THE ELEVENTH SUMMER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE Eleventh Summer Meeting of the AMERICAN MATHEMATICAL SOCIETY was held in St. Louis, Mo., on Friday and Saturday, September 16–17, 1904. The sessions opened at 10 A. M. and 2 P. M. on each day in Room 1 of the Library building of Washington University, known in connection with the Louisiana Purchase Exposition as the Hall of Congresses.

The following thirty-eight members of the Society were in attendance:

Dr. L. D. Ames, Professor H. Y. Benedict, Professor G. A. Bliss, Professor Maxime Bôcher, Professor A. S. Chessin, Dr. Saul Epsteen, Mr. E. B. Escott, Professor Fanny C. Gates, Professor G. W. Greenwood, Director J. G. Hagen, Professor G. B. Halsted, Professor Harris Hancock, Professor M. W. Haskell, Professor E. R. Hedrick, Dr. E. V. Huntington, Professor J. I. Hutchinson, Dr. Edward Kasner, Dr. H. G. Keppel, Professor James McMahon, Professor H. P. Manning, Professor J. L. Markley, Professor Heinrich Maschke, Professor E. H. Moore, Professor Simon Newcomb, Professor M. B. Porter, Mr. W. H. Roever, Mr. Oscar Schmiedel, Miss I. M. Schottenfels, Professor J. B. Shaw, Professor E. B. Skinner, Professor H. E. Slaught, Professor W. B. Smith, Professor Ormond Stone, Professor J. H. Tanner, Mr. E. H. Taylor, Professor H. S. White, Professor B. F. Yanney, Dr. J. W. Young, Professor Alexander Ziwet.

Professor Henri Poincaré, of Paris, and Professor Gino Fano, of Turin, were also present by special invitation.

In the absence of the respective officers of the Society, Professor Alexander Ziwet was elected chairman and Professor M. W. Haskell secretary of the meeting. The Council announced the election of the following persons to membership in the Society: Dr. H. A. Converse, Johns Hopkins University; Mr. E. L. Dodd, University of Iowa; Dr. Saul Epsteen, University of Chicago; Professor Tullio Levi-Civita, University of Padua; Professor J. C. Lymer, Lawrence University, Appleton, Wis; Professor W. F. Moncreiff, Winthrop College,

S. C.; Dr. Clara E. Smith, Yale University; Professor D. T. Wilson, Case School of Applied Science. Fourteen applications for membership in the Society were received.

A committee, consisting of the President of the Society, Dr. McClintock and Professors E. H. Moore, Stringham and Tyler, was appointed to prepare and report at the October meeting a list of nominations of officers and other members of the Council to be voted for at the annual election in December.

On Saturday afternoon Professor Poincaré accepted an invitation to address the Society and gave an account of certain new theorems on the existence of closed geodesics upon closed convex surfaces. During this session also the chairman introduced to the Society Professor Gino Fano, of the University of Turin, who brought greetings from Italian mathematicians.

At the close of Friday's session an excursion was made to the palace of education, where Professor Hedrick explained the exhibit of the University of Missouri, in particular certain mathematical models constructed by advanced students in his department last year. Of marked interest was a model made by Mr. Ingold to accompany his paper (12) of the list below; this gave in red and blue wire a large number of lines representing real and imaginary points of a real circle. Other models related to geodesic lines, subgroups of the modular group, and analysis situs.

It is to be regretted that the invitation of the committee, to forward early synopses of papers intended for the meeting, was so largely disregarded. Notwithstanding this, debate was in parts of the programme unusually lively, pedagogical questions naturally arousing most discussion.

The following papers were presented at this meeting:

(1) Dr. L. D. Ames: "Supplementary communication on the division of space by a closed surface."

(2) Professor G. A. Bliss: "On the problem of the calculus of variations involving several unknown functions."

(3) Dr. D. R. Curtiss: "Sur certains théorèmes de la valeur moyenne."

