SOME RECENT BOOKS ON MECHANICS.*

Vorlesungen über technische Mechanik. Von AUGUST FÖPPL. 4 Bde., 8vo. Leipzig, B. G. Teubner. Bd. 1: *Einführung in die Mechanik*, 2te Aufl., 1900, xiv + 422 pp.; Bd. 2: *Graphische Statik*, 1900, x + 452 pp.; Bd. 3: *Festigkeitslehre*, 2te Aufl., 1900, xviii + 512 pp.; Bd. 4: *Dynamik*, 2te Aufl., 1901, xv + 506 pp.

Einführung in das Studium der theoretischen Physik, inbesondere in das der analytischen Mechanik, mit einer Einleitung in die Theorie der physikalischen Erkenntniss. Von P. VOLKMANN. Leipzig, B. G. Teubner, 1900. xvi + 370 pp.

Quelques Réflexions sur la Mécanique, suivies d'une première Leçon de Dynamique. Par EMILE PICARD. Paris, Gauthier-Villars et Fils, 1902. 56 pp.

FROM the standpoint of the teacher who has to face the problem of introducing students to the elements of mechanics both theoretical and practical, probably the most interesting and suggestive work which has appeared during the last few years, is that of Dr. August Föppl, professor at the school of technology (Technische Hochschule) at Munich. The books are especially noteworthy because Professor Föppl teaches those who are to be engineers — practical men — and yet at the same time insists on teaching in such a manner that his pupils shall acquire not the applications alone but a firm grasp of the general theoretical principles which underlie those applications. This method of instruction unfortunately may not appeal to some in this country who care only for what our engineers can do and not at all for what they may know. But certainly it is much better to teach the thought and theory in addition to the routine and practice, or in conjunction with it. In fact, as Professor Föppl points out, the engineer can not be thoroughly master of his work and capable of handling whatever new difficulties may arise unless he knows the why as well as the how.

What then is the sort of lectures that a thorough scientist finds feasible when addressing a class of engineering students at one of the leading German scientific schools? This question

* Continued from the June number of the BULLETIN.

may be answered by looking into Professor Föppl's four volumes —for happily the author has followed his lectures closely. That the work is found practical and acceptable in Germany is shown clearly by the demand for a second edition within about two years of the time of first publication. In this review we shall lay especial stress on the first and fourth volumes, which are less technical than the second and third and which consequently are more interesting to us here.

The first volume, Introduction to Mechanics, discusses the elementary portions of the kinetics and statics of particles and rigid bodies with or without friction, of the theory of elasticity, and of hydromechanics. Throughout, the object of the author is to give his pupils a general knowledge of mechanical theory such that later the more advanced or special branches will be seen to fit on in their proper places. The language is natural and simple. The only mathematical prerequisite is a fair knowledge of differential and integral calculus. Every attempt is made to relieve the reader from difficulties of presentation in order that he may concentrate his undivided attention upon the new difficulties of the subject itself.

There is one possible exception to this statement. The author throughout all four volumes uses vector analysis wherever it is appropriate. The notations are those which he has previously employed in his Einführung in die Maxwell'sche Theorie der Electrizität. The vectors are printed in heavy faced German type. If we may be permitted to change to English Clarendons, let **a**, **b** represent two vectors and **e** a vector of unit length normal to **a** and **b** upon that side of the **ab**-plane upon which rotation from **a** to **b** appears counterclockwise. Then the scalar and vector products of **a** and **b** are defined as

$$\mathbf{ab} = -S\mathbf{ab} = \mathbf{a}\cdot\mathbf{b} = [\mathbf{a}|\mathbf{b}] = ab\cos(\mathbf{a}, \mathbf{b}),$$

$$\mathfrak{V}\mathbf{ab} = V\mathbf{ab} = \mathbf{a}\times\mathbf{b} = [\mathbf{ab}] = ab\cos(\mathbf{a}, \mathbf{b})\,\mathbf{e}.$$

