

THE THIRTEENTH ANNUAL MEETING OF THE
AMERICAN MATHEMATICAL SOCIETY.

THE thirteenth annual meeting of the Society was held in New York City on Friday and Saturday, December 28–29, 1906, forming a part of the general gathering of scientists in attendance at the meetings of the American Association for the Advancement of Science and the numerous affiliated societies. A noticeable effect of this reinforcement of interest was an enlarged and more widely representative attendance, considerably exceeding that of any previous meeting of the Society. Friday morning was devoted to a joint session with Section A of the Association and the Astronomical and Astrophysical Society. Professor Simon Newcomb presided; seven papers selected from the programmes of the participating organizations were presented before a large audience. The remaining three sessions were taken up with the regular programme, to which were also added several papers from Section A. Unfortunately the limited time did not permit of adequate presentation and discussion of even the most important papers announced, several being finally read by title. The productive capacity of the Society is constantly outrunning the facilities for presentation; a stricter enforcement of the rules regarding the early submission of titles and abstracts of papers, and a rigorous observance of the order of presentation announced on the printed programme are demanded to relieve the congestion and preserve the orderly conduct of the meetings. It has also been strongly urged that to promote intelligent discussion of the papers abstracts should be included in the printed programmes. This would certainly be a desirable feature. Its practical working out would require some increase and redistribution of administrative forces and responsibilities. The whole subject will receive the serious consideration of the Council.

The attendance at the several sessions of the annual meeting included the following eighty-one members of the Society:

Mr. William Betz, Mr. W. C. Brenke, Professor E. W. Brown, Professor W. G. Bullard, Dr. W. H. Bussey, Dr. W. B. Carver, Dr. J. E. Clarke, Professor F. N. Cole, Miss E. B.

Cowley, Dr. H. N. Davis, Dr. W. S. Dennett, Professor T. W. Edmondson, Professor F. C. Ferry, Professor J. C. Fields, Professor W. B. Fite, Professor A. S. Gale, Dr. F. L. Griffin, Miss Ida Griffiths, Professor G. B. Halsted, Professor Harris Hancock, Professor J. G. Hardy, Professor H. E. Hawkes, Professor E. R. Hedrick, Professor C. S. Howe, Professor L. S. Hulburt, Professor E. V. Huntington, Professor J. I. Hutchinson, Professor Harold Jacoby, Mr. S. A. Joffe, Professor Edward Kasner, Professor C. J. Keyser, Professor W. W. Landis, Dr. G. H. Ling, Mr. L. L. Locke, Dr. W. R. Longley, Professor T. E. McKinney, Professor J. McMahon, Professor James Maclay, Dr. Isabel Maddison, Professor H. P. Manning, Professor Max Mason, Professor Mansfield Merri- man, Professor W. H. Metzler, Professor H. B. Mitchell, Dr. C. L. E. Moore, Dr. R. L. Moore, Professor Frank Morley, Professor Richard Morris, Professor Simon Newcomb, Professor G. D. Olds, Professor James Pierpont, Professor R. W. Prentiss, Miss Amy Rayson, Mr. H. W. Reddick, Miss S. F. Richardson, Professor E. D. Roe, Miss Ida M. Schottenfels, Professor Arthur Schultze, Mr. E. I. Shepard, Mr. L. P. Sicheloff, Professor D. E. Smith, Professor P. F. Smith, Professor Virgil Snyder, Professor H. F. Stecker, Dr. R. P. Stephens, Professor Ormond Stone, Professor W. E. Story, Dr. W. M. Strong, Professor H. D. Thompson, Mr. M. O. Tripp, Professor H. W. Tyler, Professor E. B. Van Vleck, Professor J. M. Van Vleck, Professor Oswald Veblen, Professor L. A. Wait, Mr. H. E. Webb, Professor A. G. Webster, Professor L. G. Weld, Professor H. S. White, Professor E. B. Wilson, President R. S. Woodward.