(4) Dr. D. R. Curtiss: "Sur la théorie des fonctions hypergéométriques."

(5) Mr. E. B. Escott: "The expression of a quadratic surd as a continued fraction."

(6) Professor Maxime Bôcher: "On certain pseudo-mathematical or logical paradoxes."

(7) Dr. W. B. FITE: "The successive commutators of a group."

(8) M. MAURICE FRÉCHET: "Sur les opérations linéaires."

(9) Professor E. R. HEDRICK: "The necessary and sufficient conditions for the inverse problem of the calculus of variations."

(10) Professor J. B. SHAW: "Composition of a linear associative algebra."

(11) Dr. E. V. HUNTINGTON: "A set of postulates for real algebra, comprising postulates for a one-dimensional continuum and for the theory of groups."

(12) Mr. LOUIS INGOLD: "Real representation of the real and imaginary portions of a plane locus."

(13) Dr. F. S. MACAULAY: "On a method of dealing with intersections of plane curves."

(14) Professor G. A. MILLER: "Note on Burnside's Theory of Groups."

(15) Miss I. M. SCHOTTENFELS: "On a set of generators for certain substitution and Galois field groups."

(16) Professor E. R. HEDRICK: "The differential notation."

(17) Dr. OSWALD VEBLEN: "The fundamental theorem of analysis situs."

(18) Professor HENRI POINCARÉ: "Closed geodesics on a closed convex surface."

(19) Professor H. S. WHITE: "Certain quartic and quintic surfaces admitting infinitesimal collineations."

(20) Dr. J. W. YOUNG: "On the use of hypercomplex numbers in certain problems of the modular group."

(21) Mr. H. S. VANDIVER: "On reduction algorithms for the solution of the linear equation in a finite field."

(22) Professor T. J. I'A. BROMWICH: "The classification of quadrics."

(23) Professor L. E. DICKSON: "Explicit exhibition of all the subgroups of orders the three highest powers of p in the group G of all m-ary linear homogeneous transformations modulo p."

(24) Professor L. E. DICKSON: "Determination of all the subgroups of the group of all binary linear homogeneous transformations of determinant unity in the $GF[p^n]$."

M. Fréchet's paper was communicated to the Society through Professor E. H. Moore. Mr. Vandiver was introduced by Professor Crawley. In the absence of the authors, Dr. Curtiss's papers were read by Professor Bôcher, Dr. Fite's paper by Professor Hutchinson, M. Fréchet's and Dr. Macaulay's papers by Professor Moore, Mr. Ingold's paper by Professor Hedrick, Dr. Veblen's paper by Professor Bliss, and the papers of Professor Miller, Professor Bromwich, and Professor Dickson were read by title.

Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. In the BULLETIN for March, 1904, Dr. Ames published a communication on the theorem of analysis situs relating to the division of the plane or of space by a closed curve or surface. While no fundamental changes have been made in the methods sketched in that paper, certain minor alterations have beenf ound advisable in completing the details of the proof. In particular, the definition of a solid angle has been made to depend upon a certain surface integral, in place of the tentative definition given in the previous article. This, and a few other unimportant changes are explained in the present paper, which is to appear together with the previous paper, in extended form, in the *American Journal of Mathematics.*

2. The paper read by Professor Bliss concerned the problem of the calculus of variations corresponding to the integral

$$I = \int f(x;\ y_1, y_2, \cdots, y_n;\ y_1', y_2', \cdots, y_n')dx.$$

The necessary conditions of Legendre and Jacobi for this problem have been treated by many writers, but usually in a very complicated fashion. Professor Bliss endeavored to establish these as simply as possible by making use of the existence theorems for a set of linear differential equations, and by an extension of a method used by Schwarz for $n = 1$. In addition he gave a proof of the condition (Weierstrass) which becomes necessary if the minimum is to be a so-called *strong* one.

