: vice-president, C. J. Keyser; secretary, G. A. Miller; councilor, G. B. Halsted; member of the general committee, F. R. Moulton; sectional committee, E. O. Lovett, C. J. Keyser, G. A. Miller, E. B. Frost, Harris Hancock, F. R. Moulton, E. W. Brown. On the recommendation of the sectional committee the following thirty-six members of the AMERICAN MATHEMATICAL SOCIETY were elected fellows of the Association: Joseph Allen, R. B. Allen, Oskar Bolza, W. H. Bussey, B. E. Carter, Abraham Cohen, E. H. Comstock, H. A. Converse, S. A. Corey, F. F. Decker, C. C. Engberg, F. C. Ferry, William Gillespie, C. O. Gunther, U. S. Hanna, A. E. Haynes, W. J. Hussey, Kurt Laves, A. H. McDougall, Max Mason, W. F. Osgood, J. M. Page, M. T. Peed, James Pierpont, W. J. Rush, P. L. Saurel, G. T. Sellew, E. B. Skinner, D. E. Smith, P. F. Smith, R. P. Stephens, H. D. Thompson, E. B. Van Vleck, Oswald Veblen, H. S. White, F. S. Woods.

In addition to the address of the vice-president, the following sixteen papers were read before the section:

1. Mr. V. M. SLIPHER: "The spectrum of Mars."
2. Professor E. B. FROST and Mr. J. A. PARKHURST: "Spectrum of comet Morehouse."
3. Professor E. E. BARNARD: "On the changes in the tail of comet Morehouse."
4. Professor FRANK SCHLESINGER: "The orbit of the Algol type variable Delta Libræ."
5. Professor MILTON UPDEGRAFF: "The 6-inch transit circle of the U. S. Naval Observatory."
6. Professor F. R. MOULTON: "On certain implications of possible changes in the form and dimensions of the sun, and some suggestions for explaining certain phenomena of variable stars."
7. Mr. R. H. BAKER: "On the spectra of Alpha Virginis and similar stars."
8. Mr. F. C. JORDAN: "The orbit of Alpha Coronæ Borealis."
9. Professor E. B. FROST: "Radial velocities in Professor Boss's star stream in Taurus."
10. Messrs. PHILIP FOX and GEORGIO ABETTI: "The interaction of sun spots.'
11. Professor HARRIS HANCOCK: "Elliptic realms of rationality."
12. Dr. ARTEMAS MARTIN: "Algebraic solution of the 'three point' problem."
13. Professor J. B. WEBB: "Esperanto and a sexdecimal notation."
14. Professor J. A. MILLER and Mr. W. R. MARRIOTT: "Comet Morehouse."
15. Dr. L. A. BAUER: "On the interpolation formula of geophysics."
16. Mr. H. W. FISK: "A graphical aid to the determination of latitude and azimuth from Polaris observation."

In the absence of their authors the paper by Mr. Slipher was read by J. A. Miller; the joint paper by E. B. Frost and J. A. Parkhurst was presented by W. S. Eichelberger; the three papers by Frank Schlesinger, E. H. Baker and F. C. Jordan, respectively, were read by J. A. Brashear; F. R. Moulton's paper was read by E. D. Roe; G. F. Hull presented the two papers by E. B. Frost and by Philip Fox and Georgio Abetti, the papers by E. E. Barnard and Artemas Martin were read by title. The remaining papers were read by their authors. The following abstracts of those which were of mathematical interest bear numbers corresponding to those of the titles in the list. The abstracts of the others appeared in the report of Section A of this meeting published in *Science*, January 22, 1909.

6. The problems treated in the paper by Professor Moulton are: (1) The theoretical shape of the sun, (2) the character and period of its possible gravitational oscillations, (3) the effects of changes of its dimensions upon its rate of rotation, (4) its energy of rotation, (5) its potential energy, (6) its temperature and rate of rotation, and (7) applications of the same ideas to variable stars.

