reason why a different term should be needed when coefficients are replaced by cogrédient coefficients. The reason why the operation is invariant is the same in both cases. As illustrating the degree of elegance attained by M. Andoyer may be cited from § 3, Chapter I, the proof (susceptible of improvement, though not of simplification in form) that absolute invariants exist; and the entire § 2 of chapter III, which sets forth the relation of a homography to a binary bilinear form.

In a text prepared especially to emphasize geometrical applications, the purely algebraic side of any problem is naturally of only secondary importance. Here the student learns to think of invariance as based on reasons rather than on specific normal forms; it is not important for the beginner to reduce the canonizant, for example, to typical form, and it is important to see clearly that every eliminant of a set of equations must be an invariant, regardless of the notation used to express it. This principle leads to a preponderance of reasoning over reckoning, and the postponement of purely algebraic problems. While complete form systems are given for each special stem form, Gordan's theorem is simply stated without proof. Nor is there any mention of Hilbert's theorems (though on p. 101 it is stated that "Le théorem de Gordan subsiste dans le domaine ternaire,") nor of the enumeration problems solved by Sylvester and Deruyts. This marks the elementary character of the work, and leaves much to be looked for from the larger compend which, the preface tells us, the author has had in preparation already for some years.

HENRY S. WHITE.

Sur les lois de réciprocité. Par X. STOUFF, professeur à la Faculté des Sciences de Besançon. Paris, Hermann, 1898. 8vo, 31 pp.

THE laws of reciprocity have been the object of numerous mathematical researches, the chief of which are the memoirs of Jacobi, Eisenstein, and Kummer in *Crelle's Journal.* Stouff proposes to apply n-dimensional geometry to the subject following the examples of Minkowski's geometry of numbers.* At bottom the question of the laws of reciprocity appears to have a natural and close connection with the fuchsian polygons of Poincaré generalized for space of any number of dimensions. The intervention of these polygons appears implicitly in Gauss's memoir on biquad-

* Minkowski, Geometrie der Zahlen, Leipzig, Teubner, 1896.

ratic residues; the works of Stieltjes appertain here also. Certain differences between the various orders of residues explain themselves by difference in properties of spaces of different dimensions. The geometric reasoning of the author presupposes familiarity with transcendental geometry on the part of the reader; where the theorems admit of immediate extension to space of any number of dimensions the exposition is made for five dimensional spaces. It is beyond the scope of this short notice to recapitulate the results, which, in the nature of the subject, resist compression.

E. O. Lovett.

Leçons sur la théorie analytique des équations aux dérivées partielles du premier ordre. Par Étienne Delassus, chargé de conférences à l'Université de Lille. Paris, Hermann, 1897. 8vo, 88 pp.

The theory of partial differential equations has reached a high degree of perfection, but the theory in its present form lacks unity, its various parts resulting from particular properties and remaining almost independent of one another.

Thus the integration of linear systems rests on a partial extension of the theorem of Cauchy; in non-linear systems it is necessary to distinguish two cases according as the unknown function does or does not enter, since the method of Jacobi and Mayer experiences modifications in passing from one to the other; further the method of Lie is demonstrated only for systems in which the unknown does not figure, and is the consequence of a theorem which results from the theory of characteristics or from a method of Jacobi.

The object of the memoir of Delassus is to unify this system of details by determining a canonical form altogether general and establishing a general existence theorem. In the case of systems in which the unknown does not enter, this canonical system coincides with the form in involution, but it is no longer the same when the unknown enters explicitly. Upon this general canonical form Delassus establishes a fundamental existence theorem which he calls for short the generalized theorem of Cauchy; the demonstration of it yields the following theorem which plays a capital rôle throughout the theory: The integration of a system of partial differential equations of the first order having but one unknown can always be referred to that of a single equation of the first order. This theorem was already known for the case where the unknown function is absent, having been demonstrated by Lie by means of the general theory of characteristics and then by Mayer who employed a method of integration of Jacobi.