CATALOGUE OF THE ASTRONOMISCHE GESELLSCHAFT.

Catalog der Astronomischen Gesellschaft. Erste Abtheilung. Catalog der Sterne bis zur neunten Grösse zwischen 80° nördlicher und 2° südlicher Declination für das Aequinoctium 1875. Drittes Stück, Zone + 65° bis + 70°, beobachtet auf der STERNWARTE CHRISTIANIA. Viertes Stück, Zone + 55° bis + 65°, beobachtet auf den STERNWARTEN HELSINGFORS und GOTHA. Vierzehntes Stück, Zone + 1° bis + 5°, beobachtet auf der STERNWARTE ALBANY. Leipzig, 1890. 3 vols., 4to.

ASTRONOMERS frequently need the positions of so-called fixed stars. They are wanted when a clock is to be regulated to true sidereal or true mean time; when, again, the astronomer is on his travels and desires to fix his latitude and longitude, and the direction on the earth of his meridian; or when he is observing some planet's or comet's course, and wishes to settle the various right ascensions and declinations it occupies from day to day and hour to hour, in order from them to calculate its orbit, predict its future course, and test the law of gravitation.

Thus accurate star places are the basis in one sense of all astronomy of position; but they have an interest of their own which is more prominent now than it ever has been, and is yearly increasing.

For no star is absolutely fixed; and the small motions of the stars which have been long detected are slowly accumulating their effects, and giving evidence that will in time throw much light on the structure of the universe.

The star which has by some been called the "runaway" (Groombridge 1830) moves over half a degree, as seen from the earth, in about two centuries and a half; so that a sharp-sighted observer in a dry mountain region (where the air is transparent enough) could readily detect its motion with the naked eye in about a life-time. Other stars move so slowly, in appearance, that a hundred years of the closest telescopic observation are necessary to detect any slight deviation from their former position. The constellations exhibit to the eye substantially the same appearance from century to century; it is only in very small details that they seem to alter during fifty years.

The study of these trifling motions (as they seem to us) is extremely fascinating to those who have undertaken it; and a vast amount of human effort has been spent in the acquisition of this form of knowledge.

The Alexandrine Greeks did something in mapping and listing the stars, by such simple devices as they possessed;

the Tartar prince Ulugh Beigh had an observatory at Samarcand devoted to the same object; Tycho Brahe the Dane and Hevelius of Dantzic added to the stock of good observations of star-places ; but the earliest work which is now of much scientific value was done at Greenwich by the English Astronomers Royal. Flamsteed, who labored in the latter part of the seventeenth, and Bradley, who held his office near the middle of the eighteenth century, were eminent in their time; and Bradley especially did more to make precise observations than any one who had preceded him. With instruments of clumsy build, and inferior in power to those which are now carried from place to place over our Far West to fix a basis for our maps, he succeeded in putting this branch of astronomy upon a solid foundation. It was not till quite lately, however, that his observations were made fully available ; for Bessel, whose immortal *Fundamenta Astronomiæ* appeared in 1818, reduced only a part of them, and the results which he obtained lacked the minute accuracy which is now needful and possible. With all these drawbacks Bessel's study of a part only of Bradley's work gave the science a prodigious impulse.

Since Bradley's time the art of observation has progressed very greatly. He was hampered by the intractable qualities of matter; his telescopes were small and unachromatic, his divisions roughly made, his means of reading them crude ; but his capacity for practical astronomy unrivalled ; *incomparabilis* Bessel calls him.

The instrument-makers Graham and Bird were succeeded in England by Ramsden and Troughton ; in Germany by Reichenbach and the eldest Repsold. The opticians who made single lenses were distanced by Dollond with the achromatic object-glass, which Fraunhofer improved and nearly perfected. The telescope-tubes became shorter, retaining the same power; object-glasses of greater and greater aperture became practicable with moderate dimensions in the machinery. The micrometer-microscope was used both in dividing and reading off divisions, so that a circle of 10 inches in diameter is now as good as or better than a quadrant of 8 feet radius; and the spirit-level of a few inches in length is far more accurate and trustworthy than the plumb-line running through two stories of a house. In fact the mechanical construction of astronomical instruments has become a true fine art.

Astronomical observation of the first order now requires a subtle psychological analysis ; the human brain and body have become tools for the mind to investigate and employ ; just as we search into the minute errors of division which the finest workmen leave in our instruments—errors thought too large if a line is misplaced by a twenty-thousandth of an inch—so must we inquire most scrupulously and carefully into the possibility that our senses may lead us astray when we are under the completest strain of attention.