Professor E. B. Van Vleck presided at the afternoon session on Friday, being relieved by Professor Morley. President H. S. White, Professor Morley, and Vice-President P. F. Smith occupied the chair at the Saturday sessions. The Council announced the election of Mr. E. I. Shepard, of Harvard University, to membership in the Society. Eight applications for membership were received.

Owing to recent illness, President Osgood was unable to deliver his address; it will be presented at the April meeting of the Society.

The organization of a new Section, to be known as the Southwestern Section of the Society, was authorized by the Council. It was decided to hold the next summer meeting at Cornell

University. The next annual meeting was fixed for Friday and Saturday, December 27–28, 1907. An amendment to the constitution was adopted by which the Editorial Committee of the *Transactions* are included in the membership of the Council.

Reports were submitted by the Treasurer, Librarian, and Auditing Committee. These reports will appear in the forthcoming Annual Register. The membership of the Society has increased during the year from 504 to 548, including at present 50 life members. The number of papers presented at all meetings during the year, including the preliminary meeting of the Southwestern Section was 176, as against 147 in 1905. The total attendance of members was 350; 192 members attended at least one meeting during the year. The library now contains about 2,500 bound volumes; a list of the journals and of the accessions during 1905–1906 is printed in the Annual Register. The Treasurer's report shows a balance of \$5,195.80 on hand December 14, 1906; of this balance \$2750.06 is credited to the life-membership fund. Sales of the Society's publications during the year amounted to over \$1,500.

The publication of the lectures delivered by Professors Moore, Mason, and Wilczynski at the New Haven Colloquium, September, 1906, has been generously undertaken by Yale University.

At the annual election, which closed on Saturday morning, the following officers and other members of the Council were chosen :

<i>President,</i>	Professor H. S. WHITE.
<i>Vice-Presidents,</i>	Professor HEINRICH MASCHKE, Professor P. F. SMITH.
<i>Secretary,</i>	Professor F. N. COLE.
<i>Treasurer,</i>	Dr. W. S. DENNETT.
<i>Librarian,</i>	Professor D. E. SMITH.

Committee of Publication,

Professor F. N. COLE,
Professor ALEXANDER ZIWET,
Professor D. E. SMITH.

Members of the Council to serve until December, 1909,

Professor G. A. BLISS, Professor M. W. HASKELL,
Professor E. W. BROWN, Professor A. G. WEBSTER.

The informal dinner, always arranged in connection with each meeting of the Society, adds much to the pleasure of these occasions. Despite other distractions connected with the general gathering of scientists, over forty members attended the dinner on Friday evening and passed a few pleasant hours in social intercourse and renewal of old acquaintance.

The following papers were read at this meeting:

(1) Professor S. E. SLOCUM: "The rational basis of mathematical pedagogy."

(2) Dr. F. L. GRIFFIN: "On the law of gravitation in the binary systems."

(3) Professor JAMES McMAHON: "A differential property of the lamellar vector field."

(4) Professor J. I. HUTCHINSON: "A method of constructing the fundamental region of a discontinuous group of linear transformations."

(5) Professor JAMES PIERPONT: "Multiple integrals" (preliminary communication).

(6) Professor OSWALD VEBLEN: "Collineations in a finite projective geometry."

(7) Dr. W. R. LONGLEY: "Some particular solutions in the problem of n bodies."

(8) Professor MAX MASON: "The expansion of an arbitrary function in terms of normal functions."

(9) Professor R. D. CARMICHAEL: "On Euler's ϕ -function."

(10) Dr. ARTHUR RANUM: "On the group of classes of congruent matrices."

(11) Dr. W. B. CARVER: "Sets of quadric spreads connected with the configuration $\Gamma_{n,r}^v$."

(12) Professor C. J. KEYSER: "Circle range transversals of circle ranges in a plane: a problem of construction."

(13) Professor C. J. KEYSER: "Concerning the analytic treatment of geometric involution."

(14) Dr. A. B. COBLE: "A generalization of the plane Hesse configuration."

(15) Dr. A. B. COBLE: "Involutory Cremona transformations."

(16) Professor W. E. STORY: "Denumerants of double differentials."

