The author states in the preface that his aim is to make his path easier for the beginner by "introducing him to the theory of these series in such a way that he sees at each step precisely what the question at issue is, and never enters on the proof of a theorem till he feels that the theorem actually requires proof." In carrying out this plan he makes use of diagrams and graphical illustrations, shows where possible the *a priori* probability of the theorem in question, and draws many of his illustrations from applied mathematics.

The subject matter is arranged in 5 sections: (1) convergence, (2) series as a means of computation, (3) Taylor's theorem, (4) algebraic transformations of series, (5) continuity, integration, and differentiation of series. There is also an appendix on a fundamental theorem in limits and a table of the more elementary series. Where all is good it is difficult to single out special parts for commendation. Perhaps the most valuable characteristics are the *repeated* insistence on the importance of never forgetting that the sum of an infinite series is really the limit of a sum; the distinct formation of the theorem on the existence of a limit; the distinction between Taylor's series with the remainder and the infinite series with the accompanying remark and proofs that "it is desirable that the form should be applied much more freely than has hitherto been the custom in works on the infinitesimal calculus, both because it affords a simple means of proof in a vast variety of cases and because many proofs usually given by the aid of the latter can be simplified or rendered rigorous by the aid of the former;" and finally the treatment of continuity, etc., by graphic methods in section V. The reviewer cannot agree with Professor Osgood's statement that Taylor's theorem is proved satisfactorily in all good treatises on the differential calculus.

This pamphlet is a valuable addition to the literature on convergence of series and will be of great use to those who are called upon to teach this difficult branch of pure mathematics.

J. HARKNESS.

RECENT TEXT-BOOKS OF THE CALCULUS.

Elements of the Differential and Integral Calculus with Applications. By WILLIAM S. HALL, Professor of Technical Mathematics in Lafayette College. New York, D. Van Nostrand Company, 1897. 8vo, pp. xi+249.

This work is a conventional American text-book of the better sort. In size, appearance, and contents it resembles somewhat Osborne's well-known text-book. In the first few chapters the student's attention is concentrated upon the derivative, and the fundamental formulas are deduced strictly by the method of limits. Differentials are then introduced, being defined as infinitely small increments. The method of infinitesimals is said to differ only in phraseology and notation from that of limits. Among the special features claimed for the work are the introduction of integration immediately after differentiation and before the applications, and the use of the symbol ∂ to denote partial differentiation. In turning over the pages we have noticed several errors or slips. The definition of limit on page 3 states that a variable cannot reach its limit. The author follows Edwards's Differential Calculus and adopts Homersham Cox's proof of Taylor's theorem. That gentleman's first name, however, suffers somewhat at his hands. On page 147 a singular point is defined to have some peculiarity not depending on the position of the coördinate axes; we are then told that the most important singular points are maxima and minima.

Elements of the Differential and Integral Calculus, with examples and practical applications. By J. W. NICHOLSON, President and Professor of Mathematics in the Louisiana State University and Agricultural and Mechanical College. New York and New Orleans, University Publishing Company, 1896. 8vo, pp. xi+256.

This book, while of about the same size as the preceding one, is more suggestive, an attempt being made to introduce some of the more precise ideas of modern mathematics. The differential of a function is defined to be that part of its increment which varies proportionally with the increment of the independent variable. This definition is the one now recognized by most authorities as the most satisfactory. The author, however, denoting the derivative by m_1, writes $\Delta y = m_1 h + m_2 h^2$, a form which appears to involve the existence of the second derivative as well as the first. In fact, in a note (A_7) on the last page of the work he says that such a form implies not only existence but continuity of the second derivative. In another note (A_8) at the end of the work the author criticises the grounds assigned by Byerly and by Rice and Johnson for making $d\,(dx) = 0$. He contends that the differential of dx is zero, because dx as a variable is independent of x. This, of course, is not sound. If a variable y is independent of another variable x, it is true that we may still write

$$dy = \frac{dy}{dx} dx ;$$

but the coefficient of dx is not a partial derivative, and dy, therefore, instead of being zero, is indeterminate. In order that $d\ (dx)$ may be zero, we must assume that dx takes the same value for all values of x. This assumption, however, does not prevent our varying dx from one instant to another in a perfectly arbitrary manner.

Elements of Differential Calculus. By EDGAR W. BASS, Professor of Mathematics in the U. S. Military Academy. New York, John Wiley & Sons, 1896. 8vo, viii+354.

In regard to its logical development and its applications, both analytic and geometric, this text-book is one of the most complete yet published in America. The subject is introduced by a modified method of rates. Some of the author's definitions will sufficiently reveal his procedure.

Any variable which approaches zero as a limit is called an infinitesimal. The measure of the relative degree of rapidity of change of a function and its variable at any state, is called the rate of change of the function with respect to the variable for that state. An arbitrary amount of change assumed for the independent variable is called the differential of the variable. The differential of a function of a single variable is the change that the function would undergo from any state, were it to retain its rate of change at that state while the variable is changed by its differential. The total differential of a function of two variables is the change that the function would undergo from any state, were it to retain its rate at that state with respect to each variable while both variables changed by their differentials.

It follows that Δx is an infinitesimal, but that dx is not. This is contrary to the prevailing tendency which makes $\Delta x = dx$.* We notice two respects in which the author departs without sufficient warrant, we think, from the prevailing usage. He symbolizes the statement that x approaches a as a limit by placing between x and a an arrow pointing toward a. Further in writing partial derivatives,

* Byerly's Differential Calculus (1879) was, we believe, the first text-book in the English language to give this equation as an immediate consequence of the definition of a differential.

he uses the symbol ∂ as a prefix only for functions, retaining d with the independent variables of the denominator.