(The notations are those of Föppl, Sir W. R. Hamilton, Gibbs, and Grassman in order.) Although the author had found in his own experience that vector methods were better suited to pedagogical purposes than cartesian analysis, he had grave fears at first lest the public at large, especially in Germany, should disagree with him and dislike his method. Yet he went boldly ahead and published it. The result was a little surprising to him. Those who used the volumes were pleased, and even the reviewers could not make objections without adding the qualifying statement that: "The presentation of the chief general theorems of mechanics in terms of vectors possesses a simplicity and intelligibility which could not be obtained by the old method of cartesian coördinates." The great popularity of Professor Föppl's work seems to bear out this judgment and to indicate further that the use of vectors certainly does not increase the difficulty of learning mechanics.

What style shall be adopted in presenting elementary mechanics is one of the most difficult questions an author or a teacher must meet. The students' previous training in algebra, trigonometry, analytic geometry, and calculus as it is generally taught has been necessarily quite formal. These mighty algorithms of formal mathematics must be learned so that they may be applied with readiness and precision. But with mechanics comes the application of these algorithms and formal do-by-rote methods, though often possible, yield no results of permanent value. How to elicit and cultivate thought is now of primary importance. There are two opposite methods. First. Give the student a syllabus of the subject in which the general theory, the bare facts, are presented with as little discussion as possible. The student must then supply his own discussion, his own details. He must think the subject over until he sees its meaning. If he has patience, ability, and a little guidance the result is good. Otherwise he gets little or nothing. Second. Enter into a discussion of the subject, touch on this side and on that, show what is known and what is not, point out where definitions are needed, where experiment must be appealed to, and thus by merely dwelling on the different aspects of the subject allow the true significance of it all to become apparent to the student. This method is often very successful in introducing new ideas such as mass, time, and force. It is practically taking them as innate ideas after talking long enough to insure that this innateness has been recognized. But it also has its disadvantages. For unless it is well handled the student is apt to be bewildered by the lack of definiteness, and to lose faith in his ability to work accurately. It may be remembered that we spoke favorably of Professor Gray's Treatise on Physics in that he struck a mean between these two methods.

Professor Föppl, writing for a totally different class of readers, makes an uncommonly successful use of the second—the discursive method. The discussion is half historical, half scientific, such as is found much more elaborately worked out in Professor E. von Mach's Science of Mechanics, and is sure to appeal to one who commences mechanics with his common sense alert. The introduction to Volume I, a short essay on the origin and aim of mechanics, is excellent diet for future engineers and physicists. The author points out what is meant by experience—true, useful experience—and shows how the fact that geometry was given a definitive, logical form as long ago as the time of Euclid may be interpreted not as proving that geometry is not ultimately founded on experience, but rather as indicating how long the experimental stage antedated Euclid. To give mechanics a definitive form is now possible, as the work of Hertz and others shows. Some say that this straightforward logical method of presentation is the *simplest.* That depends on what is meant by *simple.* From the standpoint of the learner the historical crude method appears easier than the logical. Here Professor Föppl touches either rightly or wrongly on a great pedagogical principle. Perhaps it is interesting and not unsuggestive to note that while teachers and authors in our universities are trying to give calculus a logical rigor and mechanics a definitive set of axioms, there is in the meantime a great commotion among teachers of elementary geometry as to whether this logical definitive method of Euclid is at all suitable for beginners. Some scientists of note, Professor John Perry and Mr. Oliver Heaviside, for example, have taken part in the discussion and placed themselves emphatically upon the negative. As far as mechanics is concerned Professor Föppl seems to agree with them.

After the introduction, twelve pages of the text are devoted to general discussion of the ideas involved in particle, inertia, and force. The author does not define; he prefers to discuss. His treatment of a particle is particularly noteworthy. On page 17 we read: "The bodies or parts of bodies which we shall regard as particles may be reasonably small, but must not be very (*i. e.*, unlimitedly) small." The earth is a particle for many problems; a molecule for few. Setting a lower limit for the size of a particle in dynamics is usually forgotten; yet, sooner or later, it is of the greatest importance if students of atomic theories are not to be confused. A particle must generally be regarded either as composed of a large number of molecules or as a small part of an atom.