(17) Professor VIRGIL SNYDER: "Birational transformations of curves of high genus."

(18) Professor T. E. MCKINNEY: "On the continued fractions representing properly and improperly equivalent real numbers in a system of continued fractions depending on a variable parameter."

(19) Professor H. E. HAWKES: "On elementary divisors."

(20) Professor E. B. WILSON: "Rotations in higher dimensions."

(21) Professor EDWARD KASNER: "Systems of extremals in the calculus of variations."

(22) Professor EDWARD KASNER: "The motion of a particle in a resisting medium."

(23) Dr. R. P. STEPHENS: "Note on a system of curves of class n and order $2(n-1)$."

(24) Dr. D. C. GILLESPIE: "On the construction of an integral of Lagrange's equation in the calculus of variations."

(25) Mr. F. R. SHARPE: "The general circulation of the atmosphere."

The following papers from Section A were also placed on the programme of the Society:

Professor G. B. HALSTED: "The sect carrier and the set sect."

Professor HARRIS HANCOCK: "On a fundamental theorem of Weierstrass by means of which the theory of elliptic functions may be established."

Professor G. A. MILLER: "On the minimum number of operations whose orders exceed two in any finite group."

Dr. Gillespie's paper was communicated to the Society through Professor Snyder. Professor Slocum's paper was read by Professor Hancock. The papers of Professor Carmichael, Dr. Ranum, Dr. Coble, Professor Snyder, Professor Hawkes, Professor Wilson, Dr. Stephens, Dr. Gillespie, and Mr. Sharpe were read by title. Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. Professor Slocum's article points out the lack of any general principle of mathematical pedagogy, and the desirability of establishing such a principle. The fundamental principles underlying general pedagogy are first discussed and shown to rest upon the natural law which Herbert Spencer expressed by saying that the genesis of knowledge in the individual follows the same course as the genesis of knowledge in the race. This

leads directly to the historical method of presentation which forms the basis of all scientific modern pedagogy. In the case of mathematics this method applies with peculiar force, being in fact the logical as well as the psychological method of development.

As a suggestive instance of the application of the historical method, Herbart's culture epoch theory is summarized and its merits and defects pointed out. In applying the historical method to mathematics, it is shown that the difficulties encountered in elementary branches of the subject, such as geometry and algebra, arise from a lack of harmony between the culture from which these subjects originated and the spirit of modern civilization. In this connection the culture of the Greeks, and also of the Hindoos, is characterized and contrasted with our own, thus making the nature of the difficulties encountered in these particular subjects clearly apparent. The length of time taken for the historical development of a subject is also shown to be an index of its difficulty, thus affording a means of making a quantitative, as well as a qualitative, estimate of the relative difficulty of various subjects. The various practical attempts to improve mathematical instruction, such as Grube's method of number analysis, the spiral method of instruction, the correlation of courses in the Prussian schools, the Perry movement, etc., are also studied, and shown to be but particular applications of the historical method, thus affording the best inductive proof of its validity. In conclusion, suggestions are made for a detailed application of the method to the elementary mathematical curriculum, based on the theory that the ideal arrangement of courses and subject matter depends simply on obtaining the proper historical perspective.

2. In studying the gravitational law in the binary systems, Dr. Griffin discards the more severe hypotheses hitherto made (*e. g.*, that the force is such as to make the orbit a conic for all initial conditions), and inquires what laws of force satisfy the observational data and the general hypotheses usually admitted. He shows that the only law of force under which the given orbits can be described, and satisfying the conditions: (*a*) the force is a single-valued function of the distance; (*b*) the force permits real orbits in all parts of the plane, is a type including the newtonian as a special case. Also, if the further condition be imposed, that within some circle the force be nowhere an in-

creasing function of the distance, the law of Newton is required.

A study of the trajectories for the more general type shows: that, for all initial velocities below a certain limit, the orbit is stable; and that through every point of the plane there passes in every direction one and but one orbit which is a conic. Various properties of the family of conics are pointed out; and certain exceptions, arising when the bodies are treated as having size, are discussed.

