THE MATHEMATICAL TABLETS OF NIPPUR.

Mathematical, Metrological and Chronological Tablets from the Temple Library of Nippur. By H. V. HILPRECHT. Vol. XX, part I, of Series A, Cuneiform texts, published by the Babylonian Expedition of the University of Pennsylvania, 1906.   Pp. xviii + 70 + (30 + xv) plates.   Price, $5.

UNTIL the middle of the last century the mathematics of the Babylonians was practically a mystery to modern European scholars.   It was asserted, it is true, in a general way by many writers on the history of the science that our sexagesimal fractions originated in the valley of Mesopotamia.   Formaleoni of Venice had suggested as early as 1788 that these were due both to the ancient division of the year into three hundred and sixty days and to the properties of the regular hexagon.   Even the Greeks had assigned these "fractiones physicæ," as they were called in the middle ages, to the Babylonians, and before their use by such writers as Hypsicles and Ptolemy the story had long been current that Pythagoras had studied mathematics with the priests along the Euphrates, where the sexagesimal system was known.   There were also numerous assertions in the later Middle Ages that even our Hindu-Arabic numerals were due to the Chaldeans, the Kaldâ of the monument inscriptions, who overran and subdued Babylonia in the first millenium, B. C.   It was this tradition that led Tonstall to say, in 1522 : " Qui a Chaldeis primum in finitimos, deinde in omnes pene gentes fluxit," and Recorde (c. 1542) to remark : " In that thinge all men do agree, that the Chaldays, whiche fyrste inuented thys arte, did set these figures as thei set all their letters.   for they wryte backwarde as you tearme it, and so doo they reade."

All this was mere fiction, or but little more than tradition, and our knowledge of Babylonian mathematics may be said to have had but little scientific foundation until Rawlinson deciphered two small and imperfect cylinders found in 1854 by W. K. Loftus at Senkereh, the ancient Larsam (Larsa), on the Euphrates, and now preserved in the British Museum.   Discovering the meaning of the key word IB–DI, square, Rawlinson was able to show that these cylinders contained tables of squares and cubes, written on the sexagesimal system.   Pro-

fessor Sayce put the date of the cylinders between 2300 and
1600 B. C., and thought that there had been a great library
at Senkereh which would probably yield other mathematical
material. Unfortunately this expectation has never been real-
ized so far as Senkereh is concerned, although a few other sim-
ilar cylinders have been found elsewhere. There has recently
been uncovered at Nuffar (the ancient Nippur), however, a col-
lection of unexpected richness, and particularly valuable for the
light which the cylinders throw upon ancient mathematics.

The excavations at Nippur began in 1889, and for eighteen
years Professor Hilprecht has been connected with the work.
He has examined, with more or less care, over 50,000 cunei-
form tablets thus far excavated there, and has secured for Con-
stantinople and Philadelphia the best treasures of what may
have been a great temple library which the Elamites twice de-
stroyed, viz., about 2150 B. C., and about 1900 B. C. Some
of these tablets are brick textbooks prepared by teachers and
possibly deposited with other works in the temple library.
Some have the teacher's model and the pupil's copy, and still
others seem to show the erasure of the latter's work. At any
rate, such are the dates and circumstances and interpretations
which Professor Hilprecht assigns to his great discovery.
There are not wanting, however, those who attack this view,
some in a spirit of apparent fairness, and others with what ap-
pears to be a spirit of somewhat captious criticism. It is
claimed that the tablets have no literary value, and that there
is no evidence of a temple library, since some of the cylinders
come from one part of the ancient city and some from another.
One writer even weakens his cause by offering the remark that
the temple may have had a system of Carnegie branches. It
is also said that several important tablets did not come from
Nippur at all, and an Arab's statement is taken against Pro-
fessor Hilprecht's argument to prove this fact. While it would
seem to a layman in matters Assyriological that there may have
been two libraries at Nippur, as there were at Nineveh, one old
and the other of later date, and that the statement of an Arab
trader would not have much weight in a scientific controversy,
it is certain that the bickering over seventeen tablets out of
some fifty thousand should not obscure the fact that here is the
greatest discovery of all time relating to Babylonian mathe-
matics. It is probable that, with the support of such eminent
scholars as Professor Zimmern of Leipzig, Professor Hommel

of Munich, and Professor Winckler of Berlin, Professor Hilprecht will not feel that his labors have been in vain. With the whole controversy the student of mathematical history need have no concern whatever. Here are the cylinders ; they are genuine ; they are ancient ; they reveal the science of Babylonia of the second or third millenium B. C., and whether there was a temple library, or whether an Arab told the truth, is from the mathematical standpoint a consideration of no moment.

It is the result of a study of over forty mathematical cylinders that appears in the volume under review. These cylinders include multiplication and division tables, tables of squares and square roots, a geometric progression, a few computations, and some work on mensuration.

