month of that time, October, studying in Italy, will not be an accomplished Italian scholar, but he will have no trouble in understanding lectures, and in living comfortably in an Italian community.

There is, lastly, the question of the vacations. It is neither necessary nor suitable to dwell here upon the inestimable advantage to a student of spending his vacations in Italy, taking little journeys hither and thither. The serious student will choose his university by what it offers him in term time, not by what he can get out of term. Yet he who decides to come to Italy will realize that he will have the chance, with little trouble and small expense, to add to his special work a wider culture which no amount of mathematical study can give him, nor yet entirely destroy.

J. L. COOLIDGE.

TURIN,
March, 1904.

VECTOR ANALYSIS.

Vectors and Rotors, with Applications. By O. HENRICI and G. TURNER. London, Edwin Arnold, 1903. xv + 204 pp.

Introduction to Quaternions. By KELLAND and TAIT. *Third Edition*, by C. G. KNOTT. London, Macmillan and Co., 1904. xvii + 208 pp.

Vektordifferentiation und Vektorintegration. Von V. FISCHER. Leipzig, J. A. Barth, 1904. iv + 82 pp.

THE first years of this century have seen an interest in vectors and related subjects, such as has never before been observed. This may be due somewhat to the activity and broad spirit of the International association for the promotion of the study of quaternions and allied systems of mathematics; but more probably it is caused by the increasing desire of physicists for a notation which will represent more briefly and more concretely the three dimensional quantities with which they are forced to deal. With this demand on their part the champions of the various systems come forward to press upon us the advantages of their particular system and especially the disadvantages of all others. Felix Klein is at the head of a commission to investigate the relative merits of different methods, and a number of articles in the recent issues of the *Jahresbericht der Deutschen Mathematiker-Vereinigung* indicates that the commission is not failing to secure plenty of advice. It only is to be feared that the commission may attempt to enact a rigid code. That would be unfortunate and certainly futile. No theorizing can determine which of the many systems shall be adopted. Usage is the only judge. It may be hoped that the commission and its advisers will turn their efforts not to saying what shall be used, but to using that which seems to them individually to be preferable. Then usage cannot fail to decide. And we may prophesy that the decision will be for one system in one place and another in another. There is a deal of sense in Heaviside's remark that for the study of quaternions, quaternions is undoubtedly the best instrument. The fact is that all these methods are but special multiple algebras, and when once one becomes accustomed to use multiple algebra he uses that one which best suits his immediate convenience. In this connection may be quoted the broad-minded, farsighted words with which Gibbs closed his vice-presidential address to the meeting of the American association for the advancement of science at Buffalo in 1886. He said: "But I do not so much desire to call your attention to the diversity of the applications of multiple algebra, as to the simplicity and unity of its principles. The student of multiple algebra suddenly finds himself freed from various restrictions to which he has been accustomed. To many, doubtless, this liberty seems like an invitation to license. Here is a boundless field in which caprice may riot. It is not strange if some look with distrust for the result of the experiment. But the farther we advance, the more evident it becomes that this too is a realm subject to law. The more we study the subject, the more we find all that is useful and beautiful attaching itself to a few central principles. We begin by studying *multiple algebras:* we end, I think, by studying MULTIPLE ALGEBRA." Pity it is that this great philosopher of algebra, sole and true successor to Grassmann, did not live to print his later reflections! Pity it is, too, that of the large number of zealots in this field so few have reached Gibbs's first stage, so many study but a single multiple algebra! In this narrowness lies most of the cause of strife between the parties.

We have at present before us three books, each with its individual merits, each written from a different standpoint. Professor Henrici teaches in City and Guilds of London Central Technical College. His students come to him direct from school; for all one can see they know little if any trigonometry. The problem of the teacher is to start their preparation in technical mechanics. Henrici finds vectors the instrument best suited for the purpose. In fact he urges the introduction of the subject into schools to accompany instruction in geometry. Of the book in question the first 48 pages are given to a detailed account of the addition and subtraction of vectors and their multiplication by a scalar. The text is amply illustrated by examples from geometry and statics of a point. The problems proposed for the student are numerous and excellent here as throughout the book. An extended treatment of centers of mass, including the graphical method of finding the center of mass by the aid of the link polygon, forms the second chapter. As an introduction to elementary work in engineering it is admirable. The following chapter takes up the scalar and vector products of vectors. Henrici distinguishes the sort of product by the kind of parentheses used. $(\alpha\beta)$ is the scalar product; $[\alpha\beta]$ the vector. Rotors, treated first as geometrical objects, second as forces applied to a rigid body, are taken up in the fourth chapter. The rotor is defined as a localized vector free to move along its own direction. It is shown that although numerous are the differences between rotors and vectors, yet great is the advantage of using the notations of vectors in treating rotors. The link polygon is constantly employed in connection with the evaluation of resultant rotors and movements. The theory of the loaded beam and of the bending movement is also touched upon. In the sixth chapter we continue the foregoing by applying the theory to stresses in frames. The reciprocal relations between the frame and the stress diagram are fully brought out. Thus these two chapters form a complete introduction to graphical statics. The exercises are for the most part to be solved by means of the drawing board. We should certainly congratulate the students who have the good fortune to begin their technical training with so broad, thorough, and yet so simple a treatment of vectorial and graphical methods.