The Calculus for Engineers. By JOHN PERRY, M.E., D.Sc., F.R.S., Professor of Mechanics and Mathematics in the Royal College of Science, London. London and New York, Edward Arnold, 1897. 8vo, pp. vii+378.

This very unconventional text-book is one of the most interesting and suggestive works that we have ever seen. It consists of three chapters, the first containing 160 pages, the second and third each one hundred pages. Chapters I. and II. are packed full of illustrations drawn from the practical problems of engineering. In the first few pages we find among the subjects discussed: mechanisms, speed, acceleration, kenetic energy, elongation of a spring, voltage of an electric circuit, and work of a steam engine. Maxima and minima follow with applications to strength of beams, economy of fuel, electric currents, indicator diagrams, electric conductors, electric traction, the suspension bridge, and heating surface of a boiler. Next under integration we find: moment of inertia, center of gravity, attraction, strength of thick and thin cylinders, gas engines, elasticity, friction, bending of beams, pressure in fluids, fluid motion, gases, liquids, magnetic field, self induction of two parallel wires, and the thermodynamical laws. Passing to the second chapter, we notice: leakage resistance of cables and condensers, Newton's law of cooling, slipping of a belt, atmospheric pressure, fly-wheel with fluid frictional resistance, electric conductor, coil of an alternator, bifilar suspension, connecting rod, valve gears, beats in music, tides, rotating magnetic field, alternating current power, mechanical vibration, forced vibrations, vibration indicator, natural vibrations, network of conductors, idle currents of transformers, more than a score of other electrical problems, and strength of struts.

A good idea of the plan and style of the book may be obtained from the introductory sentences of Chapter III. We quote: "In Chapter I. we dealt only with the differentiation and integration of x^n and in Chapter II. with e^{ax} and $\sin ax$, and unless one is really intending to make a rather complete study of the calculus, nothing further is needed. Our knowledge of those three functions is sufficient for nearly every practical engineering purpose. It will be found, indeed, that many of the examples given in this chapter might have been given in Chapters I. and II. For the differentiation and integration of functions in general, we should have preferred to ask students to read the regular treatises, skipping difficult parts in a first reading and afterwards returning to these parts when there is the knowledge which it is necessary to have before one can understand them. If a student has no tutor to mark these difficult parts for him, he will find them out for himself by trial.

"*By means of a few rules* it is easy to become able to differentiate any algebraic functions of x, and, in spite of our wish that students should read the regular treatises, we are weak enough to give these rules here. They are mainly used to enable schoolboys to prepare for examinations and attain facility in differentiation. These boys so seldom learn more of this wonderful subject, and so rapidly lose the facility in question, because they never have learnt really what dy/dx means, that we are apt with beginners to discourage much practice in differentiation, and so err, possibly, as much as the older teachers, but in another way. If, however, a man sees clearly the object of his work, he ought to try to gain this facility in differentiation and to retain it. The knack is easily learnt, and in working the examples he will, at all events, become more expert in manipulating algebraic and trigonometric expressions, and such expertness is all-important to the practical man.

"In Chapters I. and II. we thought it very important that students should graph several illustrations of

$$y = ax^n, \quad y = ae^{bx}, \quad y = a \sin (bx + c).$$

So also they ought to graph any new function which comes before them. But we would again warn them that it is better to have graphed a few very thoroughly, than to have a hazy belief that one has graphed a great number.

"The engineer discovers himself and his own powers in the first problem of any kind that he is allowed to work out completely by himself. The nature of the problem does not matter; what does matter is the thoroughness with which he works it out."

The author is sometimes quite forcible, for example: "Now surely there is no such great difficulty in catching the idea of a limiting value. Some people have the notion that we are stating something that is only approximately true ; it is often because their teacher will say such things as 'reject 16.1 δt because it is small,' or 'let dt be an infinitely small amount of time' and they proceed to divide something by it, showing that although they may reach the age of Methuselah they will never have the common sense of an engineer." For a specimen of just this kind of work, see the deduction of

$$\delta p = \left(\frac{dp}{dt}\right) \delta t + \left(\frac{dp}{dv}\right) \delta v$$

on page 138 of the book under review. Again, we read: "In some of the following integrals certain *substitutions* are suggested. The student must not be discouraged if he cannot see why these are suggested; these suggestions are the outcome of, perhaps, weeks of mental effort by some dead and gone mathematician. Indeed, some of them are no better than this, that we are told the answer and are merely asked to test if it is right by differentiation."

The book was constructed in part from lectures which the author delivered at a night school. In several places it contains traces of the excitement under which he labored in his anxiety to make his point. The intelligent reader, however, will have no difficulty in recognizing such places and in accepting his statements at their proper discount.

THOMAS S. FISKE.

ERRATA.

OUR attention has been called to several errata occurring in previous numbers of the BULLETIN. In Mr. Macaulay's article, "Newton's theory of kinetics," published in the number for July, 1897, of the last volume, the following corrections should be made:

p. 364, line 28,	*for*	in	*read*	is
" " 29,	"	is	"	as
" " 44,	"	equally	"	equably
p. 366, " 6,	"	equal	"	great.

In Dr. McClintock's article "On a solution of the biquadratic which combines the methods of Descartes and Euler," contained in the same number, a correction should be made:

p. 389, line 24, *for* $4rv = v(p + v^2) - q^2$
read $4rv = v(p + v)^2 - q^2$.

In the report by the Secretary "Fourth Summer Meeting of the American Mathematical Society," contained in the BULLETIN for October, 1897, a misprint occurs:

p. 8, line 41, p. 9, lines 1, 2, 3, 8, } in the formulæ, *for* e *read* ε.