We might point out further that the common definition of density at a point as

$$D = \frac{dm}{dv}$$

is quite meaningless even in the case of a so-called homogeneous body unless we make an assumption which is never physically correct and which need never be made, namely that mass is a continuous, not to say differentiable function of position. Might it not be better to tell the student, especially one who is to be a physicist, that density and a large number of physical quantities are averages — averages which are taken over a region so large in comparison to the phenomenon averaged that the result is sufficiently stable to be considered as a definite number for practical purposes, but over a region so small with respect to the total region in which the phenomenon exists that the methods of the differential calculus may be employed in expressing and handling the average? Whether it is better to make physical hypotheses which are ultimately untenable in order that in the meantime mathematics may be applied with absolute rigor or to state that the methods of pure mathematics, though strictly not applicable, yield results which within negligible errors agree with the phenomena observed or observable? Professor Föppl seems to believe that discussion made in the right spirit is, all things considered, more satisfactory to practical men than precise definitions made on impossible physical hypotheses.

The law of inertia is explained and illustrated by Foucault's pendulum experiment. The illustration is subtle but striking. The fact that the pendulum will beat on in its own absolute plane regardless of the earth is particularly impressive and quite intelligible to students. Force is then discussed qualitatively; but it is only after the treatment of the motion of freely falling bodies that quantitative definitions of force and mass are given. Experiment is called upon to establish the fact that the acceleration g, due to gravity, is constant at a particular point in the surface of the earth. The force which the earth exerts on a body is called the weight of that body. Weights may be compared and a unit adopted. Mass may be introduced by means of the equation

$$W = mg = m\frac{d^2x}{dt^2}.$$

Finally by making the assumption that any force is essentially like the force due to gravity in that it may be expressed numerically in terms of weight units the author arrives simultaneously at a measure of force in general and at the fundamental dynamical equation for motion in one dimension—that is, the expression

$$F = ma = m\frac{d^2x}{dt^2}.$$

Thus far only rectilinear motion has been treated. The author now proceeds to motion in two or three dimensions, to the parallelogram law, and the principle of the superposition of different motions. The discussion accorded to centrifugal force is perhaps unnecessarily long and is concerned mainly with showing how that force should be regarded as the equal and opposite reaction to the centripetal force, not by any means as a fictive force. Work and energy are introduced and the fundamental theorem that the work done is equal to the change in kinetic energy is proved. The author introduces the principle of virtual velocities and proves it from the parallelogram law. He states, however, that the inverse method of procedure is equally admissible. The principle of virtual velocities may be taken as the fundamental physical axiom instead of the parallelogram law which may then be proved from it. Finally a long treatment of moments and the solution of a number of problems complete the discussion of the dynamics of a particle — the first division of the first volume.

The second division deals with the mechanics of rigid bodies. After elementary kinematical considerations, statics and the principle of virtual velocities are taken up. This part of the work has been regarded with great disfavor by some reviewers, notably by Dr. K. Heun in the first number of the *Zeitschrift für Mathematik und Physik* for the current year. The statement which gives trouble is found on page 143: "Let us give to a body an arbitrary virtual displacement. This displacement can even be finite. It is, however, customary to apply the principle of virtual velocities in the case of a rigid body only to infinitesimal changes of position and we shall consequently derive it for such. The transference to a finite displacement offers no difficulty inasmuch as each finite displacement can be reduced to a sum of infinitesimal changes of position." If by "can even be finite," the author means can in *all* cases be finite, as Dr. Heun seems to imagine, the statement is certainly in gross error; for it makes the principle of virtual velocities applicable only to cases of neutral equilibrium. But we see no reason for not interpreting the statement as meaning can in *some* cases be finite. In fact we are wont to use the principle of virtual velocities, as far as practical applications are concerned, mainly for finding the mechanical advantage and efficiency of machines. When a machine such as an inclined plane, a screw, or a pulley system is working at a constant rate the forces acting upon it must be in neutral equilibrium. We are inclined to believe that in a larger number of the problems in which virtual velocities are useful to the engineering student the forces must likewise be in neutral equilibrium. We might suggest that the author express himself a little more clearly on this point for the sake of those who seem to have misunderstood him. There are other points upon which the author might easily be defended from his reviewers if necessity made a defense advisable.