The requirements for a permanent sign of the second variation have also been the subject of many memoirs, but the sufficient conditions for a minimum in the Weierstrassian sense, in particular for a strong minimum, have been discussed only incompletely.* In Professor Bliss's paper, the construction of a field was explained and these conditions derived.

*See Encyklopädie der mathematischen Wissenschaften, II A 8, p. 608 and II A 8a, p. 633.

3. The importance of certain theorems of mean value for functions of a real variable gives in itself an interest to analogous propositions for a complex variable. A number of theorems of this sort are presented in Dr. Curtiss's paper. To give an example of the form these assume, it is found that the familiar equation

$$f(z) - f(a) = (z - a)f'(a + \theta(z - a))$$

has, when $f(z)$ is a function of the complex variable analytic at a, a solution θ within the circle defined by the inequality $|\theta - \frac{1}{2}| < \frac{1}{2}$, provided z is sufficiently near a. This last condition is shown, by illustrations, to be essential. Similar theorems for integrals are developed. This paper will appear in the *Bulletin des Sciences mathématiques.*

4. Dr. Curtiss's second paper, which will appear in the *Annales de l'Ecole normale supérieure,* is a resumé of those portions of his thesis (published in the *Memoirs of the American Academy of Arts and Sciences,* volume 13, No. 1 (1904)) which are devoted to hypergeometric functions without apparently singular points. It is well known that Riemann's celebrated memoir omitted from consideration certain cases, some explicitly, others implicitly. Following Riemann's procedure, a new definition which includes all cases is taken as the point of departure and the chief properties of function families thus defined are developed, the author indicating what changes and additions must be made in order to apply Riemann's methods to the larger problem.

5. Mr. Escott's paper is a continuation of that on the "Expression of the square root as a continued fraction," presented at the April meeting of the Chicago Section of the Society. Any infinite continued fraction which contains a repeating cycle of partial quotients is equivalent to a number which is the root of a quadratic equation. If the number is the root of a pure quadratic equation, *i. e.*, a square root, the continued fraction is of the form $(a: b^*, c, d, \cdots, d, c, b, 2a^*)$, where the non-repeating part consists of a single quotient and the cycle consists of quotients symmetrically arranged with the exception of the last quotient, which is double the first quotient. The stars indicate the cycle. In this paper some of the problems con-

sidered are the determination of the form of the number N when the cycle of partial quotients is given, the solution of Fermat's equations $x^2 - Ny^2 = \pm 1$ (commonly called the pellian equations), also the solution in odd numbers of the related equation $x^2 - Ny^2 = 4$ which arises in studying the relative numbers of properly and improperly primitive classes of reduced forms of determinant N.

6. Professor Bôcher's paper began with a discussion of Russell's paradox concerning the class of all classes no one of which contains itself as an element. It was pointed out that this paradox would seem to invalidate Russell's theory of cardinal numbers in a point which that writer regards as essential. Another paradox was then discussed which is related to a remark attributed by Richard in his book, Sur la philosophie des mathématiques, to Tannery, according to which there exist infinite decimal fractions whose figures proceed according to no law.

7. Dr. Fite defines a second commutator of a group as a commutator formed by associating a commutator with any operator of the group. In general, an ith commutator is one formed by associating an $(i-1)$th commutator with any operator of the group. The ith commutator subgroup is the subgroup generated by the ith commutators. A necessary and sufficient condition that a group be isomorphic with a group of class i is derived. Several relations connecting the order and class of the various commutator subgroups and the class of the original group are derived. Finally, the notion of the successive commutators is made use of to determine a limit to the class of a group of order p^a (p a prime) that contains two subgroups of order p^{a-1} and of given classes.

