faces of the fourth order ; 11° geometry of the straight line ; 12° ruled surfaces, complexes, and congruences ; 13° differential geometry ; 14° non-euclidean geometry ; 15° geometry of space of n dimensions ; 16° kinematical geometry ; 17° theory of connexes.

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D'OCAGNE'S DESCRIPTIVE AND INFINITESIMAL GEOMETRY.

Cours de Géométrie descriptive et de Géométrie infinitésimale. Par MAURICE D'OCAGNE, Ingénieur des Ponts et Chaussées, Professeur à l'École des Ponts et Chaussées, Répétiteur à l'École Polytechnique. Paris, Gauthier-Villars et Fils, 1896. 8vo, xi+428 pp.

THIS work is an expansion of the course in pure geometry given by the author at the École des Ponts et Chaussées. It proposes to give an exposition of all the geometrical notions which are of interest to engineers, the material not required in the course appearing in small print. As the title indicates, the construction of the book is unique. The author deems it necessary to separate completely that which has to do with the representation of geometrical bodies from that which treats of their intrinsic properties. Accordingly his course is divided into two parts represented in the work by the two distinct divisions : *Géométrie descriptive* and *Géométrie infinitésimale.* The author believes also that the exposition of general doctrines should precede that of the details of a subject and prefaces each chapter with a body of essential principles before examining any particular case ; thus for example, in the theory of surfaces he presents an ensemble of general properties before studying surfaces of a special nature, such as the *surfaces gauches* ; this is contrary to the custom prevailing in similar courses.

1. The first part (two hundred and forty-six pages) of the work includes the first four chapters. In the first chapter, *Projections cotées*, we have the usual details relative to the representation of the ordinary relations between right lines and planes, together with certain problems concerning the round bodies, and the theory of topographical surfaces and profiles.

The second chapter, *Perspective axonométrique*, develops a special mode of plane representation of the bodies of space with a view to the application to be made of it in the theory of shadows. The theory is developed more at length than is necessary for the purposes of the theory of shadows. The axonometric perspective of the plane and that of ordinary space are naturally treated separately. The theory of d'Ocagne encounters a little difficulty in giving a rigorous definition of the apparent contour of a body. The chapter is supplemented by two notes one of which shows the identity of axonometric perspective and oblique projection, and the other indicates the application of this mode of perspective to the representation of the *motifs* of architecture.

The third chapter is occupied with the *Théorie des ombres usuelles*. We remark the careful coördination of the general definitions, methods, and theorems systematically arranged in the first section of the chapter ; the very simple rule in the second section to which the author refers the determination of the shaded parts of a polyhedron ; and the general method of the third section derived from the theory of the preceding chapter for the construction of shadows thrown upon planes. The fourth section applies the method of oblique projection to the study of shadows on surfaces, studying in order cylinders, cones, spheres, and surfaces of revolution. The chapter is amply illustrated, as in fact is the entire volume : three hundred and forty being the number of figures distributed through the text, some of which are duplicated.

The last chapter of the first part is devoted to the ordinary theory of linear perspective. The initial explanation of the object and character of geometrical perspective puts the student on his guard against current misconceptions of perspective. The author's perspective of the sphere and the application of his transformation* to the construction of conical perspective, are worthy of special notice.

2. The second division of the book is prefaced by a preamble whose purpose as stated by the author in his introduction to the volume is twofold : 1° to give certain indispensable explanations relative to the infinitesimal elements in the sequel, in such a manner as to attribute the same rigor to infinitesimal formulæ, *i. e.*, formulæ containing infinitesimals, obtained geometrically, as to those to which we are led by analysis ; 2° to define that which is meant by the geometry of variable forms, independently of every idea of displacement. With regard to this second point the

*See d'Ocagne, *Nouv. ann. de math.*, 3d series, vol. 12 (1893), p. 350.

author maintains by way of illustration that to say that a circle is the locus of points equidistant from a given point or that a circle is a curve described by a point which remains at a constant distance from a given point, is to make one and the same statement. In the one case we figure simultaneously, in the other successively, all the points of the locus.

The fifth chapter, *Courbes planes*, takes as its basis certain fundamental formulæ borrowed from Mannheim,* but stated in new form and demonstrated in novel manner conformably to the point of view taken in the preface. With the exception of the one relative to the centers of curvature of conics, the numerous applications given in the second section are taken from Professor d'Ocagne's original papers: for example, 1° a theorem on the determination of normals,† generalizing the known theorem on the construction of normals to curves and surfaces defined by a relation between the distances of their points to fixed curves and surfaces; 2° the study‡ of the envelope of a chord subtending in a curve an arc of constant length and of that of a chord viewed from a fixed point under a constant angle; 3° the author's idea of associating with a given curve another curve so related to the former that the infinitesimal elements of the first are derivable from infinitesimal elements of a lower order of the second, for example, normals of the first from points of the second, and so on; for a given curve there exists a great number of curves possessing this character; § the one seeming to possess the greatest advantage in the way of applications is d'Ocagne's adjoined || curve of normal directions.