Before leaving the principle of virtual velocities we wish to ask whether the introduction of that subject into elementary mechanics is to be commended. Almost all English and some continental books practically ignore it. When it is not ignored the treatment given it is usually indefinite and unsatisfactory (Professor Föppl's book forms no exception) owing to the fact that the necessary and sufficient conditions as to rigidity, constraints, internal and frictional actions which must be satisfied in order that the principle be applicable are not clearly stated.

The third division of the volume deals first with the statical, second with the dynamical properties of centers of gravity, and finally with the kinetic energy of a rigid body. From this point of departure the fourth division leads naturally through the undissipated transformation of energy to the fifth division which is a long and practical treatment of frictional or dissipative forces. Sliding, rolling, axial, and string or belt frictions are each discussed carefully with applications.

In the sixth division the student is made acquainted with some of the fundamental ideas of elasticity and strength of materials which he will use in the third volume; in the seventh, with the direct and oblique impact of a perfectly elastic or perfectly inelastic sphere on another sphere or on a plane; in the eighth, with those qualitative and simpler quantitative facts of hydromechanics which are deducible without a great knowledge of either mechanics or mathematics. It might have been better if Professor Föppl had included in the seventh division a discussion of imperfectly elastic impact. There is plenty of room as the volume contains only about four hundred pages.

The second volume, Graphical Statics, is about four hundred and fifty pages long and is divided into seven parts: composition and resolution of forces acting at a point or in a plane, the funicular polygon, forces in space, trusses in a plane, trusses in space, elastic deformation of trusses and statically undetermined trusses, theory of arches.

The inclination toward theoretical in addition to practical considerations is indicated in the treatment given to such subjects as the reciprocal relations between force-plans, the differential equations of a funicular curve under a load anywise distributed, the null system, the long analytic discussion of the theorem that a truss which contains only the necessary number of members and is stable must be statically determined and conversely is stable if it is statically determined for all possible loads. We miss, however, both in this volume and in the next, the interesting theorem that under any given conditions the most economical truss is one which is statically determined. We also happily miss the elaborate plates which generally accompany a work on graphical statics and which add so enormously to the price. The figures Professor Föppl gives are simple, comparatively inexpensive, and incorporated in the text. The object of the author is to give the reader a usable figure and allow him to draw one more accurate if he desires. Students of theoretical mechanics would do well to learn as much graphics as is contained in this work.

The third volume, Resistance or Strength of Materials, is also for the most part as instructive to those interested in theoretical mechanics as to engineers: for the author has not filled his book with masses of detail and tables of constants, but has dealt with the deduction of those theoretical results which must be obtained before the tables are usable. The eleven divisions are entitled; general investigations on stress, elastic deformation and specification of materials, bending of a straight bar, energy of deformation, bars with a curved line of center, bars on a yielding foundation, strength of plane plates supported along their contour, strength of vessels under external and internal pressures, torsional strength, buckling strength, and the elements of the mathematical theory of elasticity. The tendency of this volume, like the others, to develop the theory of the subject would probably be distasteful, if not unintelligible, to our engineers in this country.

Owing to the nature of the subject exact mathematical developments, even when possible, would lead to expressions too complicated for use. Professor Föppl therefore introduces approximations very freely, and in many cases discusses the maximum error due to these approximations. There is one case where we fear his assumptions may not be quite justifiable. In solving the problem of a torsion of a rectangular beam he assumes that the strain is expressible by the first three terms of a power series. He omits, however, to test his results by means of the stress equations. The complete solution is due to St. Venant and is expanded into a series of transcendental terms. The last division gives a satisfactory and not very difficult introduction to the general mathematical theory of elastic isotropic bodies. This is practically the only elaborate addition to the lectures actually delivered by the author to his students. Even this would be given in the regular course, he states, were it not for sheer lack of time!