8. Pincherle has given a development into series for a linear operator U_f applied to a considerably restricted function f. In the present note M. Fréchet, making use of the Fourier series for f, is able to obtain a development of U_f into a convergent series if f satisfies Dirichlet's conditions. A development into a series possessing a generalized limit (in the sense of Cesàro) may be given in case f is *any* continuous function. These results are applied to the discussion of such linear operators as

$$Y_f = \int_0^\pi f(y)K(y)dy,$$

where the function $K(y)$ is integrable in Riemann's sense or in the extended sense explained by Lebesgue in his recent Leçons sur l'intégration.

9. Professor Hedrick's first paper is in abstract as follows: The direct problem of the calculus of variations has never been reduced to the application of a single condition which is at once necessary and sufficient. In the inverse problem, given a two parameter family of curves which satisfy the Jacobi condition inside a certain region, integrals may be found which satisfy the Lagrange differential equation. In order that any one curve of the given family should render the integral selected a minimum, it is sufficient, by Weierstrass's condition, that the second derivative of the integrand with respect to y' should be positive for any point (x, y) on the curve and for any value of y' whatever. But since a curve of the given family passes through each point of the region in any preassigned direction, the necessary condition that every one of the curves of the family render the integral a minimum, by Legendre's condition, coincides with the sufficient condition stated above. Hence that condition, for any point (x, y) inside the region, and for any value of y' whatever, is both necessary and sufficient for the inverse problem.

10. It is shown in Professor Shaw's paper that every linear associative algebra consists of sub-algebras A_{ij} $(i, j = 1, 2, \cdots, s)$. Of these each A_{ii} consists of "direct" units and is of the form $\{Q \cdot P\}$ where Q is a quadrate of order κ_i^2, and P a non-quaternionic algebra of order ρ_i, A_{ii} being of order $\kappa_i^2\rho_i$. The algebras A_{ij} $(i \neq j)$ consist of units whose squares vanish, such that if the product of two units α_{ij}, α_{jk} from two such algebras A_{ij}, A_{jk} does not vanish then $\alpha_{ij}\alpha_{jk} = c_{ik}\alpha_{ik}$, where c_{ik} is a scalar. Also

$$\alpha_{ij}\alpha_{kl} = 0, \qquad (j \neq k),$$

$$\alpha_{ij}\alpha_{jj} = \alpha_{ii}\alpha_{ij} = \alpha_{ij}.$$

Certain theorems are given as consequences of these general theorems. Some of the sub-algebras A_{ij} $(i \neq j)$ may be absent, so that there are s^2, or less than s^2, such sub-algebras.

11. The postulates for real algebra given in Dr. Hunting-

ton's paper may be analyzed into three groups: (1) propositions concerning the relation $<$, which, taken by themselves, form a set of independent postulates for the theory of a one-dimensional continuum or a continuous scale; (2) propositions concerning the operation $+$, which, taken by themselves, form a set of independent postulates for the theory of groups; and (3) a proposition connecting the two symbols $<$ and $+$.

All these postulates are shown to be independent, that is, the list contains no redundancies; and the system which they determine is shown to be unique, that is, there is essentially only one system (namely, the system of all real numbers) in which a relation $<$ and an operation $+$ are so defined as to satisfy all the postulates. The existence of the system of real numbers proves, moreover, the consistency of the postulates.

The postulates for a one-dimensional continuum are the obvious ones (see, for example Schoenflies's "Bericht über Mengenlehre"),* but their independence has not heretofore been established.

The postulates for the theory of groups carry the analysis farther, it is thought, than the earlier sets † given by the writer and by E. H. Moore.

The list as a whole is more satisfactory than the writer's earlier set of postulates for real algebra ‡ in several respects; first, the separation of the postulates concerning $<$ from those concerning $+$ is now complete; secondly, the individual postulates are more nearly simple statements (and are hence more numerous); and finally, no assumption is now made in regard to the existence of any kind of numbers. This last improvement was suggested by a recent paper of Burali-Forti's,§ to which special attention is directed.

Dr. Huntington's paper was presented also to the St. Louis Congress of arts and sciences in the section of algebra and analysis; it will be printed in full in the *Transactions* of the Society.

