

It is of interest to notice that the type III presented itself first in connection with the general transformation theory* of the lineal elements of space. Such a transformation is defined by setting $x_1, y_1, z_1, y'_1, z'_1$ equal to arbitrary functions of x, y, z, y', z' . Lie showed that the only case in which every union is turned into a union is the extended point transformation. There may, however, be other transformations which convert some unions into unions. Examples of this sort have been given by Lie in which the unions considered are the curves of a minimal complex or some other Monge equation (II). The general result is as follows:

The most extensive systems of curves that can be converted into curves by element transformations which are not merely extended point transformations are those defined by differential equations of type III, i. e., precisely those characterized by the Meusnier property.

For any system of type III transformations of this sort may be found. If a system not satisfying an equation III is transformed into curves, the element transformation is necessarily an extended point transformation.

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ON THE DISTANCE FROM A POINT TO A SURFACE.†

BY PROFESSOR PAUL SAUREL.

(Read before the American Mathematical Society, April 25, 1908.)

IN a recent number of the BULLETIN,‡ Professor Hedrick has called attention to the fact that the normal distance from a given point to a surface may be a minimum among the distances in every normal plane section without however being a minimum among the distances to the surface. In this connection it may be of interest to observe that for surfaces of a very general type the phenomenon in question can occur only when the given point is a principal center of curvature. In fact, if the equation

* Cf. abstract of the author's paper on this subject in the BULLETIN, vol. 10 (1904), p. 492.

† The theorems established in a previous note (BULLETIN, vol. 13, p. 447) are not new, as I then believed; they are to be found in section 60 of Gour-sat's Cours d'Analyse. I take this opportunity of acknowledging Professor Hedrick's kindness in calling my attention to the fact.

‡ BULLETIN, April, 1908, p. 321.

of the surface in the neighborhood of the foot of the normal can be written in the form

$$(1) \quad z = ax^2 + by^2 + R,$$

in which the remainder R is of the third or higher order in x and y , and a and b are not both equal to zero, it is easy to establish the following theorems:

I. If the foot of the normal be an elliptic point of the surface, the distance from the nearer (more remote) of the principal centers of curvature may be a minimum (maximum) among the distances in every normal section without being a minimum (maximum) among the distances to the surface.

As illustrations of these two statements may be mentioned the distances from the point $(0, 0, \frac{1}{2})$ to the surfaces

$$z = x^2 + \frac{y^2}{2} + \frac{3}{2}x^2y, \quad \text{and} \quad z = x^2 + 2y^2 - 3x^2y + 3x^4.$$

II. If the foot of the normal be a hyperbolic point of the surface, the distance from a principal center of curvature may be a minimum among the distances in every normal section without being a minimum among the distances to the surface.

The distance from the point $(0, 0, \frac{1}{2})$ to the surface

$$z = x^2 - y^2 + 6x^2y - 3x^4$$

furnishes an illustration of this statement.

III. If the foot of the normal be a parabolic point of the surface, the distance from the principal center of curvature may be a minimum among the distances in every normal section without being a minimum among the distances to the surface.

In this statement the term parabolic point is to be understood as meaning any point for which a or b in equation (1) vanishes.

Professor Hedrick has given an example that falls under this theorem. An equally simple example is furnished by the distance from the point $(0, 0, \frac{1}{4})$ to the surface

$$z = 2x^2 + 10x^2y.$$

IV. The phenomenon discovered by Professor Hedrick cannot occur unless the given point be a principal center of curvature.

To establish the last of these theorems it is sufficient to observe that the square of the distance from a point $(0, 0, d)$ on the

normal to a point (x, y, z) on the surface is given by an expression of the form

$$d^2 + Ax^2 + By^2 + \dots,$$

in which A and B are constants and the omitted terms are of the third order at least, and that when the given point is not a principal center of curvature the coefficients A and B do not vanish. If, however, the given point be a principal center of curvature, A or B will vanish and an examination of the sign of the non-vanishing coefficient will easily yield the first three theorems.

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ON THE SOLUTION OF ALGEBRAIC EQUATIONS IN INFINITE SERIES.

BY PROFESSOR P. A. LAMBERT.

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I. *Introduction.*

The purpose of this paper is to present a general method for determining all the roots of any algebraic equation by means of infinite series. The method consists in forming three algebraic functions of x from the given equation

$$(1) \quad f(y) = 0,$$

(a) by introducing a factor x into all the terms of (1) except the first and last ;

(b) by introducing a factor x into all the terms of (1) except the first and second ;

(c) by introducing a factor x into all the terms of (1) except the second and last.

These algebraic functions are expanded into power series in x by Laplace's series. If in these power series x is made unity, the resulting series, if convergent, determine the roots of the given equation. It will be shown that all the roots of the given equation can be expressed in infinite series derived either from the algebraic function formed in accordance with (a), or from the two algebraic functions formed in accordance with (b) and (c). The method presupposes the solution of the two-term equation