3. Professor McMahon's paper is in abstract as follows: Any lamellar field has the differential property expressed by the equation $d(\log q)/dn = d\theta/ds$, in which q is the length of the vector at a point P of the field, and $d\theta$ is the differential angle through which the vector turns as P moves through a distance ds along the curve of the vector, the other differential dn being taken along the principal normal to the curve just mentioned. A similar theorem holds in any two-dimensional lamellar field; and, in the particular case of a two-dimensional laplacian field, the differentials dn and ds may be taken in *any* two rectangular directions in the plane; this is in accordance with a well known property of two conjugate functions ($\log q$ and θ) in the complex plane. If a laplacian three-dimensional field is symmetric about an axis, it has the same differential property as a laplacian two-dimensional field if the differentiations are performed in a meridian plane.

4. The paper by Professor Hutchinson gives a simple and effective method for constructing the fundamental regions for a finite or infinite discontinuous group of linear transformations on n variables which leaves any given hermitian form unaltered.

5. Professor Pierpont's preliminary communication dealt with the relation of an improper multiple integral to its iterated integrals, in continuation of the author's paper in the *Transactions*, January, 1906.

6. Professor Veblen showed that the most general collineation of $PG(k, p^n)$ is of the form $x'_i = \sum_j a_{ij} x_j^{p^m}$, where $i, j = 1, 2, \dots, k+1$; $m = 0, 1, \dots, n-1$.

7. Dr. Longley considered some special cases of the plane motion of a given number of bodies. It is supposed that the

bodies form an invariable configuration which turns about the center of mass of the system. In the problem of four bodies it was shown that the rhombus is a possible configuration provided (1) opposite masses are equal, (2) the ratio of the diagonals of the rhombus is equal to or greater than unity and less than $\sqrt{3}$. The ratio of the masses is uniquely determined by the selection of the ratio of the diagonals. If a fifth body be introduced at the center of the figure, the same limits upon the ratio of the diagonals hold. A symmetric configuration was defined as one in which the masses are distributed symmetrically with respect to every line joining a body to the center of mass. A few special cases of symmetric configurations were examined. All cases in which the bodies may move in circles can be extended to motion in any conic, the configuration being always of the same shape, but of varying size.

8. In a previous paper Professor Mason gave a proof of the expansion of an arbitrary function in terms of normal functions of a certain differential equation of the second order. In that paper the convergence of the series was established by a method analogous to one used by Schmidt in connection with integral equations. In the present paper this is replaced by a simpler and more direct method.

9. Professor Carmichael's paper appeared in the February number of the BULLETIN.

10. Dr. Ranum's paper is a development of the theory begun in his article on the same subject, to appear in the *Transactions*. Among the theorems proved are the following: (a) Every chief binary group of classes of congruent matrices, mod $(p^{a_i})(i, j = 1, 2; p = \text{prime})$, is simply isomorphic (in a certain particular way) with one and only one (normal) group, satisfying the conditions $a_{11} \cong a_{22}, a_{22} \cong a_{21} \cong 0, a_{11} + a_{22} \cong a_{12} \cong a_{11}$. (b) The great majority of groups of classes of congruent matrices cannot be directly written as linear substitution groups, but every group for which the moduli of any two columns are proportional can be so written.

11. In an earlier paper,* in which the method of treatment was synthetic, Dr. Carver called attention to certain peculiarly related

* "On the Cayley-Veronese class of configurations." *Transactions Amer. Math. Society*, vol. 6, No. 4 (October, 1905).

sets of conics connected with the plane configuration $\Gamma_{n,2}^v$. In the present paper this idea is treated analytically and is extended to the consideration of sets of quadric spreads connected with the r -dimensional configuration $\Gamma_{n,r}^v$. It is shown that, the quadric spreads being given, the configuration is determined. Special cases for $r = 1$ and $r = 3$ are then discussed.