12. Many representations of the imaginary points of a plane locus have been given, the Riemann surface representation

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* *Jahresbericht der deutschen Mathematiker-Vereinigung*, vol. 8 (1900).

† See *Transactions*, vol. 3 (1902), pp. 485–492, and vol. 4 (1903), pp. 27–30.

‡ Ibid., vol. 4 (1903), pp. 358–370.

§ *Atti della R. Accademia delle Scienze di Torino*, vol. 39 (1904).

being among the best known. It is desirable, however, to have a representation which will more satisfactorily visualize the locus. Ordinary point space being three-dimensional, it is obvious that some other space element must be chosen as the basis of the geometric representation, if the whole picture is to be given in a single ordinary space. A representation is here suggested in which the element taken is a straight line. Since line geometry in space is four-dimensional, this choice is a very natural one. Making a suitable convention regarding the correspondence of number pairs and straight lines, it turns out that the simpler loci are represented by simple configurations, the ordinary appearance of the real portion of the locus being retained.

The coordinate planes may be looked upon as complex fields analogous to the ordinary complex plane, and the relations usually developed with the aid of the complex plane, including the Riemann surface relations, can be reproduced in this new representation.

Models of simple loci may be readily constructed in which the imaginary as well as the real points of the locus are apparent to the eye, and relations which are troublesome in the ordinary representation become self-explanatory. Such a model has been constructed for the intersection of straight lines with a circle and was on exhibition in the Educational building at St. Louis.

13. The paper of Dr. Macaulay is a new investigation of certain theorems which he gave in the *Proceedings of the London Mathematical Society*, volume 31 (1899), pages 381–423, and which were later discussed in a simpler manner by Professor Charlotte A. Scott in the *Transactions.*

Noether's fundamental theorem states that certain conditions are sufficient in order that a polynomial F in two variables x, y may be of the form $Af + B\phi$, where f, ϕ are given polynomials, and A, B undetermined polynomials. These conditions may be thus expressed: Change the origin to any point a, b, *i. e.*, substitute $x + a$, $y + b$ for x, y in F, f, ϕ, and expand in ascending powers of x, y; then, whatever finite values a, b may have, F must be of the form $A'f + B'\phi$, where A', B' are undetermined infinite power series, which are entirely different for different origins. Any origin at a point of intersection of the curves f, ϕ supplies a certain number of conditional equations for the coefficients of F; the conditions which different origins supply are quite independent of one another, and there is no limit to the number of conditions which a single origin may supply. On the other hand, any origin which is not a point of intersection of f, ϕ supplies no conditions; for if f (say) does not vanish at the origin, F/f is a power series, so that F is of the form $A'f$, and therefore of the form $A'f + B'\phi$.

The theorems in question relate to the conditions which are supplied for F in the most general case when the origin is a point at which the curves f, ϕ have multiple points of any order and complexity, and contact of any kind.

14. In Professor Miller's review of Burnside's Theory of Groups (BULLETIN, volume 6 (1900), page 390) an error in regard to a fundamental theorem seems to have been noted too briefly. The theorem relates to the possible types of subgroups in an abelian group and is developed by Burnside in articles 45–47. The first two articles seem free from error, but article 47 begins with the false assumption that the condition

$$\sum_1^t n_u \leq \sum_1^t m_u \qquad (t = 1, 2, \cdots, r),$$

which has only been proved as *necessary* for the existence of a subgroup of type $(n_1, n_2, \cdots, n_s)$, is also sufficient. Hence the developments of this article and the part of Theorem III which depends upon them are unreliable. It is not difficult to see from the results which Burnside gives in the earlier articles that the necessary and sufficient condition for the existence of the subgroup in question is

$$n_t \leq m_t \qquad (t = 1, 2, \cdots, s).$$

The fundamental importance of this theorem seems to demand an explicit statement in regard to this error even if it could escape the notice only of the beginner.