The results are: (1) The sun is oblate and the theoretical difference in its polar and equatorial diameters is less than $0''.01$. (2) Its gravitational oscillations are expressible in spherical harmonics whose periods depend upon their order. Assuming the sun to be a homogeneous liquid, the longest period is 3 h. 8 m. If it has the viscosity of water, this oscillation will change to 37 per cent. of its value in 2.2×10^{15} years. (3) The change of the sun's diameter by $0''.1$ will change its period of rotation by 7.8 minutes. (4) The formula was found for the change in the rotational energy. (5) The formula for the potential of a spheroid of polar radius c, equatorial $c\sqrt{1+\lambda^2}$, and mass m upon itself is

$$V = \tfrac{3}{5}k^2\frac{m^2}{c}(1 - \tfrac{1}{3}\lambda^2 - \tfrac{7}{15}\lambda^4 - \cdots).$$

(6) The expansion of the sun by $0''.1$ will decrease its temperature (assuming its specific heat is unity) more than 1400° C., and if it obeys Stefan's law, diminish its radiation (assuming its temperature to be 6000° before expansion) by more than 65 per cent. (7) It is shown how gravitational oscillations can explain many puzzling phenomena of variable stars, such as variable periods in the so-called eclipse variables, secondary maxima and minima, varying maxima and minima, etc. It is thought that these factors are supplementary even in those cases where the binary character of the star is certain, and that perhaps in certain classes of stars they may be the only causes of variability.

11. It is known that on the Riemann surface associated with

$$s = \pm\sqrt{A(z-a_1)(z-a_2)(z-a_3)(z-a_4)},$$

in which a_1 and a_2 are connected by a canal as are also a_3 and a_4, every one-valued function of position which has everywhere a definite value is of the form $w = p + q \cdot s$, where p and q are rational functions of the complex variable z; and reciprocally every function of this form is a one-valued function of position on this Riemann surface. If we denote two such functions by

$$w_1 = p_1 + q_1 \cdot s, \quad w_2 = p_2 + q_2 \cdot s,$$

then the sum, difference, product and quotient of the two functions w_1 and w_2 are functions of the form $w = p + q \cdot s$.

Let z take all complex values and consider the collectivity of all rational functions of z with arbitrary constant real or complex coefficients. These functions form a closed realm, the individual functions of which repeat themselves through the processes of addition, subtraction, multiplication and division, since clearly the sum, the difference, the product and the quotient of two or more rational functions is a rational function and consequently an individual of the realm. This realm is denoted by (z).

It is evident that if we add (or *adjoin*) the algebraic quantity s to this realm we will have another realm, the individual functions or elements of which repeat themselves through the processes of addition, subtraction, multiplication and division. This realm includes the former realm. We shall call it the elliptic realm and denote it by (s, z).

By a theorem due to Liouville, the most general one-valued doubly periodic function is a rational function of z and s. It is consequently a one-valued function of position on the Riemann surface and belongs to the elliptic realm of rationality (z, s).

The elliptic or doubly periodic realm of rationality (z, s) degenerates into the simply periodic realm when any pair of branch points are equal, say $a_1 = a_2$; and into the realm of rational functions when two pairs of branch points are equal, say $a_1 = a_2$ and $a_3 = a_4$.

Thus the elliptic realm includes the three classes of one-valued functions: (1) the rational functions, (2) the simply periodic functions, (3) the doubly periodic functions. All these one-valued functions, and only these, have algebraic addition theorems.

In other words, *all functions of the realm (z, s) have algebraic addition theorems, and no one-valued function that does not belong to this realm has an algebraic addition theorem.*

We have thus the theorem:

The one-valued functions of position on the Riemann surface

$$s^2 = A(z - a_1)(z - a_2)(z - a_3)(z - a_4)$$

belong to the closed realm (z, s) *and all elements of this realm and no others have algebraic addition theorems.*

Professor Hancock's paper will be offered to the *American Journal of Mathematics* for publication.

12. The paper by Artemas Martin is devoted to an algebraic determination of the point within a triangle at which the sides subtend given angles. The paper is to appear in the *Mathematical Magazine*, edited by the author.

13. The paper by J. Burkitt Webb is devoted to exhibiting the advantages which would result from the adoption of a system of notation with 16 as its base. The success which has attended the movements towards a universal language has inspired the author with hope in the success of a movement towards the selection of a more useful system of notation, and he pointed out the many advantages which the base 16 would offer.

15. The rather prevalent custom of resolving or expressing every natural phenomenon — be it periodic or otherwise — by a Bessel or a Fourier series, or by spherical harmonic functions, has brought about at times, especially in geophysical and cosmical phenomena, if not direct misapplications, at least misinterpretations of the meaning and value of the derived coefficients. Instead of clarifying the situation our calculations may have actually contributed to befog it. Instead of rejecting, one must learn to consider the outstanding residuals as the *true* facts of nature and not treat them as though they were "abnormal" or contrary to nature's law.