After 1790 efforts were made, in several directions, to catalogue the stars. Piazzi at Palermo, and Bradley's successors at Greenwich, spent most of their labors upon the brighter stars; those, some ten thousand in number, which are visible to the naked eye, or approach such visibility. Lalande at Paris undertook the gigantic task of observing all that his telescopes would reach. Fortunately for him, he had but small instruments. He succeeded in observing over forty thousand. Bessel, after completing his reduction of Bradley in 1818, obtained in 1819 a meridian circle capable of dealing with all ninth-magnitude stars not below or too near his horizon; and did all that was in one man's power towards cataloguing the stars, nearly 150,000 in number, which come into this category. He confined himself to less than one-half the sphere, or about two-thirds of his range of visibility, and took as many stars into his catalogue as he could observe. His scholar, Argelander, was at first his assistant at Königsberg; but was promoted to the observatory at Abô in Finland, afterwards removed to Helsingfors, and after excellent service there in another direction went to Bonn on the Rhine about 1840. There he extended Bessel's Zones in substantially Bessel's way to the neighborhood of the North pole (the immediate vicinity of this point had already been surveyed by Schwerd at Speyer), and afterwards from the southern point reached at Königsberg towards the southern horizon. This work has since been continued by our eminent countryman Gould to the South pole, on a truly gigantic scale, and with a still greater approach to completeness.

But in 1852 Argelander took a new departure. Up to this time astronomers had generally used their instruments of observation in searching for the stars. They are somewhat ill-adapted to this purpose. They bear high powers, and their fields are small, and illuminated, so as to make visible the spider-lines on which the bisections are made. All these circumstances cause the loss of many stars which are missed in the process of sweeping. So Lalande, with a small meridian instrument, picked up many stars afterwards overlooked by Bessel and Argelander; while they found many which Lalande ought to have been able to see. Nay more: Argelander had spent his leisure at Bonn, while his observatory was in building, before he had even a temporary shed for his transit instrument, in making a catalogue of naked-eye stars. In this there were some 40 of the fifth and sixth magnitude, which had never been seen through any telescope, so far as records indicated.

His new plan was to make a working list of stars to the ninth magnitude inclusive, by a survey of the heavens (the celebrated Bonner Durchmusterung), to include stars to the tenth. In this survey no special pains were taken to get accurate places ; the aim simply was to locate every such star nearly enough to identify it and map it somewhat roughly on a chart. A small telescope of three inches aperture was used with a field lighted only by the stars themselves, and a painted scale visible in this manner to replace the fine spider-lines of the more accurate instruments. With this apparatus a star could be roughly observed every four seconds, with a student-assistant to watch the clock-timing of passage ; while in the whole work the number of stars averaged seven or eight to the minute.

This observation was done twice for every part of the Northern hemisphere, and down to 2° south of the equator, between the years 1852 and 1859; and gave a catalogue of 324,198 stars, accurate enough to find them. Later Argelander's assistant, Schoenfeld, who did a great share of the actual work, extended it to the parallel of 23° south: leaving the continuation to the South pole to be effected by Dr. Gill at the Cape of Good Hope.

From this greatest of all star catalogues in size the stars whose magnitude was 9.0 or brighter were selected for more deliberate and precise observation. There were more than 100,000 of them ; and the observations have been now almost entirely completed.

The work was accomplished according to a plan formulated by Argelander in 1868. It was to be done, as Lalande, Bessel, and Argelander himself had previously worked, in zones bounded by parallels of declination. The rule was made that each star was to be *twice* observed, and with more pains and less hurry than had been possible in the previous zone observations. Thus the whole labor was beyond the powers of one astronomer, and it was divided among a number. The first year or two saw beginnings in Helsingfors (Finland), where Krueger, another collaborator and son-in-law of Argelander, labored ; in Kazan and Dorpat (Russia), in Christiania (Norway), in several German observatories, at Cambridge in England, Chicago, and Cambridge in Massachusetts. Various circumstances interrupted, the Chicago fire and consequent financial ruin of the establishment, and the call upon some astronomers in Europe for service thought more practical. The final arrangement of zones has been as follows :

80° to 75°,	Kazan.
75 to 70,	Dorpat.
70 to 65,	Christiania.
65 to 55,	Helsingfors and Gotha.
55 to 50,	Cambridge, Mass.
50 to 40,	Bonn.
40 to 35,	Lund, Sweden.
35 to 30,	Leyden.
30 to 25,	Cambridge, England.
25 to 15,	Berlin.
15 to 5,	Leipsic.
5 to 1,	Albany.
1 to —2,	Nicolaief.

The space around the North pole had been, meanwhile, again surveyed in the most thorough manner by Carrington and others, so that the limits here given covered the necessity in the Northern hemisphere. German observatories have done nearly half the work, the remainder being divided in somewhat unequal proportions between Russia, America, Scandinavia, England, and Holland. The great Russian observatory of Pulkova furnished the indispensable fundamental catalogue. Of course with equal breadth the polar zones are the smaller; thus of the three catalogues already published, Christiania (5° wide) contains 3,949 stars, Helsingfors (10° wide) 14,680, and Albany (only 4° wide) 8,241. This makes, in less than one-fifth of the Northern hemisphere, or one-tenth of the whole heavens, a grand total of 26,870; but some hundreds of these may be duplicates (between Helsingfors and Christiania), as the zones are made to overlap at the edges.

