SHORTER NOTICES.

Sur le Développement de l'Analyse et ses Rapports avec diverses Sciences: Conférences faites en Amérique. Par EMILE PICARD. Paris, Gauthier-Villars, 1905. 167 pp.

La Science moderne et son Etat actuel. Par EMILE PICARD. Paris, E. Flammarion, 1908. 301 pp.

PICARD'S lectures delivered here in America, three at the decennial of Clark University in 1899 and one at Saint Louis in 1904, were published soon after their delivery. Those in celebration of the decennial of Clark appeared in the volume devoted to that celebration and again in the *Revue générale des sciences* early in 1900. The lecture at Saint Louis had its place in the Proceedings of the congress of arts and sciences. It was not, however, until 1905 that the author collected the four lectures in a separate monograph readily accessible to all.

In the general introduction to the first lecture the author states: "Je lis toujours pour ma part . . . le *Bulletin* de la Société mathématique américaine . . . qui tient ses lecteurs au courant des travaux les plus récents." It is much to be feared that the present review is too tardy to come under this generous compliment. There is, however, in all that Picard writes, and particularly in these lectures, that finish and that wisdom which makes his work immediately a classic and compels, not at the time merely but for many years thereafter, the attention and interest of the student who would really be "au courant des travaux les plus récents."

The chief thoughts in the first lecture are grouped about the genesis and growth of the general conception of function. The subject takes one far and wide in the history of the development of mathematics and there is scarcely a reputable mathematician since Euler whose name does not deserve mention in the sketch; for although the functions of real and of ordinary complex variables naturally take most of the attention, the possibility of functions of other hypercomplex numbers and the various geometric fields therewith connected are not overlooked. That the author is not so exclusively concerned with the scientific side of his subject as to forget the fundamental pedagogic ideas, is brought into evidence by the two remarks: "Si Newton et Leibniz avaient pensé que les fonctions continues n'ont pas nécessairement une dérivée, ce qui est le cas général, le calcul différentiel n'aurait pas pris naissance; . . . " and in reference to the extremes to which rigor may be pushed "ces spéculations raffinées ont même pénétré dans l'enseignement élémentaire, ce qui est, a mon avis, très regrettable." And such matters are not merely mentioned; they are treated as worthy of some elaboration.

The second lecture is on the theory of differential equations, whether partial or ordinary. Many interesting results are cited from the work of the previous decade, and not a few of them are due to the author or to others who have followed where he has led. One fact that is rarely seen in print is the discovery of Borel that every analytic integral of a partial differential equation with analytic coefficients may be expressed by a formula containing only one arbitrary function of a single real variable. In regard to ordinary equations, the work of Painlevé on equations which are not linear and of Poincaré on the configuration of the real solutions are among the foremost of the matters noticed. The author very naturally records a protest against the tendency to regard the theory of ordinary equations as a chapter in the theory of analytic functions.

That Picard would hardly subscribe to the motto "Es giebt nur Potenzreihen" appears evident from the fact that much of the discussion in the first two lectures is connected with the name of Fourier and that it is only in the third lecture that he comes to speak especially of analytic functions and of the various special functions which have been introduced into mathematics. After a preliminary reference to the origin of the theory of analytic functions and an extended mention of the recent work of the French school, among whom Mittag-Leffler must be included, on the various developments of these functions and on the special properties of integral functions, the author draws the general outlines of the automorphic functions, the new transcendents of Painlevé, the hyperautomorphic functions, and finally his own algebraic functions of two variables. Desiderata are not forgotten amid the mass of results already acquired.

The concluding lecture, that at Saint Louis, wherein the general development of mathematics in connection with its applications is reviewed from early times through the century of d'Alembert's bon mot "allez en avant, et la foi vous viendra" down to the present rigorous days, cannot be sub-reviewed: it must be read. One idea, however, which will appear again in

La Science moderne, may be noted: that of heredity as against non-heredity, of functional equations as against differential equations.

Modern science as it is is a subject so vast that no one person can hope to be an "Ausgelernter" in science alone, to say nothing of all other domains of thought, as might have been possible in the days of Faust. Yet if there is a place where such an accomplishment is most favored it is Paris, where the five academies flourish and where intellectual pursuits are centralized as they are nowhere else. Picard's book, La Science moderne, may well take its place among the fair flowers of this hothouse of thought.