With the fourth volume, Dynamics, the author returns to more purely theoretical discussions. In fact, the first and fourth volumes, with the last division of the third, form a complete treatise on the elements of mechanics of a particle, a rigid body or a system of rigid bodies, of fluids, whether perfect or not, and of elastic bodies. The titles of the five divisions of this last volume are: dynamics of a particle, dynamics of a rigid body and a swarm of particles, relative motion, dynamics of systems of bodies (Lagrange's equations, etc.), hydrodynamics. The discussion is mainly a completion and extension of that in the first volume.

In the first division, central motion, potential, and harmonic vibrations are treated. The fact that the author does not overlook forced vibrations is noteworthy. Electrical developments of the last fifteen years have caused forced vibrations to grow constantly more and more important. Yet in few, even of the recent text-books, is any mention made of the subject. The treatment of the conservation of moment of momentum in the second division is enlivened by such interesting examples as the fall of a cat, the shrinkage of the earth, the ability to accelerate one's motion while swinging, and the effect on the motion of a ship due to the rotation of the machinery. The motion of a top, impulses applied to a rigid body, and the theory of oscillations are some of the other subjects treated. The third division offers two proofs of Coriolis's theorem, and discusses some applications of relative motion.

In the fourth division that too great carelessness in stating restrictions, which was mentioned earlier in connection with the presentation accorded to the principle of virtual velocities in the first volume, obscures and mars the development of Lagrange's equation and Hamilton's principle. The method in which generalized forces are introduced is hardly the best, hardly convincing. A discussion of mechanical similarity and its applications closes the division. The treatment of hydrodynamics, like that of the mathematical theory of elasticity, is a valuable introduction to the subject. Perhaps they both are more nearly abstracts than introductions. For this includes, in addition to Euler's equations, the motion of a sphere through a perfect fluid, Helmholtz's theorem, the equations of a viscous fluid, and that contains not merely the fundamental equations, but also to a considerable extent the theories of Boussinesq, Hertz and St. Venant.

In conclusion let us state that the value of each volume is materially enhanced by the summary which concludes it and contains a list of the principal formulæ. The third volume has already been published by Gauthier-Villars in a French translation. We could wish that the entire work might be translated into English with the hope that it might stimulate our teachers and students of engineering to be more thorough and complete in their theoretical considerations. But it is doubtful whether the venture would prove much of a financial success. Engineers would think the work insufficiently practical. Physicists would think it too practical. In differing with the former we should call attention to that suggestive little book "Made in Germany," which was published not long since in England for the purpose of setting forth the reasons why Germany was coming so rapidly to the front in all lines of practical endeavor. One of the chief reasons given was the German's thoroughness and determination to know his field from beginning to end. In differing from the physicists we should point out that in those two volumes—the second and third—which they would probably scorn most, the name of Maxwell appears frequently. The presence of that name might aid in convincing students of theoretical and practical physics that they could doubtless find quite as much profit in becoming acquainted with the contents of these volumes as in examining the more abstruse and unpractical parts of their science.

Professor Volkmann's Introduction to Theoretical Physics is a protest against the tendency toward the more or less artificial presentations given to mechanics by Kirchhoff, Hertz, and even Boltzmann. It is an attempt to show how by harking back to Newton and the English method of treating mechanics a greater clearness, a no less deep insight, and with sufficient care a no less scientifically accurate presentation may be obtained. The whole work is such a curious medley of history, philosophy of scientific method, foundations of mechanics, theoretical mechanics, and applications that we are at a loss to know for what sort of readers it can be intended. For those who already do know the subject the applications are not necessary; for those who do not, the philosophy is confusing. The author's explanation to his readers that "You have already busied yourselves to a certain extent with physics and mechanics and thereby commence these lectures with certain ideas and expectations which may be just as much of a help and value as of a hindrance and harm to you," seems on the whole to be of little use in trying to decide who his readers may be. We fear Professor Volkmann has fallen into an altogether too common German habit of writing to please himself.

