RECENT TEXT-BOOKS OF GEOMETRY.

Elements of Geometry. By GEORGE C. EDWARDS, Associate
Professor of Mathematics in the University of California.
New York, The Macmillan Company, 1895.   8vo., pp.
xvi + 293.

Plane and Solid Geometry. By WOOSTER WOODRUFF BEMAN,
Professor of Mathematics in the University of Michigan,
and DAVID EUGENE SMITH, Professor of Mathematics in
the Michigan State Normal School.   Boston, Ginn & Co.,
1895.   8vo., pp. ix + 320.

Plane and Solid Geometry.   Suggestive Method.   By C. A.
VAN VELZER, Professor of Mathematics in the University
of Wisconsin, and GEO. C. SHUTTS, Professor of Mathe-
matics in the Wisconsin State Normal School.   Madison,
Wis., Tracy, Gibbs & Co., 1894.   8vo., pp. viii + 395.

These geometries are written by authors whose attain-
ments prepare them for careful and thorough work.   The
author of the first is associate professor of mathematics in
the University of California, and the remaining authors are
professors of mathematics in the Universities and Normal
Schools of the States of Michigan and Wisconsin, respec-
tively.   This combination of joint authorship in each of
these States shows that the two great schools of each State
work in harmony with each other.

1. Professor Edwards has followed an unfortunate plan
of logical development which mars an otherwise excellent
and original work.   Several popular text-books in the
United States employ the directional method of defining a
straight line, and apparently get rid of or deduce Euclid's
axiom of parallels from it just as Professor Edwards has
done.   The authors of these books do not seem to be aware
of the startling position in which they have thus placed
themselves.   The directional definition accomplishes no
more for the straight line than Euclid's or others that have
been employed, and simply makes straight lines such lines
that, if two of them are placed with any two points in
coincidence, they will coincide throughout.   A hemispher-
ical surface furnishes a moderately good analogue of a non-
Euclidean plane.   Suppose the physical conformation of the
light medium were such that rays of light traveled in arcs
of great circles on this surface and were absorbed at the
boundary.   Then, according to the directional definition,
arcs of great circles would be straight lines, and all that can
be logically derived from the definition holds for this spher-

ical surface. In particular, Professor Edwards's proof that the sum of the exterior angles of a triangle is four right angles, holds word for word for spherical triangles, and hence proves that the sum of the angles of any spherical triangle is exactly two right angles. According to the Autocrat of the Breakfast Table, "logic is logic," and the authors of the directional method have given us a new phase of it equal in every respect to the logic of the wonderful "one-hoss-shay."

Professor Edwards's book contains many noteworthy features. What may be called the descriptive method of proof that is common in higher mathematics is adopted throughout the work, all formal proofs being discarded. This method has many advantages. It offers opportunities for bringing out the leading lines of thought and methods of attack, and of making clear to the student those parts that are most important and that require greatest emphasis. The examples, explanations and illustrations are excellent. The error in logical development will, in the hands of a good teacher, prove of educational value in bringing out the points I have named, and will serve to forcibly illustrate the exceeding delicacy of logical reasoning and the care that every student must exercise in avoiding whatever seems plausible until assured that it has a sound logical basis.

2. Professors Beman and Smith have written a geometry that has many merits. The plan of development is essentially that laid down by the Association for the Improvement of Geometrical Teaching, of England. The book introduces many modern ideas. Thus, the principle of duality is introduced in connection with the first propositions of Book I., and the student is shown how to use it in pairing his propositions and in obtaining suggestions of new propositions. The principle of symmetry is also introduced early, and central and axial symmetry are shown to be dually related. The principles of continuity and of positive and negative relations are also explained in the first book. These are all important and modern principles of geometry. The properties of similar figures are presented in such a way as to lead naturally into the properties of perspective. The logic is generally good and clear. The construction of formal proofs is particularly emphasized, and large lists of examples are furnished for original exercise. The student who uses this book will have better conceptions of modern geometrical methods and principles than can be obtained from the usual text-books. In the writer's opinion elementary text-books should go even far-

ther than the authors have done and not stop short of enabling the student to handle the compass and rule in the modern way.

3. The geometry of Professors Van Velzer and Shutts is characterized by their phrase, "suggestive method." In general style and arrangement it is very much after the pattern of Chauvenet's geometry. Although the authors give credit in this respect to the revised edition of Chauvenet, they have very properly avoided the directional method which renders that revision considerably inferior in point of logic to the old edition. This book seems to be an excellent successor of that much-valued treatise. The important innovation of the suggestive method consists in giving, in connection with each proposition, suggestions as to proof, which the student is expected to expand into a formal proof after the pattern of a few "models" that are given in each book. The idea is an excellent one, and cannot fail to develop geometrical insight and originality. The regular sequence of propositions is supplemented by lists of original exercises.

It will be seen that each of the above text-books embodies new and distinctive features. The first and second make use of algebraic analysis much more extensively than has hitherto been common in works on elementary geometry. With the exception of the errors due to the use of the directional method by Professor Edwards, they are all superior to text-books that are in common use, in which numerous errors of this kind are often contained. The logical development of the foundations of a subject is a very delicate matter and requires the most careful and diligent thought of able minds to clearly elucidate it. Elementary writers and students cannot, therefore, always understand the necessity for this or that logical arrangement, since the amount of work and study that has been put upon it does not show upon the surface. A correct line of demonstration may be extremely difficult to elementary students because the whys and wherefores of it extend far beyond their depth, but this furnishes no excuse for presenting them with a false line of demonstration whose plausibility is less exacting upon their mental powers.

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