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Super Yangian of superstring on $\mathrm{AdS}_5 \times \mathrm{S}^5$ revisited

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Abstract

We construct infinite number of conserved nonlocal charges for type IIB superstring on the $\mathrm{AdS}_5 \times \mathrm{S}^5$ space in the conformal gauge without assuming any κ gauge fixing, and show that they satisfy the super Yangian algebra. The resultant algebra is the same as our previous work [8], where a special gauge was assumed in such a way that the Noether current satisfies a flatness condition. However the flatness condition for the Noether current of a superstring on the AdS space is broken in general. We show that the anomalous contribution is absorbed into the current where fermionic constraints play an essential role, and a resultant conserved nonlocal charge has different expression satisfying the same super Yangian algebra.

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1 Introduction and summary

Integrability of the AdS/CFT correspondence [1] has a possibility to broaden its application range from weak to strong coupling. Yangian symmetry is a symmetry responsible for integrable system [2, 3]. Then Yangian symmetry is widely studied for a superstring on AdS spaces [4, 5] as well as spin chain systems [6] and CFT duals [7]. Supersymmetry is one of the guiding principles to establish the quantum integrability. However, it has not been confirmed yet whether nonlocal charges for a superstring on AdS spaces satisfy the super Yangian algebra, because treatment of fermions is still not clear. We presented a classical super Yangian algebra for a superstring on the $\mathrm{AdS}_5\times\mathrm{S}^5$ in the canonical formulation in [8], where a special gauge was assumed in such a way that the Noether current satisfies a flatness condition. The existence of this κ gauge is not justified yet, but this gauge is required for the gauged coset model as a consistency condition. In this work, we have reexamined the flatness condition to construct conserved nonlocal charges. Then we evaluate brackets of the nonlocal charges showing that they satisfy the super Yangian algebra as same as [8].

Our starting point is a superstring action, which has the global super-AdS symmetry. The global invariance guarantees the existence of the Noether current J^R_μ satisfying $\partial^\mu J^R_\mu = 0$. The index R stands for right invariant (RI). This Noether current does not satisfy the flatness condition without assuming any κ gauge fixing,

$$\mathcal{D}_\mu = \partial_\mu - 2J^R_\mu, \quad [\partial^\mu, \mathcal{D}_\mu] = 0, \quad \epsilon^{\mu\nu}[\mathcal{D}_\mu, \mathcal{D}_\nu] = -4\Xi, \tag{1.1}$$

where anomalous term is bilinear of "fermionic" current q_μ[1]

$$\Xi = \tfrac{1}{2}[q_\tau, q_\sigma]. \tag{1.3}$$

[1]This "fermionic" current is not fermionic, but it is G-valued as

$$\begin{aligned} q_\mu &= Z\left(Z^{-1}\partial_\mu Z\right)|_{\rm fermi} Z^{-1} \\ &= \begin{cases} q_\tau = -Z\begin{pmatrix} 0 & (\bar{j}_\sigma)_{\bar{b}a} \\ (j_\sigma)_{b\bar{a}} & 0 \end{pmatrix} Z^{-1} \approx 2Z\begin{pmatrix} 0 & D_{a\bar{b}} \\ \bar{D}_{\bar{a}b} & 0 \end{pmatrix} Z^{-1}, \\ q_\sigma = Z\begin{pmatrix} 0 & (j_\sigma)_{a\bar{b}} \\ (\bar{j}_\sigma)_{\bar{a}b} & 0 \end{pmatrix} Z^{-1}. \end{cases} \end{aligned} \tag{1.2}$$

Z is a coset parameter of G/H with "global super-AdS group" G and "local Lorentz group" H, transforming $Z \to gZh$ with $g \in$ G and $h \in$ H. In canonical formulation τ derivative is determined by a bracket with the Hamiltonian, so τ components of fermionic left invariant (LI) currents are j_σ and $\bar{j}_\sigma$ as familiar in a flat case. We denote $\approx$ for the use of fermionic constraints.