The first division of the book is an introduction to the theory of physical knowledge and methods of physical thought. There are forty-four pages of octavo text with numerous footnotes referring to Newton, Kant, Lotze, Thomson and Tait, Helmholtz, and especially Volkmann. Then follows a bibliography of twenty-six titles of which ten are to previous articles by the author. To criticise the author's theory which is itself presented in compact form would take too much space. The theory is interesting to read; but how valuable it may be to either a theoretical or practical physicist is not clear.

The second division, about seventy pages, systematically lays the foundations for mechanics according to Galileo and Newton and works out the application to the case of a particle. The author postulates a conception of time, mass, and a universal ether which is to act as a fundamental system of reference. Just what these conceptions are we are unable to determine; but the necessity of postulating them apparently becomes evident after reading the author's discussion, if indeed it were not evident before. Next, the ideas of inertia, vector velocity, vector accelerations, and physical dimensions are introduced and applied to the motion of a projectile in vacuo.

As a foundation for kinetics, Newton's laws are stated and discussed critically. The analytic statement of the laws and the development of the dynamics of a free or constrained particle complete this division of the book. We are unable to ascertain precisely what a particle (Massenpunkt) is. The term does not appear in the index and a definition seems lacking in the text. Fortunately, however, we are told in due season that the postulates of space, time, and mass are: There is a *space* of entirely determinate geometric characteristics in which reality transpires —the euclidean space. There is a *time* which flows on uniformly and is independent of all phenomena. There is a quantity—the *mass*—which preserves its identity unchanged through all changes of phenomena. The word phenomenon has been used in its etymological meaning as a translation for Erscheinung.

The third division develops the mechanics of a system of particles whether the distribution be discontinuous or continuous. Especial attention is given to the general theorems of motion of the center of gravity, rate of change of moment of momentum, and energy. The student ought to be able to get a firm grasp on these fundamental principles and be ready to understand their application to those various vibrating physical instruments which are discussed theoretically in the fourth division.

In the fifth division, after twenty pages of preliminary remarks on fluids and how to treat them, the author settles down to hydrostatics first in its usual aspect of phenomena on the interior of a fluid and then in that part of the subject which takes into account the effects of surfaces of discontinuity and is more often known as the theory of capillary action. The sixth division contains such questions introductory to geophysics as the attraction of spheres, the relation of gravitation and weight, methods of determining the gravitation constant G and the density of the earth.

The discussion of the general principles of mechanics forms the seventh and last division of the work. The author had here an opportunity to collate and compare the various general principles—those of D'Alembert, Lagrange, Gauss, Maupertuis, and Hamilton. This opportunity he accepted. The result can hardly be considered any material improvement over other treatments whether in definiteness or completeness. The volume ends with an index which is an absolute necessity for a book of such varied propensities.

If we have given an intelligible account of the contents of Professor Volkmann's work we shall feel satisfied. To review it connectedly is next to impossible. We had hoped that we might find in it a careful, precise, intelligible and connected treatment of theoretical physics or analytic mechanics. The precision and connection, at least, are for the most part lacking. This very lack, however, furnishes to those who are acquainted with mechanics, a stimulus to thought and thus the book may become more interesting and suggestive than one less indefinite — which is about as much as we can say for it.

Professor Picard's little volume consists of two parts — an essay on Mechanics and the Science of Energy and a First Lesson in Dynamics. The former is merely an extract from the general report on science which the author wrote in connection with the Exposition in 1900. The latter was delivered in 1894 and has hitherto been accessible only in periodicals. That they are now printed together ought to be a cause for congratulation among students and teachers of mechanics.

The first noticeable feature of the work is the perfect, the transcendent clearness of the style. It is unnecessary to inquire for whom the book was written. Any one can read it with ease, pleasure and profit.