15. Miss Schottenfels's paper determines that the following substitutions

$$C_h\colon\ x_i' = x_{i+1},\ x_k' = x_1 + hx_2 \quad (i = 1, 2, \cdots, k-1;\ h = 0, 1)$$

are generators for (1) all N-ary linear substitution groups of determinant unity, with integral coefficients, (2) all linear and linear fractional groups in the Galois field $[2^1]$.

16. While the differential notation is now presented in an unobjectionable form in many books on the calculus, there still remains a tendency, either on the part of the student or on the part of the author, to depart from the correct definitions immediately after they are given. In Professor Hedrick's second paper the possibility of substituting a consistent and simple derivative notation in the place of the ordinary differential notation is insisted upon, and the advantages of doing so are pointed out. It is shown that the advantages of symmetry, compactness, and clearness may be retained, and that any proof can be followed as readily as in the differential form. The notation suggested involves the introduction of variable parameters in terms of which the variables are expressed.

17. Dr. Veblen's paper contains a proof of Jordan's theorem that a simple closed curve decomposes a plane in which it lies into two regions. The definition of the curve and the proof of the theorem do not involve any metrical hypotheses, *i. e.*, there is no reference to coördinates or to axioms of congruence. If P is any point not on a curve c, then any point C of c which can be joined to P by a broken line not meeting c except in C is said to be *finitely accessible* from P. That the points of c finitely accessible from P are everywhere dense on c is one of the principal lemmas. The paper is to be published in the *Transactions.*

18. Considering an everywhere convex closed surface, Professor Poincaré first observed that the closed geodesics upon such a surface might be divided into two classes according as the number of double points of the geodesics were even or odd, since under a continuous deformation of the surface such double points could be made to disappear, not singly, but only in pairs.

The determination of these closed geodesics depends on the determination of the maxima, minima and minimaxima of a certain function ; in general, therefore, the total number of such geodesics will be an odd number and there must always exist at least one. Professor Poincaré stated as his belief that there are always at least three.

To look at the subject from another point of view, we might ask what is the shortest closed curve on a closed convex surface. This would of course in general be infinitesimal, but not if the curve is to be a geodesic. The (gaussian) total curvature of a closed convex surface is 4π. If the surface be separated into two parts by a closed curve, it may happen that the total curvature of each part is 2π; that this may be the case, the curve must be a geodesic. The converse is true for closed geodesics without double points, and also under suitable modification for geodesics with double points.

The investigation is of importance in connection with the discussion of trajectories in celestial mechanics.

19. Picard's identical differential equation for a quartic surface possessing a linear exact differential of the first kind is

$$\theta_1 \frac{\partial f}{\partial x_1} + \theta_2 \frac{\partial f}{\partial x_2} + \theta_3 \frac{\partial f}{\partial x_3} + \theta_4 \frac{\partial f}{\partial x_4} \equiv 0.$$

Professor White interpreted this geometrically as stating that the quartic $f = 0$ is invariant under the infinitesimal collineation $x_i = y_i + \lambda \cdot \theta_i$ $(i = 1, 2, 3, 4)$. This interpretation leads by inspection method to Poincaré's two quartics of the kind described. The class of quartics and quintics admitting such collineations of the type (1 1 1 1) is examined in some detail. (Much of this work has no doubt been anticipated by Professor H. B. Newson, whose paper, presented at the ninth summer meeting of the Society has not yet been published.)