The multiplication tables are all arranged on the column plan, the same that was used by the medieval Italians in their "per colonna" operations. They also resemble the Italian tables in that the multiplicands are not successive, the European tables giving only those products needed for the current measures. Professor Hilprecht feels sure that this reason did not, however, influence the Babylonian mathematicians, although this hypothesis might easily be in harmony with his discovery that the multiplicands are all factors of 60^4. The tables are carried much farther than those found in medieval Europe, the multiplicands extending to 180,000.

More valuable than the multiplication tables are those of division, and of these the cylinders numbered 22 and 25 are the most interesting. In the former the quotients of 60^4 by 1, 2, $\cdots$, 18 are given, as follows :

IGI   1   GAL–BI   8,640,000   A–AN
IGI   2   GAL–BI   6,480,000   A–AN
IGI   3   GAL–BI   4,320,000   etc.

The meaning will be understood by knowing that IGI–GAL (literally, "having an eye," hence the decider, determinator, denominator) means, practically, the divisor or simply "divided by" ; that BI means "its," and refers to the dividend, which in this case is the mysterious 60^4 ; and that A–AN means "each." Hence the second line may be translated, "60^4 divided by $2 = 6,480,000$ each." In other words we have here the equivalent of a table of unit fractions of 60^4, in which the

first is not $\frac{1}{1}$, but $\frac{2}{3}$.   Now why, asks Professor Hilprecht, was $\frac{2}{3}$ thus indicated as a multiplier?   In reply, it seems not unreasonable to consider the correspondence between these Babylonian remains and the Egyptian mathematics of the (nearly) contemporary Ahmes.   This at once throws a good deal of light upon both of the cylinders here discussed.   This No. 22 simply means the first fraction of 60^4 is 8,640,000 ; the second, 6,480,000, and so on.   Now this first fraction in the Egyptian hieroglyphic and hieratic systems seems, from Ahmes, to have been $\frac{2}{3}$, the only non-unit fraction known to him, and apparently the first one known to the Babylonians.   It was the only one, save $\frac{1}{2}$, to have a special symbol in Egypt, and it probably had a special name, at least, in Babylonia.   The very fraction name, IGI–GAL, "having an eye," suggests the eye-shaped fraction symbol of the Egyptians, and raises the question whether the original might not have been an eye instead of *ro*, a mouth. Nevertheless it must be admitted that the tablet in question does not contain the well-understood cuneiform fraction symbols for $\frac{1}{2}$, $\frac{2}{3}$, etc., and that there is, therefore, no direct connection between it and the Ahmes manuscript.   As to the fraction symbol, too, the relation between the form of eye and mouth is probably merely fanciful, although the Semitic (as distinguished from the Sumerian) Babylonian and the Hebrew both use "mouth" for fraction, as the Egyptians did.   Thus the Babylonian for $\frac{2}{3}$ is *shínipu*, probably from *šiná* (two) and *pû* (mouth), the "two-fraction," a form also found in Hebrew.

The most interesting of all the cylinders, however, is No. 25, which is transliterated as follows :

Line 1 :	125	720	Line 9 :	2,000	18
2 :	IGI–GAL–BI	103,680	10 :	IGI–GAL–BI	6,480
3 :	250	360	11 :	4,000	9
4 :	IGI–GAL–BI	51,840	12 :	IGI–GAL–BI	3,240
5 :	500	180	13 :	8,000	18
6 :	IGI–GAL–BI	25,920	14 :	IGI–GAL–BI	1,620
7 :	1,000	90	15 :	16,000	9
8 :	IGI–GAL–BI	12,960	16 :	IGI–GAL–BI	810

Of this Professor Hilprecht has, with much ingenuity, unraveled part of the meaning, viz : that $60^4 \div 103{,}680 = 125$, which accounts for two figures in lines 1, 2, and similarly for other pairs.   He also notices that $125 = 2 \cdot 60 + 5$, and that

3,600 ÷ 5 = 720, and similarly for other odd numbered lines.
Now is there any further explanation? In reply it may be
noticed that the unit fraction already mentioned, a form met
not only among the Egyptians but among the early Greeks and
other peoples about the Mediterranean, may have played a
part hitherto unrecognized among the Babylonians. This is
also seen in the table on page 27, where certain unit fractions
of 195,955,500,000,000 are given, and in numerous other
places. Therefore instead of lines 1, 2, meaning 60^4:
103,680 = 125, the interpretation may be as follows:

$$\frac{1}{60^2} \cdot \frac{1}{103,680} \cdot 60^4 = \frac{1}{60^2} \cdot 125 = \frac{1}{30} + \frac{1}{720}.$$