Contrast to (1.1) we found a flat current by adding q_μ with an imaginary coefficient as

$$\tilde{J}^R_\mu = J^R_\mu + \frac{i}{2}\epsilon_{\mu\nu}q^\nu, \tag{1.4}$$

$$\tilde{\mathcal{D}}_\mu = \partial_\mu - 2\tilde{J}^R_\mu, \quad [\partial^\mu, \tilde{\mathcal{D}}_\mu] = 4i\Xi, \quad \epsilon^{\mu\nu}[\tilde{\mathcal{D}}_\mu, \tilde{\mathcal{D}}_\nu] = 0, \tag{1.5}$$

where its conservation is broken The question is how to make conserved nonlocal charges from these two covariant derivatives, and whether they satisfy super Yangian algebra.

After deriving these above relations in Section 2.1, we construct a set of infinite number of conserved nonlocal currents in Section 2.2. In Section 3.1, we construct the nonlocal charge in the form of the sum of the Bena–Polchinski–Roiban (BPR) connection [4] and fermionic constraint in such a way that it commutes with the fermionic constraint. The modification of the BPR connection by Hamiltonian constraints is expected in [13]. The property that the nonlocal charge commutes with the fermionic constraints is crucial for the practical computation of the algebra where the Poisson bracket is allowed to use instead of the Dirac bracket. This also confirms the κ-symmetry invariance of the super Yangian charges. In Section 3.2, we compute the super Yangian algebra, which is the same as our previous work with different expression of generators.

2 Super Yangian generators

In this section, we construct nonlocal charges of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ superstring as super Yangian generators. At first we derive several current relations such as flatness and conservation in the conformal gauge without assuming any other gauge fixing. Using these relations we construct conserved nonlocal currents.

2.1 Flat currents for $\mathbf{AdS_5{\times}S^5}$ superstring

The notation follows from [8]. We use the Roiban–Siegel action for a superstring on $\mathrm{AdS}_5{\times}\mathrm{S}^5$ [9], which is based on a coset G/H with G = GL(4|4) and H = $[\mathrm{Sp}(4)\mathrm{GL}(1)]^2$. A coset parameter $Z_M{}^A$ which is transformed as $Z \to gZh$ with $g \in$ G and $h \in$ H. LI currents are denoted by

$$(J^L_\sigma)_A{}^B = (Z^{-1})_A{}^M \partial_\sigma Z_M{}^B = \begin{pmatrix} \mathbf{J}_\sigma & j_\sigma \\ \bar{j}_\sigma & \bar{\mathbf{J}}_\sigma \end{pmatrix}, \tag{2.1}$$

where notation of components of supermatrices are in footnote[2] . The canonical conjugate to ${Z_M}^A$ is ${\Pi_A}^M$ satisfying $[{Z_M}^A, {\Pi_B}^N\}_{\rm P} = (-)^A \delta_B^A \delta_M^N$. The bracket is the graded Poisson bracket $[A, B\}_{\rm P} = \frac{\partial A}{\partial Z}\frac{\partial B}{\partial \Pi} - (-)^{\sigma(Z)}\frac{\partial A}{\partial \Pi}\frac{\partial B}{\partial Z}$, and should not be confused with the commutator of matrices $[A, B] = AB - BA$. The LI supercovariant derivative is given as

$$
{D_A}^B = {\Pi_A}^M {Z_M}^B = \begin{pmatrix} \mathbf{D} & D \\ \bar{D} & \bar{\mathbf{D}} \end{pmatrix}. \tag{2.3}
$$

The Hamiltonian of the system in the conformal gauge is given by [10]

$$
\begin{aligned}
\mathcal{H} = -\int d\sigma \ \mathrm{tr}\Bigg[&\frac{1}{2}\{\langle\mathbf{D}\rangle^2 + \langle\mathbf{J}_\sigma\rangle^2 + -\langle\bar{\mathbf{D}}\rangle^2 - \langle\bar{\mathbf{J}}_\sigma\rangle^2\} \\
&+(\bar{D}\bar{j}_\sigma - Dj_\sigma + j_\sigma\bar{j}_\sigma)\Bigg].
\end{aligned} \tag{2.4}
$$

We use full GL(4|4) parameters ${Z_M}^A$ by gauging H components, so ${Z_M}^A$ is constrained by H-gauge symmetry. In addition, fermionic constraints exist whose half generate the κ-symmetry. H-gauge constraints and fermionic constraints are

$$
\begin{aligned}
(\mathbf{D})_{(ab)} = \mathrm{tr}\mathbf{D} = (\bar{\mathbf{D}})_{(\bar{a}\bar{b})} &= \mathrm{tr}\bar{\mathbf{D}} = 0, \\
F_{a\bar{b}} = E^{1/4} D_{a\bar{b}} &+ \frac{1}{2}E^{-1/4}(\bar{j}_\sigma)_{\bar{b}a} = 0, \\
\bar{F}_{\bar{a}b} = E^{-1/4}\bar{D}_{\bar{a}b} &+ \frac{1}{2}E^{1/4}(j_\sigma)_{b\bar{a}} = 0,
\end{aligned} \tag{2.5}
$$

with $E = \mathrm{sdet}Z$. Poisson brackets between these constraints and Hamiltonian in (2.4) are zero.