Dr. Bauer exemplified these statements in a brief discussion of two cases that are typical in geophysical investigations — the one involving an application of spherical harmonic functions to the representation of the distribution of the earth's magnetism over the earth, while the other involved the use of Fourier series in the representation of certain diurnal geophysical phenomena.

The chief purpose of the paper was to call renewed attention to the limitations, from a physical standpoint, of the form of "interpolation formulas" usually employed in the representation of natural phenomena. The formula of Gauss is simply a mathematical approximation to the actual law of distribution. In spite of all the work done, no one up to date has succeeded in giving any physical interpretation of the coefficients of the harmonic terms beyond those of the first order involving three coefficients, and these three stand for the simplest possible case of a first approximation to the actual state of the earth's magnetism: that of a uniform magnetization about a diameter inclined to the axis of rotation.

16. Mr. H. W. Fisk considers the formula for latitude,

$$\phi = h - p \cos t + \tfrac{1}{2}p^2 \sin 1' \sin^2 t \tan h + \cdots$$

from Chauvenet's Astronomy I., § 176, and the formula for azimuth,

$$A = p \sin t \sec \phi + \tfrac{1}{2}p^2 \sin 1' \sec \phi \tan \phi \sin 2t + \cdots$$

from Jordan, "Zeit und Orts-Bestimmung," page 122. The first terms of these formulas are readily computed. The last terms, called correction terms, are arranged as a set of curves from which the value is quickly taken by inspection. The geographical limits within which this method may be used, as well as the expected accuracy under different conditions are discussed. Attention is given to the change in correction terms due to the progressive change in the value of p. Assuming, for instance, that p will continue indefinitely to change at its present rate $0'.34$ per annum, we may find the period during which the curves could be used within the error $0'.1$. The latitude curves are found to be good 83 years at latitude 10° and about 8 years at latitude 60°. The azimuth curves are correspondingly useful for 80 years at 10° and 4 years at 60°.

The next regular meeting of the association will be held in Boston under the presidency of President D. S. Jordan, Stanford University. No summer meeting will be held during 1909, in order to permit the members to attend the meetings of the British Association at Winnipeg, which has extended a cordial official invitation to the members of the American Association. Professor E. W. Brown, Yale University, was elected vice-president and chairman of Section A, and Professor G. A. Miller, University of Illinois, continues as secretary. The section elected Professor Winslow Upton, Ladd Observatory, as member of the sectional committee for five years.

G. A. MILLER,
Secretary.

SOME SURFACES HAVING A FAMILY OF HELICES AS ONE SET OF LINES OF CURVATURE.*

BY MISS EVA M. SMITH.

IN a recent paper, Forsyth † gives a general method for the determination of surfaces with assigned lines of curvature, and he solves completely the case where both sets are circles. We apply the method to the case where one of the given sets consists of helices, and it appears that surfaces do exist having as one set of lines of curvature general helices ($\rho/\tau =$ constant along each curve), but there are always limitations on the forms of ρ and τ. In particular, ρ and τ cannot both be constant along every curve. The complete solution seems to be too wide for analytic discussion, but there are two particular cases for which definite results can be obtained. This note contains a discussion of these cases.

1) Assuming that ρ and τ are constant along each curve of the set (regular helices), we obtain the result that: *There are no surfaces with regular helices as one set of lines of curvature.*

2) If ρ/τ is constant along each curve of one set of lines of curvature, and the other set consists of geodesics, we can obtain a complete solution ; the equations of the resulting surfaces in parametric form are given at the end of this paper. The notation and equations used are those given in Darboux, Théorie des surfaces, volume 2, but derivatives with respect to u and v are here denoted by suffixes 1 and 2 respectively.‡

§ 1.

Consider the case where the helices are all regular. The curves $v =$ constant are helices, and therefore ρ and τ for these curves are functions of v only, and we denote τ/ρ by k.

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* For the suggestion of this subject I am indebted to Prof. A. R. Forsyth.

† *Messenger of Mathematics*, vol. 38 (1908), pp. 33–44.

‡ Note that p_1 is not $\partial p/\partial u$. To express derivatives of the rotations we use parentheses, *e. g.* $(p)_1 = \partial p/\partial u$.