In the introduction, after shutting out metaphysical considerations and limiting himself to a discussion of the methods of investigation which are essentially scientific, the author points out the constantly growing field of science and the constantly increasing correlation of different parts of the field, until "Il semble que dorénavant, dans la vie scientifique comme dans la vie sociale, l'association s'imposera de plus en plus. Tel travail ne pourra être effectué que par la collaboration d'un mathématicien et d'un physicien, et tel autre demandera le concours d'un chimiste et d'un physiologiste." This is a generalization and at the same time a reminiscence of the idea with which he closed his lectures at Clark — an idea which in practice is constantly and emphatically outraged by the training that is provided by some of our foremost schools as preparation for the doctorate — "En terminant, je me permettrai de donner un conseil aux étudiants mathématiciens qui m'ont fait l'honneur de m'écouter: je leur recommanderai de ne pas se cantonner trop tot dans des recherches spéciales. Il leur faut acquérir d'abord des vues générales sur les diverses parties de notre science, sans lesquelles leurs recherches risqueraient de rester stériles, et qui leur coûteraient plus tard un bien plus grand effort."

The first chapter is a summary of the lecture at St. Louis, explained so far as may be without technicalities and adapted to the comprehension of a more popular audience. The second chapter bears the title, the mathematical sciences and astronomy. There is much more in it than the title would indicate; for a start is made from the modern theories of number and of analysis and from some rapid indications concerning recent advances in geometry, with especial reference to the theory of groups (it should be remembered that the new program of public instruction in France introduces the notion of groups into geometry at a very early stage) and of the popularly attractive non-euclidean geometries. And some further words on the development of analysis are inserted before the author finally settles down to celestial mechanics and astronomical physics. So much for the venerable sciences of mathematics and astronomy.

The next chapter logically comes to the subjects of mechanics and energetics. Here there is an account of mechanics from its early formative stages through the time of Newton; some mention of the deductive side of mechanics from that day to the present, with especial emphasis on the ideas of Hertz in regard to hidden motions and hidden masses; and then the statement and discussion of the problem of the mechanical explanation of the universe. There is a concluding section on energy, in which the laws of thermodynamics and the relation of mechanics to the wider fields of physics receive attention. The fourth chapter is on the physics of the ether. The way in which the author manages to set forth the interplay of optics with electricity and magnetism, to tell of the recent discoveries in cathode and X-rays, and to expound the chief phenomena and theories of the radioactive substances is to be admired and beheld with wonder.

That Picard is proceeding from the more abstract and definite toward the more phenomenological and less thoroughly correlated sciences may be seen from the titles of the remaining five chapters: material physics and chemistry, mineralogy and geology, physiology and biological chemistry, botany and zoology, and medicine and the microbic theories. Whether or not the first four chapters would be readily understood by persons who, though educated, were not specialists in mathematics or physics might at first be a serious question: but if the explanation there is as clear and interesting as it surely is in these five last chapters, which are certainly not directly in our province, the answer could only be in the affirmative. To read, in the author's delightful French, of the accomplishments in all these fields by such a variety of authorities as Moissan, Suess, Koch, Lehmann, Haeckel, Pasteur, Loeb, Marsh, de Vries, and many others whose names are less familiar, is highly interesting and leaves the same literary and romantic impression as Maeterlinck's L'Intelligence des fleurs, less the fantasy — but would scarcely find a proper place for review in this periodical. Besides, it should appear that even if one man did write of all these matters, one reviewer could not be expected to treat all of them in other than the perfunctory way in which almost all offerings of scientific work are "reviewed" in the daily or weekly press.

E. B. WILSON.

Algebraic Equations. By G. B. MATHEWS, M.A., F.R.S., Fellow of St. John's College. Cambridge University Press, 1907. 64 pp.

THIS is one of a half dozen "Cambridge Tracts in Mathematics and Mathematical Physics," issued by the Cambridge University Press. In this tract the author is confronted with the problem of giving an exposition of the Galois theory of equations within the compass of about sixty pages. This limitation makes it necessary for him to present the subject more or less in outline and to confine himself to a very few illustrative examples. An outline presentation, prepared by an eminent author, is certain to bring out in bold relief interesting view points. Such is the case in this booklet. And yet we are of the opinion that the real value of this book to beginners would have been enhanced by more abundant illustration and a somewhat fuller detail of explanation.

To save space, the author does not put down a definition in a sentence by itself; the definition is to be inferred from a condensed statement made as part of a sentence occurring perhaps in the body of a demonstration. Thus, the definition of an intransitive group (page 14) is given in course of a proof, as follows: "First suppose G is intransitive: this means that a certain number of roots $x_1, x_2, \cdots, x_r$ $(r < n)$ are only interchanged among themselves by the substitutions of G." Less easily comprehended are the definitions of simple groups and self-conjugate factor groups, similarly interpolated on pages 16 and 17. Despite the effort to secure extreme condensation, there occur redundancies, such as "absolutely undetermined" (page 2), "absolutely unaltered" (page 57), "perfectly definite" (page 6). These are less objectionable in oral exposition than in a printed outline.

Here and there are evidences of hasty composition. Thus, the tract is encumbered with some heterogeneous terminology. The author speaks in different places of the "arithmetically