[2] A supermatrix is denoted by boldfaced letters for bosonic components and small letters for fermionic components as

$$
M_{AB} = \begin{pmatrix} \mathbf{M}_{ab} & m_{a\bar{b}} \\ \bar{m}_{\bar{a}b} & \bar{\mathbf{M}}_{\bar{a}\bar{b}} \end{pmatrix}, \quad \mathbf{M}_{ab} = (\mathbf{M})_{(ab)} + \langle\mathbf{M}\rangle_{\langle ab\rangle} + \frac{1}{4}\Omega_{ab}\mathrm{tr}\mathbf{M} \tag{2.2}
$$

with symmetric part (ab), traceless-antisymmetric part $\langle ab\rangle$, and trace part $\mathrm{tr}\mathbf{M} = \Omega^{ab}\mathbf{M}_{ab}$, respectively. Ω_{AB} is antisymmetric $\mathrm{Sp}(4)^2$ invariant metric.

Equations of motions are determined by the Poisson bracket with the Hamiltonian in (2.4) as $\partial_\tau \mathcal{O} = [\mathcal{O}, \mathcal{H}]$,

$$\partial_\tau Z = [Z, \mathcal{H}]_{\rm P} = Z \begin{pmatrix} \langle \mathbf{D} \rangle & -\bar{j}_\sigma^T \\ -j_\sigma^T & \langle \bar{\mathbf{D}} \rangle \end{pmatrix}. \tag{2.6}$$

The τ derivative of LI currents in (2.1) and (2.3) are given as

$$\begin{aligned} &\partial_\tau \langle \mathbf{D} \rangle = \partial_\sigma \langle \mathbf{J}_\sigma \rangle + [(\mathbf{J}_\sigma), \langle \mathbf{J}_\sigma \rangle], \quad \partial_\tau \langle \bar{\mathbf{D}} \rangle = \partial_\sigma \langle \bar{\mathbf{J}}_\sigma \rangle + \left[(\bar{\mathbf{J}}_\sigma), \langle \bar{\mathbf{J}}_\sigma \rangle\right], \\ &\partial_\tau \langle \mathbf{J}_\sigma \rangle = \partial_\sigma \langle \mathbf{D} \rangle + [(\mathbf{J}_\sigma), \langle \mathbf{D} \rangle], \quad \partial_\tau \langle \bar{\mathbf{J}}_\sigma \rangle = \partial_\sigma \langle \bar{\mathbf{D}} \rangle + \left[(\bar{\mathbf{J}}_\sigma), \langle \bar{\mathbf{D}} \rangle\right], \\ &\partial_\tau j_\sigma/2 = \partial_\sigma D + \mathbf{J}_\sigma D - D\bar{\mathbf{J}}_\sigma + \{j_\sigma \langle \bar{\mathbf{D}} \rangle - \langle \mathbf{D} \rangle j_\sigma\}/2, \\ &\partial_\tau \bar{j}_\sigma/2 = \partial_\sigma \bar{D} + \bar{\mathbf{J}}_\sigma \bar{D} - \bar{D}\mathbf{J}_\sigma + \{\bar{j}_\sigma \langle \mathbf{D} \rangle - \langle \bar{\mathbf{D}} \rangle \bar{j}_\sigma\}/2. \end{aligned} \tag{2.7}$$

In general, the right-hand side of the first line contains bilinear of fermionic currents $\langle j_\sigma \bar{D} \rangle - \langle D \bar{j}_\sigma \rangle$; however, it vanishes in this case by fermionic constraint and its antisymmetric property. For example, $\langle j_\sigma \bar{D} \rangle \approx (j_\sigma)_{\langle a}{}^{\bar{a}}$ $(j_\sigma)_{b\rangle}{}^{\bar{b}} \epsilon_{\bar{b}\bar{a}} = 0$.