It has long been admitted that, everything considered, the best introduction to quaternions is Kelland and Tait's. Tait's own treatise is too stiff and unyielding toward the reader. Hamilton's, though easier, is long, and until recently it has been difficult to obtain. If any book has popularized or is to popularize quaternions it is this one under review. Without attempting to speculate concerning the possible future use of quaternions to physicists, we may well agree that, physicist or no physicist, quaternions eminently deserves to be studied for its own sake. For this purpose Kelland and Tait's Introduction is admirably suited — owing doubtless to the extraordinary teaching ability of Kelland. In preparing the third edition Professor Knott has exercised a wise conservatism toward the greater part of the text. He has, however, introduced a great improvement in chapters 3 and 4 by bringing out clearly the true standpoint and the logical development of the Hamilton system. This will be of service to one who is trying to get into the spirit of the algebra. Another valuable change is the greater attention paid to the dynamical and physical applications. Although nothing can really supplant Tait's chapter on this subject in his treatise, the ordinary student of physics will now find Professor Knott's ninth chapter about sufficient to his greatest needs. The only change we could suggest in the work is that the editor keep right on a little further and replace some of the more extended applications to geometry by a treatment of some physical problems such as systems of forces, the geometry of strains, and the propagation of light in crystalline media. But we recognize the delicacy one feels in recasting the work of others and may well be content with what Professor Knott has already accomplished.

Dr. Fischer's work is in no sense a text-book. It is in reality an original monograph serving to develop the ideas of Gibbs. The author thinks, and with considerable right, that space differentiation which is usually represented by ∇ might with better analogy be represented by $d/d\mathbf{r}$. As this idea depends on division, we define the reciprocal of $\mathbf{a}$ to be $\mathbf{a}/a^2$, and $\mathbf{a}/\mathbf{b}$ to be the product of $\mathbf{a}$ into the reciprocal of $\mathbf{b}$. If then V is a scalar function of position in space, $dV/d\mathbf{r} = \nabla V$. This is a vector quantity. The derivative of a vector function V is a linear vector function, that is, a dyadic. The scalar and vector of this dyadic, $(d\mathbf{V}/d\mathbf{r})_s$ and $(d\mathbf{V}/d\mathbf{r})_\times$ according to the notations of Gibbs, are respectively the divergence and curl of $\mathbf{V}$. These the author represents by $d\cdot\mathbf{V}/d\mathbf{r}$ and $d\times\mathbf{V}/d\mathbf{r}$. The work in the text is developed with great care and with ample geometric illustration. It is well worth reading by all who are interested in vector analysis and especially by those who are acquainted with the methods of Gibbs. One may note that in a pamphlet on Vector Analysis printed in 1884, but not published, Gibbs suggested as possible and perhaps preferable the notation $d/d\mathbf{r}$ in place of ∇. This idea he never developed, at least so far as is known, and consequently Dr. Fischer's monograph fills an evident gap in the theory.

E. B. WILSON.

CAMBRIDGE, MASS.,
June 23, 1904.

THE MATHEMATICS OF INSURANCE.

Versicherungsmathematik. Von ALFRED LOEWY. Leipzig, Sammlung Göschen, 1903. 145 pp.

IN publishing this book the "Sammlung Göschen" has certainly followed out successfully its expressed policy of giving to the public a brief, yet clear and up-to-date development of one of the most interesting applications of mathematical theory. While a reader who is unacquainted with the subject of life insurance would find Professor Loewy's exposition somewhat too condensed, anyone with a knowledge of elementary algebra who has some acquaintance with the business aspect of the subject cannot fail to appreciate the value of this little pocket edition which contains in its 145 pages the development of all the important formulæ needed by the actuary.

While one recognizes at once the meanings of many of the words such as Nettoprämie = net premium, Sterblichkeitstafel = mortality table, the significance of some of the German expressions, of which a glossary of 15 follows, is not at all evident. Indeed, a few are not to be found in the average German-English dictionary and their meaning can only be learned from the context.

Zinsfuss = rate of interest,
Zinseszins = compound interest,
Barwert = present value,
Leibrente = annuity,
Erlebensversicherung = endowment,
postnumerando = payable subsequently,
pränumerando = payable in advance,
Rückversicherung = reinsurance,
Bruttoprämie = gross premium,
Prämienrückgewähr = return (of part or whole) of premium,
Rückkaufpreis = surrender value,
Passiva = liabilities,
Aktiva = assets.