12. Professor Keyser's first paper deals with an important detail in the plane geometry that arises from employing the circle range as primary element. A circle range is the ensemble of circles having two points in common. Two circle ranges having a common circle are transversals of each other. Four arbitrary circle ranges r_1, r_2, r_3, r_4 have in common two and but two transversal ranges, ρ_1 and ρ_2 . These may be real and distinct or coincident or conjugate imaginary. The problem solved is that of constructing ρ_1 and ρ_2 in the case where they are real. The four ranges taken in sets of three determine four semiquadrics of ranges, viz., $Q_{123}, Q_{124}, Q_{134}, Q_{234}$. The transversals of the ranges of a Q constitute a complementary semiquadric Q' of that Q . The four Q 's have ρ_1 and ρ_2 in common. The four Q 's have no common range, but any three of them have one, any two of them two, common ranges. Any Q and its Q' together constitute a hyperboloidal configuration H of ranges, having thus two sets of generating ranges, those of Q and those of Q' . The four H 's have ρ_1 and ρ_2 , but no other ranges, in common. The radical axes of the ranges of a Q envelop a conic section. The like is true of a Q' . If a Q and a Q' belong to the same H , the two conics coincide. The four conics thus associated with the four H 's have the radical axes of ρ_1 and ρ_2 in common. The conics are the intersections of the H 's and the special congruence composed of the lines (infinite circles) of the plane. The conic as envelope is thus a second order locus of infinite circles. In like manner the point conic is the envelope of circle ranges of infinite circles. The four conics may be constructed by using the radical axes of ρ_1 and ρ_2 and those of the given r 's taken three at a time. The problem of constructing ρ_1 and ρ_2 is *logically* the same as that of constructing the transversals of four lines of space, but the construction makes no use in euclidean fashion of any circle configuration analogous to the plane, a fact supporting the hypothesis that the plane need not be employed as element in the three-dimensional constructions.

13. The second paper by Professor Keyser deals with the question of a suitable algebra for geometric involution. Connecting the exposition (for the sake of convenience) with a range of points, the problem is to provide an analytic machinery that shall handle the point pair as element instead of the points of the pair. This is done in a manner analogous to that of Darboux and Klein in the matter of pentaspherical point and sphere coordinates in space. Three point pairs in mutual involution are taken for configuration of reference. The power of a point as to a pair is the product of the distances between the points and the points of the pair. If π_1, π_2, π_3 be the powers of P as to the fundamental pairs, and if $\alpha_1, \alpha_2, \alpha_3$ be the reciprocals of the half distances of the points of the fundamental pairs, then $\sum(\alpha_i \pi_i)^2 = 0$. If (1) $\lambda x = \alpha_i \pi_i$, then (2) $\sum x_i^2 = 0$. Equations (1) and (2) suffice to determine the x_i as a redundant system of point coordinates. An equation of the form $\sum c_i x_i = 0$ represents a point pair and conversely. Two point pairs ξ_i and η_i are in involution when and only when (3) $\sum \xi_i \eta_i = 0$. If the ξ_i be fixed and the η_i variable, (3) defines an involution ξ_i . The ξ_i are also the coördinates of the focal pair. If ξ_i be variable and η_i fixed, (3) defines a pencil η_i of involutions. The η_i are also the coordinates of the two focal involutions of the pencil. Thus is evidenced the bidimensionality of the range in involutions and in pencils (of involutions) and the dual relationship of these entities as elements. The geometry thus arising in the line (or other like space) is quite as rich as and is algebraically identical with the range-pencil geometry of the plane or any similar doctrine. Ordinarily involution appears as a mere detail of projective geometry. That is because, as ordinarily presented, it is but the intersection of the theory here indicated and the usual projective geometry of the plane. In the geometry here in question, every involution contains one infinite pair (composed of a finite point and the infinite point of the line) and two zero-radial pairs (points of pair falling together). If the ensemble of infinite pairs be taken for absolute, the geometry will be euclidean or parabolic. It will be lobachevskian or hyperbolic if the ensemble of zero-radial pairs be taken for absolute.