20. By the modular group Γ is meant the totality of substitutions

$$z' = \frac{\alpha z + \beta}{\gamma z + \delta},$$

where α, β, γ, δ are integers such that $\alpha\delta - \beta\gamma = 1$. If the substitutions of Γ are transformed by the substitution

$$W(z) = \frac{az + b}{cz + d},$$

where a, b, c, d are numbers of a commutative hypercomplex number system E such that $ad - bc$ is neither zero nor a divisor of zero, a group Γ' simply isomorphic with Γ is obtained whose coefficients are numbers of E. Such a hypercomplex representation of Γ may be used to study the subgroups of Γ. If E be the two-unit system whose units e_0 and e_1 satisfy the relations $e_0^2 = e_0$, $e_0 e_1 = e_1 e_0 = e_1$, $e_1^2 = 0$, the substitutions of Γ' in which all the coefficients of e_1 are zero clearly form a group. The subgroups thus defined by various devices W are all cyclical; and in fact all cyclical subgroups of Γ may be defined in this way. Further, the totality of substitutions in Γ' in which all the coefficients of e_1 are integers congruent to zero mod. n (n any integer) form a subgroup. The subgroups obtained in this way are all congruence groups. For different W's Dr. Young finds that it is possible to obtain very simply an arithmetic definition of a large series of the congruence subgroups of Γ in a form apparently never before exhibited.

21. Since any finite field can be represented as a Galois field, the arithmetic significance of the latter is considerable. Mr. Vandiver's paper gives a new algorithm (in reduction form) for the solution of the linear equation in such a field. It has the peculiarity that the same moduli are retained throughout the reduction, and is simpler in theory than the well known continued fraction method, as well as entirely different in character.

In reference to the other fields the residue classes with respect to an ideal prime in an algebraic field are considered in connection with a linear congruence.

22. Mr. J. L. Coolidge has recently given a classification of quadrics in hyperbolic space (*Transactions*, volume 4, page 161), basing his work on Clebsch's discussion of the mutual relations of two quadrics.

Professor Bromwich examines the same problem with the help of the Sylvester-Weierstrass classification of quadratic forms, using invariant factors. It appears that certain of Coolidge's reduced forms can be still further simplified, as they contain coefficients which are not invariants of the two quadrics. The types of quadrics in elliptic and parabolic spaces are then briefly discussed, to show the features in which they agree with those in hyperbolic space; there is also prefixed to the paper a short summary of those results relating to the reduction of quadratic forms which are used in the subsequent work.

23. The first paper by Professor Dickson exhibits all the subgroups of order p^μ, $p^{\mu-1}$, $p^{\mu-2}$ of G, where $\mu \equiv \frac{1}{2}m(m-1)$

is the exponent of the highest power of p dividing the order of G. A fortunate choice of notation gives the results and proofs a more compact and luminous form than is usual in so general an investigation in group theory. Repeated use is made of the theory of commutator subgroups. The paper gives a wide generalization, by different methods of proof, of the results in the author's article in the BULLETIN for May, 1904.

24. The second paper by Professor Dickson forms a preliminary chapter in his investigation, as research assistant to the Carnegie Institution of Washington, on the resolvents for the p-section of the periods of hyperelliptic functions of $2m$ periods. The group for this p-section contains subgroups isomorphic with the binary group Γ of determinant unity. Having determined all the subgroups of Γ, we derive at once the subgroups of the quotient group of linear fractional transformations. As the latter are required for the elliptic modular theory, the present procedure is an instance of mathematical economy.

M. W. HASKELL,
H. S. WHITE.

THE OCTOBER MEETING OF THE SAN FRANCISCO SECTION.

THE sixth regular meeting of the San Francisco Section of the AMERICAN MATHEMATICAL SOCIETY was held on Saturday, October 1, 1904, at the University of California. The following fifteen members were present:

Dr. E. M. Blake, Professor H. F. Blichfeldt, Professor G. C. Edwards, Professor R. L. Green, Professor M. W. Haskell, Professor L. M. Hoskins, Professor D. N. Lehmer, Professor A. O. Leuschner, Dr. J. H. McDonald, Professor G. A. Miller, Professor H. C. Moreno, Professor C. A. Noble, Dr. T. M. Putnam, Professor Irving Stringham, Professor A. W. Whitney.

Major P. A. MacMahon presided during the morning session and Professor Stringham during the afternoon session. During the morning session the following officers were elected for