The Noether current, which is RI, is given by

$$\partial^\mu J_\mu^R = 0, \; J_\mu^R = \begin{cases} J_\tau^R = Z\Pi = ZDZ^{-1}, \\ J_\sigma^R = Z(J_L + \mathcal{A})Z^{-1} = Z\langle J_\sigma^L \rangle Z^{-1}. \end{cases} \tag{2.8}$$

with

$$\begin{aligned} &\langle J_\sigma^L \rangle \equiv \begin{pmatrix} \langle \mathbf{J}_\sigma \rangle & \frac{1}{2} j_\sigma \\ \frac{1}{2}\bar{j}_\sigma & \langle \bar{\mathbf{J}}_\sigma \rangle \end{pmatrix}, \quad \mathcal{A} = \begin{pmatrix} \mathbf{A} & -\frac{1}{2} j_\sigma \\ -\frac{1}{2}\bar{j}_\sigma & \bar{\mathbf{A}} \end{pmatrix}, \\ &\begin{cases} -\mathbf{A} = (\mathbf{J}_\sigma) + \frac{1}{4}\Omega_{ab} \, {\rm tr}\mathbf{J}_\sigma, \\ -\mathbf{A} = (\bar{\mathbf{J}}_\sigma) + \frac{1}{4}\Omega_{\bar{a}\bar{b}} \, {\rm tr}\bar{\mathbf{J}}_\sigma. \end{cases} \end{aligned} \tag{2.9}$$

The bosonic part of $\mathcal{A}$ is gauge field for gauged H-symmetry of the coset G/H, whereas fermionic part of $\mathcal{A}$ is reflection of the fermionic constraint so is not able to gauge away.

In order to calculate the flatness condition of the Noether current, the following relation is used from (2.7) and (2.9) as:

$$\begin{aligned} &\partial_\tau \langle J_\sigma^L \rangle = \partial_\sigma D + [D, \mathcal{A}] - \left[\begin{pmatrix} 0 & D \\ \bar{D} & 0 \end{pmatrix}, \langle J_\sigma^L \rangle\right] + \xi, \\ &\xi = \left[\begin{pmatrix} 0 & D \\ \bar{D} & 0 \end{pmatrix}, \begin{pmatrix} 0 & j_\sigma \\ \bar{j}_\sigma & 0 \end{pmatrix}\right]. \end{aligned} \tag{2.10}$$

In the previous paper, ξ was absent, since the fermionic constraints make $\xi = (\xi_{(ab)},\ \xi_{(\bar{a}\bar{b})})$ to be elements of H, which might be gauged away consistently. In this paper, we keep this term and recalculate the conserved nonlocal currents and the super Yangian algebra. The flatness condition is broken by the ξ term as

$$\partial_\tau J^R_\sigma - \partial_\sigma J^R_\tau - 2(J^R_\tau J^R_\sigma - J^R_\sigma J^R_\tau) = Z\xi Z^{-1}. \tag{2.11}$$

This flatness anomaly is recognized as Ξ in (1.3) by the use of the fermionic constraints in (2.5),

$$Z\xi Z^{-1} = \frac{1}{2}(q_\tau q_\sigma - q_\sigma q_\tau) = \Xi.$$

On the other hand, we found a modified current in (1.4), which is flat[3]

$$\partial_\tau \tilde{J}^R_\sigma - \partial_\sigma \tilde{J}^R_\tau - 2(\tilde{J}^R_\tau \tilde{J}^R_\sigma - \tilde{J}^R_\sigma \tilde{J}^R_\tau) = 0, \tag{2.12}$$

but it is not conserved

$$\partial^\mu \tilde{J}^R_\mu = -2i\Xi. \tag{2.13}$$

The fact that conservation anomaly Ξ in (2.13) is the same function appeared in the flatness anomaly in (2.11) leads to another nontrivial flat current

$$\partial_\tau q_\sigma - \partial_\sigma q_\tau + q_\tau q_\sigma - q_\sigma q_\tau = 0. \tag{2.14}$$

But it is not conserved

$$\partial_\tau q_\tau - \partial_\sigma q_\sigma = 2(J^R_\tau q_\tau - q_\tau J^R_\tau - J^R_\sigma q_\sigma + q_\sigma J^R_\sigma). \tag{2.15}$$

The flatness of q_μ is essential to construct nonlocal currents, where the flatness anomaly in (2.11) is converted into divergence of a current as

$$\partial_\tau \left(J^R_\sigma - \frac{1}{4} q_\sigma \right) - \partial_\sigma \left(J^R_\tau - \frac{1}{4} q_\tau \right) = 2(J^R_\tau J^R_\sigma - J^R_\sigma J^R_\tau). \tag{2.16}$$

This modified flatness condition is nothing but the conservation law of the first-level nonlocal current.