14. In Dr. Coble's first paper it is noted that two interchangeable plane collineations of period 3, which are not powers of each other, and which have unique fixed triangles, uniquely

define a plane Hesse configuration. An immediate generalization can be made in spaces of $p - 1$ dimensions S_{p-1} , p being any prime number greater than 2. The resulting configuration is made up of $p^2 S_{\frac{1}{2}(p-3)}$'s and $p^2 S_{\frac{1}{2}(p-1)}$'s so situated that the $S_{\frac{1}{2}(p-3)}$'s lie, in sets of p , in $p(p + 1) S_{p-2}$'s; and the $S_{\frac{1}{2}(p-1)}$'s meet, in sets of p , at $p(p + 1) S_0$'s. The collineation group associated with the configuration has an invariant subgroup of order $2p^2$, and the factor group is isomorphic with the linear group $LF(2, p)$.

15. Involutionary Cremona transformations have been divided by Bertini into four classes according as they are reducible by a Cremona transformation to one or another of four "types." The object of Dr. Coble's second paper is to study the general case of a class by using a construction of the involution which is unaltered by Cremona transformation. In the cases considered the dependence of the fundamental points upon each other is given. The invariant curves and allied line loci are treated in some detail.

16. An algebraic form (homogeneous and isobaric polynomial) in any number of variables x, y, \dots and in the coefficients of any number of quantics of any orders that is not altered by such a linear transformation as simply adds an arbitrary constant multiple of x to y may be called an xy -differentiant belonging to the system of quantics, and the property just described may be called the differentiant character xy . In 1854 Cayley published a formula for the number of linearly independent xy -differentiants of any given type that belong to a single binary quantic of any order, and this formula was proved and extended to any number of binary quantics by Sylvester in 1877. The object of Professor Story's paper is to extend Cayley's formula to forms in any number of variables and in the coefficients of any number of quantics of any orders that have any *two* given differentiant characters. The denominants actually given are those for xy - and xz -differentiants, for xz - and yz -differentiants, for xy - and yz -differentiants (which are also xz -differentiants), and for xy - and zy -differentiants, without regard to the whole number of variables involved. The denominant for in- and covariants of any system of ternary quantics is a particular case of that for xy - and yz -differentiants. Incidentally, a number of properties of differentiants are estab-

lished. The general method of determining denumerants here developed is applicable to any number of differentiant characters, and the writer hopes before long to extend his formulas to triple differentiants, including in- and covariants of quaternary systems.

17. By means of plane sections of ruled surfaces, and of special quadric spreads in hyperspace, Professor Snyder proves that when the points of two nonsingular curves are in (1, 1) correspondence they are projectively equivalent. More generally, if a curve c_n has an i -fold point ($i \leq n - 3$) and no other singularity, it cannot be transformed birationally into itself or any other curve of order n except by collineation; similarly, if it have a k -fold point ($k \leq n - 4$) and a double point, or not more than $E[(\frac{1}{2}(n-1))^2] - 2$ distinct double points, $E(k)$ being the largest integer in k .

18. In Professor McKinney's paper the axis of reals is divided into intervals of the types $a) a_i - 1 + \lambda \cdots (a_i + \lambda)$, $a_i > 0$; $b) (a_i - \lambda) \cdots a_i + 1 - \lambda$, $a_i < 0$; $c) (a_i - \lambda) \cdots (a_i + \lambda)$, $a_i = 0$, where a_i is an integer, λ a variable parameter, $0 < \lambda \leq 1$, and the effect of the parentheses is to exclude the inclosed number from the interval. The system of intervals is symmetric with respect to the origin. Every number in the interval is said to correspond to a_i , the representative number of the interval. The defining equation takes the form $x_i = a_i - 1/x_{i+1}$, where x_i , real and not zero, corresponds to a_i .

For all values of λ , $\frac{1}{2}(-1 + \sqrt{5}) < \lambda \leq 1$, the continued fractions representing two properly equivalent numbers are ultimately alike. For $0 < \lambda \leq \frac{1}{2}(-1 + \sqrt{5})$ they are ultimately alike except under specified conditions. In the case excepted the continued fraction representing one of the numbers may, from elements of a certain rank on, be readily derived from that representing the other number. The results for improper equivalence follow immediately from the relation of the continued fractions representing x and $-x$.

