present article he points out, among other things, the connection between this matrix and the linear substitutions by means of which H and its isomorphic groups may be represented.

In a recent number of *Liouville* Laurent published an "Exposé d'une théorie nouvelle des substitutions,"* in which he proposes to create an algorithm by means of which the general theory of substitutions may be presented. He employs not only the product and powers of substitutions but also their sums and differences. The term *new theory* should perhaps be understood to mean that the author is dealing mostly with facts that have not appeared in treatises. References would seem to have made the article more useful. The author adds: "Ceux qui voudront bien lire les pages qui suivent se convaincront que je n'ai fait qu'effleurer un sujet très vaste."

In closing we would repeat what was stated at the beginning of this report that we have aimed to call attention to only a few of the important recent advances in the theory of groups. In almost all parts of higher mathematics the group theory is continually taking a more prominent position† and it would require a man of riper years and much wider attainments than those possessed by the writer to give a harmonious and extensive account of the marvelous recent progress in this field. If our humble efforts shall be of service to some beginner in leading him to problems whose solution will assist him to penetrate the rich fields of this theory they will be amply rewarded.

CORNELL UNIVERSITY,
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NOTE ON BURNSIDE'S THEORY OF GROUPS.

BY DR. G. A. MILLER.

IT is well known that Professor Cayley published an enumeration of the possible substitution groups whose degree does not exceed eight‡ and that Professor Cole pub-

* Laurent, *Liouville's Journal*, vol. 4 (1898), pp. 75-119; cf. recent articles by the same author in *Nouvelles Annales*.

† Klein-Fricke, Vorlesungen über die Theorie der automorphen Functionen, 1897, p. 1.

‡ Cayley, *Quar. Jour. of Math.*, vol. 25 (1891), pp. 71 and 137.

lished a large supplementary list of these groups.* Under the guidance of the latter the writer examined all these groups and observed that an intransitive group of degree 8 and order 16 should be omitted from these lists and that two transitive groups of the same degree and of orders 24 and 1344 respectively should be added.† No additional errors have been published.

In the recent work of Burnside some of these groups are employed for the sake of illustrations. It seems desirable to explain the three instances where the results of Burnside differ from those of the given lists since this may assist the reader of his valuable work. These differences occur in the enumeration of the imprimitive groups of degree 6, the intransitive ones of degree 7, and the primitive ones of degree 8. We proceed to consider them in order.

While seeking all the possible imprimitive groups of degree 6 our author speaks (p. 182, l. 8) of an imprimitive group of degree 6 and order 12 that contains operators of order 4. It is easy to prove that such a group cannot be constructed. It would have to contain a subgroup of order 6 since it contains two systems of imprimitivity. Hence it would contain only one subgroup of order 3, and, as it could not be Abelian, it would contain 3 subgroups of order 4. Its operators would have to transform these three subgroups according to the symmetric group of degree 3; *i. e.*, the group could contain only one subgroup of order 2 and could therefore not be represented as a transitive group of degree 6.

By omitting "or IV" on p. 182, l. 8 and writing seven in place of eight in the following line the results of our author in regard to the imprimitive groups of degree 6 agree with the older lists and we have just proved that these changes should be made. In seeking the possible intransitive groups of degree 7 which contain 4, 3 as their systems of intransitivity, pp. 163 and 164, our author evidently includes a case in which the constituent of degree 4 is intransitive. It is therefore necessary to omit paragraph VI near the bottom of p. 164 and to write 18 in place of 22 in the first line of p. 165.

Among the primitive groups of degree 8 those containing the Abelian group of order 8 which includes 7 operators of order 2 are of special interest since they include all the solv-

*Cole, *Bulletin of the New York Mathematical Society*, vol. 2 (1893), p. 184.

† Miller, *Bulletin of the New York Mathematical Society*, vol. 3 (1894), p. 242.

able primitive groups of degree 8.* As the group of isomorphisms of this Abelian group is the simple group of order 168† all the groups in question must be subgroups of the triply transitive G^8_{1344} and they must correspond to subgroups of the given group of isomorphisms.

The substitutions of G^8_{1344} that do not contain a given element constitute a G^7_{168} and may conveniently be regarded as representing the given group of isomorphisms. To the two transitive types of subgroups in G^7_{168} there must correspond doubly transitive subgroups of G^8_{1344}. We thus obtain the two solvable primitive groups of degree eight, viz., G^8_{56} and G^8_{168}.

If an intransitive subgroup of G^7_{168} were contained in a primitive group it would be of degree 7 and contain one transitive constituent of degree 3 and one of degree 4. As such an intransitive group could not include any substitution whose degree is less than 4 it would have to include the transitive four group. This is clearly impossible.‡

Hence G^8_{56} and G^8_{168} are the only solvable primitive groups of degree 8 and the two other groups given by our author, p. 210, l. 29, whose orders are $2^5 3$ and $2^6 3$ respectively, are not primitive. Lines 28 and 29 on p. 211 which relate to the same groups should therefore be omitted. The differences between the lists of our author and the earlier lists mentioned above are therefore not due to any error in the latter. It is necessary to add that while we considered these groups in our recent Chicago seminar two of the members, Mr. Grant and Miss Schottenfels, found the first two errors mentioned above.

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* Jordan, Traité des substitutions (1870), p. 398.
† Moore, BULLETIN, vol. 1 (1894), p. 61.
‡ Netto, Theory of substitutions (Cole's edition, 1892), p. 95.