That is, lines 1, 2 give the important numbers in connection
with all these unit fractions, viz.: 125, 720, and 103,680, as-
suming, of course, that the computer knew the nature of the
problem with respect to the powers of 60. In the same way
lines 3, 4 may mean

$$\frac{1}{60^2} \cdot \frac{1}{51,840} \cdot 60^4 = \frac{1}{60^2} \cdot 250 = \frac{1}{15} + \frac{1}{360},$$

and so on. In every case the denominator of the last unit
fraction is given. E. g., lines 7, 8 give the unit fractions
$\frac{1}{15} + \frac{1}{15} + \frac{1}{15} + \frac{1}{90}$; lines 9, 10 give $\frac{1}{2} + \frac{1}{8}$, and so on. All
of the unit fractions save the last are readily seen, at least after
the initial $\frac{1}{30}$ is found, so that only the last denominator seemed
necessary. It is evident, however, that the cylinder does not
actually contain these unit fractions, and the explanation here
suggested may be far from the true one; but it is consistent,
and if not exact it seems to show at least a connection between
the number concepts of Babylonia and the Mediterranean coun-
tries. Moreover the cylinder has three other features of in-
terest: (1) Passing from lines 7, 8 to 9, 10 the fractions (if we
consider them as such) would become, following an apparent
law of doubling, $\frac{8}{15} + \frac{1}{45}$; but, to maintain the unit fraction
idea, they are written $\frac{1}{2} + \frac{1}{18}$ the denominator 18 appearing in
line 9, all of which recalls the Egyptian treatment; (2) The
table gives both an increasing and decreasing geometric pro-
gression; (3) It is evidently based upon $60^4 = 12,960,000$
which, as Professor Hilprecht notes, underlies all the mathe-
matical texts described in this work, and which is nothing less

than the mystic Platonic number, the "lord of better and worse births," the number of days in the "magnus Platonicus annus" of 36,000 years (of 360 days each), and the number at the basis of all the multiplication and division tables of Nippur, Sippar, and the library of Ashurbânapal.

The historic interest in this Platonic number does not relate to any extent to the supposed mysticism involved in it, but to the fact that Plato not improbably received it from the Pythagoreans, and they from their master the tradition of whose sojourn by the Euphrates is thus in some slight degree confirmed. At any rate, the whole matter seems to show a relation between the East and the West, both in the underlying idea of the unit fraction and in the number mysticism of the ancient philosophers.

A further debt which historians of mathematics owe to Professor Hilprecht arises from his contribution to the subject of ancient geometry.   The results of his investigation of several cylinders on mensuration, together, no doubt, with the Scheil-Eisenlohr investigation * of the Ine-Sin tablet show "that at this early period the Babylonians must have been familiar with the following theorems: 1. The area of a rectangle is equal to the product of its base and altitude.   2. The area of a square is equal to the square of its side.   3. The area of a right triangle is equal to one-half the product of its base and altitude. 4. The area of a trapezoid is equal to one-half the sum of its bases multipled by its altitude."   He also shows that they must have known either how to find the volume of a parallelepiped (and hence a cube) or else of a circular cylinder (and hence the area of a circle).

Still another inference of great importance, and supported by evidence that seems sufficient, is that the Babylonians knew, in some form or other, the law of the expansion of $(a + b)^2$, although whether this was derived through a study of geometric forms or by induction with numbers, it is impossible as yet to tell.

A final point of much interest may be mentioned, viz., the fact that the subtractive principle of the Romans is also found in the Babylonian remains.   The Romans spoke of nineteen as "undeviginti," writing it as IXX or XIX, and Professor Hilprecht gives no less than twelve cuneiform methods of indicating the same idea (20–1).   It seems to be certain that the

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* Ein altbabylonischer Felderplan nach Mittheilungen von F. V. Scheil, herausg. u. bearb. von A. Eisenlohr, Leipzig, 1896.

Babylonians applied this principle to 2 as well as 1, and even to higher numbers.

The work is illustrated by numerous drawings of mathematical tablets, and by a series of carefully executed photographic plates.

Altogether there has not appeared since the publication of the Eisenlohr translation of Ahmes such a valuable contribution in the way of source material for the study of ancient mathematics. It is earnestly to be hoped that Professor Hilprecht will continue in this important line, and that he will be able to assist still further in clearing up a number of vexed questions relating to the early mathematics of Mesopotamia. In particular it would be helpful if he could throw some light upon ancient calculation,* upon the Babylonian abacus (if one existed), and upon the relation (if any) between the number names of Phoenicia, Egypt, and Babylon. It is also to be hoped that he may succeed in giving us some information about the mathematics of the Shumeri (?), those non-Semitic inhabitants of the Euphrates valley whose language, the Sumerian, should, in the natural course of events, have influenced the mathematical terminology of much of the ancient world. It is to these people that there seems due some of the first work in mathematics and astronomy, and it is probable that the sexagesimal system itself had birth among them.

DAVID EUGENE SMITH.

OSGOOD'S THEORY OF FUNCTIONS.

THE BULLETIN has received for review both parts of the first volume, now completed, of Professor W. F. Osgood's Lehrbuch der Funktionentheorie (Leipzig, Teubner, 1905 and 1907). Pending the publication of a critical review, which is in preparation, we present herewith the author's own lucid and interesting summary of the contents, translated, with Dr. Osgood's permission, from the preface.

In the first volume of this work it is our purpose to develop systematically the theory of functions upon the basis of the infinitesimal calculus, in intimate contact with geometry and with mathematical physics. The first special developments are

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* His cylinder 25a may, when fully deciphered, contribute something.