The chapter closes with an application to kinematics¶ leading to the theory of Peaucellier's cell and of the articu-

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* Mannheim, Cours de géométrie descriptive de l'École Polytechnique, 2d edition, pp. 203–205.

† d'Ocagne, *Comptes rendus*, 2d semester, 1889, p. 959; *Nouv. ann. de math.*, 1890, p. 289; 1894, p. 501.

‡ *Nouv. ann. de math.*, 1883, p. 252; 1886, p. 88.

§ d'Ocagne, "Remarques sur la géométrie infinitésimale des courbes planes," *Jour. de math. spéciales*, 1888.

|| In the plane of the curve (M) take two poles O and D; draw the radius vector OM and through the pole D, called the pole of normal directions, a parallel DM' to the normal MN; the locus (M') of the point M' is the adjoined curve of normal directions of the curve (M). See *Amer. Jour. of Math.*, 1888, p. 55; 1892, p. 227; *Bulletin de la Soc. Math. de France*, 1892, p. 49.

¶ d'Ocagne, *Nouv. ann. de math.*, 1881, p. 456; 1882, p. 40; 1884, p. 199.

lated systems† of Hart and Kempe. The author remarks in the introduction that this last application shows how the science of kinematics can be attached to the geometry of variable figures when the latter is conceived without the intervention of the notion of displacement, a conception, in in the author's opinion, more satisfactory from a philosophical point of view.

The sixth chapter studies the theory of *Courbes gauches.* It consists of two sections, one on the general principles and one on the helix. The elements of a space curve, osculating plane, curvature, torsion, osculating sphere, osculating helix, and so on, are presented and applied completely to the general helix. There is a very elegant demonstration given of the theorem that the oblique projection of a circular helix on a plane perpendicular to its axis is an cycloid.

The seventh chapter deals with *Surfaces en général.* The author very properly calls attention to his elementary demonstration of the theorem of Malus in the first section. In studying the curvature of lines traced on a surface in the second section, considering first the sense of the curvature, the indicatrix of Dupin is introduced to determine the changes of sense; considering the magnitude of the curvature; the author establishes first the formula for any curve of double curvature, using the same order of procedure as Jordan adopts in his Cours d'Analyse, and then refers the question, by the theorems of Meusnier and Euler, to the determination of the principal radii of curvature, finally employing Dupin's indicatrix to determine the variations in the magnitude of the curvature. After introducing the notions, axes of curvature and deviation, together with the theorem of Sturm and the formulæ of Bertrand and Bonnet, the author presents an interesting paragraph on the curvature of surfaces including Casorati's recent definition* along with those of Gauss and Sophie Germain. The third and last section attacks lines traced on a surface by defining geodesic curvature and torsion; the fundamental formulæ relative to the latter element are deduced immediately from those relative to deviation derived in the preceding section. By a very elementary process the geometric theory of lines of curvature presents itself and the author establishes Dupin's theorem on triply orthogonal systems in a simple and

* Liguine, Ibid., 1882, p. 153.

† *Acta Math.*, vol. 14, p. 95. Casorati's form is equal to one half the sum of the squares of the reciprocals of the principal radii of curvature; for this reason d'Ocagne proposes that it be called the mean quadratic curvature.

elegant manner. The property of the osculating plane's tangency to the surface is adopted as the defining property of asymptotic lines; their curvature is determined by means of a theorem of Beltrami, the theorem of Meusnier failing for these lines; their torsion is found by a theorem of Enneper. Geodesic lines are defined as those whose osculating plane is normal to the surface; their essential property of minimum length and their analogies with straight lines of the plane are fully indicated. The author suggests that those curves whose points are at a constant geodesic distance from a fixed point be called geodesic circumferences. This change of terminology is worthy of remark; the term geodesic circle has been ambiguously used to designate the curves of constant geodesic distance studied by Gauss and those of constant geodesic curvature first considered by Minding. After an account of curvilinear coördinates and an historical résumé of the details of the theory of minimal surfaces the chapter concludes with a brief summary of the theory of applicability.

The eighth and last chapter of the work is entitled *Surfaces de nature spéciale*. In coördinate geometry surfaces are classified according to the nature of their equations; similarly in infinitesimal geometry surfaces are arranged into certain families with reference to their infinitesimal properties. Of these families the envelopes of spheres and in particular surfaces of revolution are taken up in the first section. The treatment of skew surfaces in the second section is characterized by a careful consideration of the parameter of distribution which results in full rigor, generality, and precision in the delineations of the problems treated. The author employs his method of orthogonal tangents to construct tangent planes to skew surfaces having a plane director. The study of skew surfaces having as director a cone of revolution appears in a new form. Another original stroke in this chapter is the complete determination, by linear constructions, of the indicatrix of every point of a skew helicoid having a cylindrical core. The third section concludes the chapter with developable surfaces treated as a particular variety of ruled surfaces.

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