The author shows how the idea held at the end of the eighteenth century that mechanics had attained a definite form has proved illusory, and how modern scientists, especially Hertz have been able to reduce mechanics to a more logical set of axioms and a more entirely deductive form. Yet clearly behind a great admiration for Hertz and his work there stands out the evident feeling on the part of the author that something more practical, more anthropomorphic is quite as convenient and valuable.

There is one point which seems a trifle misleading. Upon page 14 we find the statement that dynamics of a particle will probably not become obsolete for the reason that atomic hypotheses play and will perhaps always play a preponderating rôle in science. The reason is not convincing. When an atom is treated as a definite object, it is not necessarily and not usually regarded as a particle. It is either a rigid or a fluid body. A particle, as was pointed out above on page 28 of this review, may be a large aggregate of atoms or a small part of an atom, but not often the atom itself.

In his discussion of what is meant by the possibility of a mechanical explanation of all natural phenomena, Professor Picard points out that some mean the possibility of reducing the phenomena to one type of differential equations or another; some, the existence of a universally applicable principle like that of least action; and some, the construction of a dynamical model, as an example of which Kelvin's model of the ether may be mentioned.

The chapter on the science of energy states the different points of view of two classes of scientists, of which one believes energy to be nothing else that an abstract conception without real existence and of which the other believes in the real existence of different kinds of energy commutable into each other. Particular attention is given to the relation of known experimental results to the various laws of energy. Mention of the law of dissipation affords an opportunity to discuss whether or not that law is compatible with a mechanical explanation of the universe.

Professor Picard's first lesson in dynamics contains numerous departures from other introductory presentations of the subject. His particle is something small enough to be regarded without negligible error as a geometric point. This seems unnecessarily indefinite and yet unnecessarily definite. The important thing is that the motion does not differ appreciably from translation. Whether the particle may be regarded as a point has little to do with it. Inertia is presented in the Newtonian manner. Force is the name for any action which causes motion.

With the introduction of the measure of force there appears the first novelty. The author commences with a *field of force.* In particular, a *constant* field of force is such that particles left to themselves at different points of the field describe congruent paths in the same manner. The acceleration in a constant field is constant and may be taken as the measure of the field. Constant fields, or any fields at a definite point, may be compounded according to the parallelogram law. If force be defined statically by means of any dynamometer, experiment will show that the static and dynamic measures of force are proportional. In the same field, for example in the field of gravity near the earth's surface, the forces acting on two masses are proportional to the masses. The unit of force or mass may therefore be chosen arbitrarily. The lesson closes with the determination of the fundamental differential equation of motion for a variable force and the discussion of work done by a force.

The mental acumen of a student who can understand and appreciate the whole of this, his first lesson, must be considerable. No further comment is necessary on this presentation of the fundamental ideas, which seems sufficiently accurate and modern, yet quite practical, and withal not far from Newton's classic treatment.

We have now passed in review about a dozen recent volumes on mechanics. Many different standpoints are represented, and often represented very well. But what is more noteworthy and valuable is the steady general improvement over older works, not only in the matter presented, but in the method of presentation. There is yet much to be done in producing treatises on mechanics. Perfection has not been reached, nor even nearly enough approached. There is one field in particular in which a thorough and careful work is greatly to be desired. That is the field of general dynamical theory. Sooner or later — let us hope it will be sooner — there must appear a work which shall take such general principles as D'Alembert's, Lagrange's and Hamilton's, and state just what are the restrictions under which each may be applied in practice. For instance, in Lagrange's Mécanique Analytique we find at least four distinct ways of writing the general equations of motion. Each has its peculiar advantages and restrictions. Who will set these forth? There is also plenty of room for a brief, almost syllabic, treatise on dynamical theory. Few students are able to appreciate how extremely limited purely theoretical considerations are in dynamics. Six or eight hours is quite sufficient to carry a lecturer from the beginning through the derivation of Lagrange's equations and Hamilton's principle, if all applications be omitted. And it is only thus that a thoroughly comprehensive grasp of dynamics can be obtained.

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YALE UNIVERSITY,
June 10, 1902.