The limits of the present article are too narrow to enter into the technical details of the observations and reductions. The admirable introduction by Professor Boss to the Albany zone (printed in English) can be referred to as the best account of Argelander's plan in our language; the original instructions are given in the *Vierteljahrschrift der Astronomischen Gesellschaft*, Vol. II. The whole undertaking is in fact almost the original cause of the formation of the Society, which has since undertaken many other serious problems, and has become the leading astronomical society of the world. The results, reduced to 1875, are extremely accurate. Of course more time spent on every single observation would have rendered them still better; but all indications show that, in the great majority of cases, each coördinate of a star's place will be found accurate within a second of a great circle; so that if a star changes place but two or three seconds in a century, its motion can be detected before the year

2000. And as all Bessel's and Lalande's stars, however faint, have been reobserved, there is a vast mass of material now ready for the study of proper motions. A few years now will see the printing of the row of stately volumes which will contain the results of several centuries (where all the work is combined as if done by one astronomer) of human labor.

A continuation to the declination $-23°$ is now in progress in America, Algeria, and Austria; Gould's great work, about the same time, defers the necessity of going farther, although it does not render it superfluous. Photography will doubtless be called in to make this problem easier; or, rather, the Southern zones will be included in the present photographic survey, and perhaps repeated later by the same method.

The comparison of the three volumes mentioned at the head of this article is in many respects instructive. The astronomers were of different nations, employed widely varying instruments, and in one respect a different method. Fearnley of Christiania, and Krueger of Helsingfors and Gotha were pupils of Argelander, and employed the old "eye and ear" method (elaborated by the Greenwich astronomers of the last century). In this the transits of the stars across the meridian are watched by the astronomer, who continually counts by the ear the beats of his clock. If this makes too little sound, he can reinforce it by an electro-magnet. He notes where the star is at the integral second (or half-second) before it passes the wire, and where at the second or half-second after; and estimates the tenths by comparing a second or two afterwards what psychologists call the "traces" on his memory. The method is not always the most precise possible, as it requires long training so to regulate the mental processes that uniform results shall be obtained; but in high declinations, where the stars appear to move much slower, it has certain advantages, and is always free of the annoyance that the sheets or tapes of the chronographic method must be read off. Argelander himself never used the more modern American method, which is, other things being equal, the more accurate, but is not always the one which produces equally accurate results with the least labor.

The American is the telegraphic method. The star is seen approaching the wire, and the observer touches a telegraph key when he estimates that it has reached it. This instant is mechanically recorded on a chronograph. In one point this seems to be less accurate than the other; a very faint star is usually misplaced by the fact that the observer lingers in his judgment that the phenomenon has taken place when the effect is hard to see; so that the right ascensions of faint stars are too large when chronographically determined.

The Albany zone was so observed. Professor Boss of course determined how much each star was delayed in observation by this process ; using an ingenious method invented by Bessel of artificially diminishing the light of the stars as seen through the telescope without altering the character of the image, and so found that his own mental processes delay his judgment by about a hundredth of a second per magnitude ; that is, he would observe a star of the eighth magnitude seven-hundredths of a second later than one of the first in the same place ; and so put it forward a second of arc and a small fraction in right ascension.

On the other hand, the Albany observations of right ascension are rather better, one by one, than those made at Helsingfors. This was probably in part due to Krueger's anxiety about his declinations, which gave him more trouble, owing to the weakness of his instrument in that respect. Fearnley, on the other hand, had a zone so far north (65° to 70°) that with the old method he was able to equal the quality of Boss' work in right ascension with the new, while his employment of verniers instead of reading microscopes has somewhat impaired his declinations.

But, all told, the uniformity of the three catalogues, due to the excellent plan formulated by Argelander, is more sensible and far more important than the trifling discrepancies in execution. The plan is in fact the quintessence of modern practical astronomy in the subject with which it deals. That it has been so warmly welcomed and so thoroughly executed by astronomers over the whole civilized globe is at once a proof of the excellence of their training and of the great advance which has been made in giving the human mind control over its own processes and over material objects.

TRUMAN HENRY SAFFORD.

A PROBLEM IN LEAST SQUARES.

BY PROF. MANSFIELD MERRIMAN.

To determine, by the method of least squares, the most probable values of a and b in the formula $y = ax + b$ when the observed values of both y and x are liable to error.

I. LET x_1 and y_1, x_2 and y_2, ... x_n and y_n be n pairs of observed values of two variables known to be connected by the relation

$$y = ax + b.$$