19. Professor Hawkes's paper gave a proof of the necessary and sufficient conditions for the equivalent of two families of bilinear forms when the determinant of the family is not zero. The proof depends on the reduction of a matrix to normal forms, in

which appear numbers that are precisely the exponents of the elementary divisors of the characteristic determinant of the matrix.

20. Professor Wilson places the theory of rotations in connection with the general theory of matrices or dyadics from the point of view of Gibbs. This leads to an easy and immediate treatment of such questions as those treated in the case of four dimensions by F. N. Cole, *American Journal of Mathematics*, volume 12 (1890). The treatment given by Professor Wilson leads to the general classification and reduction of n -dimensional pairs in case the matter of reality is regarded as important.

21. Professor Kasner's first paper appears in the present number of the BULLETIN.

22. In recent papers published in the *Transactions* Professor Kasner has investigated the characteristic geometric properties of the system of trajectories connected with any positional field of force. In the present paper the particle is acted upon by such a force and also by a resisting medium. Distinct discussions are necessary for the two-dimensional and three-dimensional problems. In both discussions the case where the resistance varies as the square of the velocity is especially interesting since then and only then it turns out that certain of the properties which hold for free motion hold also in the resisted motion. These are the properties numbered I and II in the papers referred to above.

23. Dr. Stephens showed that, if a point t of a circle be projected at a fixed inclination upon the n Wallace lines which arise from n points t_i of the same circle taken $n - 1$ at a time, then these n projections lie in a straight line whose envelope, as t varies, is a curve of class n and order $2(n - 1)$. If the angle of projection be allowed to vary, there is formed a system of curves whose centers lie on a line. If $n = 3$, Dr. Converse's case results at once; if the points t_i form a regular polygon, we have the case considered by Professor Steggall.

24. If the vanishing of the first variation of two known integrals each containing one dependent variable gives the same differential equation, then by a theorem of Darboux it is possible to write immediately an intermediary integral of this

differential equation. The paper of Dr. Gillespie shows that if two known integrals, each containing n dependent variables, give through the vanishing of their first variation the same system of differential equations, it is then possible to write immediately an integral of this system of differential equations.

25. The general circulation of the atmosphere has been discussed by Ferrel and Oberbeck. The former made no attempt at a complete solution. The latter assumed that the air was incompressible but satisfied Boyle's law. He also surrounded the atmosphere with a spherical boundary of indefinite height. In Mr. Sharpe's solution it is shown that when there is no inequality of temperature between the equator and the poles the air is in both adiabatic and conductive equilibrium. Consequently when there is an increase of temperature from the pole to the equator the temperature still satisfies Fourier's law of conduction and a circulation exists which vanishes at the upper limit of the atmosphere as well as on the earth's surface. This meridional circulation modifies the east and west motion of the air, which in turn changes the pressure distribution and the circulation.

F. N. COLE,
Secretary.

THE DECEMBER MEETING OF THE CHICAGO SECTION.

THE twentieth regular meeting of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY was held at the University of Chicago, Chicago, Ill., on Friday, December 28, 1906. The total attendance was thirty-five, including the following twenty-nine members of the Society :

Mr. G. D. Birkhoff, Professor G. A. Bliss, Professor Oskar Bolza, Professor D. R. Curtiss, Professor L. E. Dickson, Dr. Otto Dunkel, Professor T. F. Holgate, Mr. Louis Ingold, Professor O. D. Kellogg, Professor Kurt Laves, Mr. N. J. Lennes, Dr. A. C. Lunn, Mr. W. S. MacMillan, Professor H. Maschke, Professor G. A. Miller, Professor E. H. Moore, Dr. J. C. Morehead, Professor F. R. Moulton, Dr. L. I. Neikirk, Professor H. L. Rietz, Professor N. C. Riggs, Mr. W. J. Risley, Mr. A. R. Schweitzer, Professor J. B. Shaw, Dr. C. H.