# NOTE ON A CERTAIN EQUATION INVOLVING THE FUNCTION $E(x)$.

BY PROFESSOR R. D. CARMICHAEL.

RECENTLY J. V. Pexider has studied the equation*

$$(1) \qquad E\left(\frac{n+\alpha}{x}\right) - E\left(\frac{n+\alpha}{x+1}\right) = d,$$

where $E(s)$ is the greatest integer $\leqq s$, and where $\alpha$ is zero or a positive quantity less than 1, $n$ is a known positive integer, $d$ is zero or a known positive integer, and $x$ is an unknown positive integer. M. Pexider confines himself chiefly to the case in which $d = 0$ and $x$ is less than $n$. He finds the values of $x$ which satisfy the equation subject to these restrictions.

In the present note it is proposed to exhibit a simple working method by which the roots can be found in any case. When $d > 0$, $x$ is always less than $n$, except that $x$ may equal $n$ when $d = 1$. In what follows $x$ is taken always less than $n$.

If $n + \alpha$ is divided by an integer $i$, giving the quotient $q + \beta$ where $\beta$ is zero or a positive quantity less than 1; then if $n$ is also divided by $i$, the quotient will evidently be $q + y$ where $y$ is zero or a positive quantity less than 1. Hence

$$E\left(\frac{n+\alpha}{i}\right) = E\left(\frac{n}{i}\right).$$

Therefore, the equation

$$(2) \qquad E\left(\frac{n}{x}\right) - E\left(\frac{n}{x+1}\right) = d$$

has the same roots as (1). We may then confine ourselves to the solution of the latter equation as being somewhat the simpler of the two.

Represent $n$ in the form

$$(3) \qquad n = ax + c \qquad (c < x, \ a \neq 0).$$

* *Rendiconti del Circolo Matem. di Palermo*, vol. 24, no. 1, pp. 46–64. For convenience I write the equation in a form somewhat different from that of M. Pexider.

Then

$$E\left(\frac{n}{x}\right) \equiv E\left(\frac{ax+c}{x}\right) = a;$$

and therefore

$$(4) \qquad E\left(\frac{n}{x+1}\right) \equiv E\left(\frac{ax+c}{x+1}\right) = a-d$$

is the necessary and sufficient condition that $x$ is a root of (2). Now (4) will be satisfied if and only if

$$ax+c-(x+1)(a-d) < x+1 \quad \text{and} \quad \geqq 0.$$

This readily reduces to

$$(5) \qquad a-c > (d-1)(x+1) \quad \text{and} \quad \leqq d(x+1).$$

From these results we have the following working method for finding the solutions of (2) and hence of (1):

*Write $n$ in every possible way in the form*

$$(3) \qquad n = ax + c \qquad (c < x,\ a \neq 0)$$

*and examine whether both inequalities*

$$(5) \qquad a-c > (d-1)(x+1) \quad \text{and} \quad \leqq d(x+1)$$

*are fulfilled at the same time; if so, $x$ is a root of the equation; otherwise, it is not such a root.*

Applying this to the case when $d = 0$, we have to determine $x$ subject to the conditions

$$n = ax + c, \quad (c < x,\ a \neq 0,\ c-a < x+1 \text{ and } \geqq 0).$$

M. Pexider (l. c., page 57) shows that the number of such roots ($x$ being less than $n$) is

$$A(n) = E(n) - E(\sqrt{n}) - E\left(\frac{n}{E(\sqrt{n})+1}\right).$$

He exhibits also a better method than the above by which the roots may be determined in the present case.

If $d = 1$, the roots of our equation are to be found from

$$n = ax + c, \quad (c < x,\ a \neq 0,\ a-c > 0 \text{ and } \leqq x+1).$$

This will give all the roots of (1) and (2) except $x = n$ which has been definitely excluded from this result by previous assumption that $x < n$.

For other values of $d$ the expressions in (5) do not simplify, and we have to determine $x$ from the general conditions

$$n = ax + c,\ c < x,\ (a \neq 0,\ a - c > (d-1)(x+1) \text{ and } \leqq d(x+1)).$$

It may be verified by actual trial that the solutions of (1) and (2) may in this way be effected with a degree of readiness which will make the method serviceable in a large number of cases.

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# THE INNER FORCE OF A MOVING ELECTRON.

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1. *Introduction.* — Sommerfeld* has given a general method of determining the force which an electron exerts on itself when its motion is known. Schott† has applied the retarded potential to the same problem. In the present paper the known vector expression‡ for the force with which a moving point charge acts on another point charge is used to determine the action between any two elements of the electron. A double integration over the volume of the electron gives the inner force of the electron. The Abraham-Sommerfeld expressions for the longitudinal mass and Abraham's value for the transverse mass are thus very simply determined and the limitations on the solutions are made manifest.

2. *The Force Between Two Moving Point Charges.* — The electron is assumed to be a uniformly charged solid sphere which is moving in a straight line without rotation. Take the $x$-axis in the direction of motion. Measure the time from the instant at which the force is to be determined and choose the velocity of light as the unit of velocity. Let $(x_0, y, z)$ be the coordinates of the point charge $de_2$ relative to the point charge $de_1$. The distance moved by the electron in time $t$ is

* *Göttinger Nachrichten*, 1904.
† *Ann. der. Physik*, 1908, No. 1.
‡ Abraham : Theorie der Elektrizität, vol. II, p. 98.