Mathematicians will be grateful to Professor Rudio for this very complete and attractive report. The book contains so much of general interest that it will be welcome to all.

JAMES PIERPONT.

The Mathematical Theory of the Top. Lectures delivered on the occasion of the Sesquicentennial Celebration of Princeton University. By FELIX KLEIN, Professor of Mathematics in the University of Göttingen. New York, Charles Scribner's Sons, 1897. 12mo, iv + 74 pp.

THESE lectures "were reported and prepared in manuscript form by Professor H. B. Fine, of Princeton University, and the manuscript was revised by Professor Klein." The twelve figures in the text, some of which required great delicacy in construction, were drawn by Professor H. S. S. Smith. The subject matter of the lectures is divided into four parts: the geometric introduction; the dynamic relations; the general analytic discussion when the independent variable is complex; the toy top with moving point.

First of all, the double system of coördinates used in considering the problem of the rotation of a body about an axis with one point fixed is considered, and the attempt is made "to throw a method already well known into the most convenient form for application to mechanics." As the leading expounder today of the ideas of Riemann it is fitting that Professor Klein should use the transformations of the system derived from the stereographic projection of the points of the sphere, rather than the system employed by Euler, or the system which may be designated as the quaternion system. In the beginning the transformations are expressed much more symmetrically in these latter systems, but the problem is not one which is symmetrical, since the body rotates about one axis. And it would therefore seem (as the result shows) that the system of stereographic projection, which considers one line as distinguished from all others, should be superior, and that the expressions for this movement will have a kind of symmetry in this system. Moreover, such a system has been used with great power by French writers, notably Darboux. It is especially emphasized that, in this system of parameters, the parameters, as well as the magnitude ordinarily called the variable, are functions of the time. Since these parameters are complex numbers, and are functions of t, the analytic problem which naturally emerges is to consider the rotation as depending upon a complex variable t. Certain of the properties dependent on this problem are stated in terms of the non-euclidean geometry, but "the non-euclidean geometry has no metaphysical significance here, but is used solely because it is a convenient method of grouping in geometric form relations which must otherwise remain hidden in formulas."

In considering the dynamical question, it is noted that the simpler movement under consideration is a Poinsot movement, and then the equations of Lagrange are introduced. It is only by the inversion of the integrals that the problems can be solved and this has been done by the methods of Jacobi. These considerations will be much simplified by the use of the parameters here introduced, and by the introduction of the Riemann method of conform representation, by means of which the parameters can be expressed as quotients of sigma functions which may be called multiplicative elliptic functions. In the same way the curve on the sphere about the point of the top, which may be called the polhode, is defined by a multiplicative elliptic function, as is also the herpolhode.

By means of the diagram of the stereographic projection of a Riemann surface on the sphere the functions of a complex t are represented, and the determination of this surface is explained by taking a single portion of this surface at one time.

Poinsot was the first to show that the motion of a top on a plane leads to hyperelliptic functions, but in the dynamic problem in hand Jacobi's method of inversion involves many difficulties of calculation. Professor Klein closed these lectures with the introduction of the expressions for the parameters in terms of the automorphic prime functions as these are better adapted to the purpose of inversion than the functions of Jacobi.

In an address before a special meeting of the American Mathematical Society, held in Princeton, on Saturday, October 17, 1896, Professor Klein continued his contribution to the subject in considering the question of the stability of the sleeping top. An abstract of this address has been printed in this Bulletin, January, 1897; 2d Series, vol. 3, pp. 129–132.

H. D. Thompson.