[3]Our notation is $\epsilon^{\mu\nu}\epsilon_{\mu\rho} = \delta^\nu_\rho$, $\epsilon^{\tau\sigma} = \epsilon_{\tau\sigma} = 1$. Then $\epsilon^{\mu\nu} q_\mu q_\nu = -\epsilon_{\mu\nu} q^\mu q^\nu$.

2.2 Conservation of nonlocal currents

Conserved nonlocal currents are constructed quite analogous to the inductive method by Brezin, Izykson, Zinn-Justin and Zuber [12] (BIZZ) with our non-flat covariant derivative $\mathcal{D}_\mu$ in (1.1). The zeroth level of conserved current is the Noether current $(\mathcal{J}_0)_\mu = J^R_\mu$. It can be written as $J^R_\mu = \epsilon_{\mu\nu}\partial^\nu \chi_0$. Let us set $\chi_{-1} = -\frac{1}{2}$ in such a way that $(\mathcal{J}_0)_\mu = \mathcal{D}_\mu \chi_{-1}$. According to the BIZZ procedure the first-level conserved current includes $\mathcal{D}_\mu \chi_0$. This term is not conserved

$$\partial^\mu \left(\mathcal{D}_\mu \chi_0\right) = -\frac{1}{2}\epsilon^{\mu\nu}\left[\mathcal{D}_\mu, \mathcal{D}_\nu\right]\chi_{-1} = 2\Xi\chi_{-1} = \frac{1}{4}\partial^\mu \left(\epsilon_{\mu\nu} q^\nu\right),$$

but the anomalous term is converted into divergence of a current. The obtained conserved current is

$$\begin{aligned}(\mathcal{J}_1)_\mu(\sigma) &= \mathcal{D}_\mu \chi_0 + \frac{1}{2}\epsilon_{\mu\nu} q^\nu \chi_{-1} \\ &= \epsilon_{\mu\nu}\left(J^R - \frac{1}{4}q\right)^\nu (\sigma) - 2J^R_\mu(\sigma)\int^\sigma d\sigma' \, (J^R)_\tau(\sigma') \\ &\Rightarrow \partial^\mu (\mathcal{J}_1)_\mu = 0,\end{aligned} \tag{2.17}$$

where $\chi_0(\sigma) = \int^\sigma d\sigma' \; J^R{}_\tau(\sigma')$ is used. The integration path, denoted by $\int$ and $\int^\sigma$, must be chosen to make well-defined functions where a cut in a closed string worldsheet is required [8, 11].

The second-level conserved current includes $\mathcal{D}_\mu \chi_1$ with $(\mathcal{J}_1)_\mu = \epsilon_{\mu\nu}\partial^\nu \chi_1$, which is not conserved

$$\partial^\mu \left(\mathcal{D}_\mu \chi_1\right) = \partial^\mu \left(-\frac{1}{2}\epsilon_{\mu\nu} q^\nu \chi_0\right).$$

The conserved current is obtained as

$$\begin{aligned}(\mathcal{J}_2)_\mu(\sigma) &= \mathcal{D}_\mu \chi_1 + \frac{1}{2}\epsilon_{\mu\nu} q^\nu \chi_0 \\ &= \left(J^R - \frac{1}{4}q\right)_\mu (\sigma) \\ &\quad - 2\epsilon_{\mu\nu}\left(J^R - \frac{1}{4}q\right)^\nu (\sigma)\int^\sigma d\sigma' \, (J^R)_\tau(\sigma') \\ &\quad - 2J^R_\mu(\sigma)\int^\sigma d\sigma' \left(J^R - \frac{1}{4}q\right)_\sigma (\sigma')\end{aligned} \tag{2.18}$$

$$+ 4J^R_\mu(\sigma)\int^\sigma d\sigma'\,(J^R)_\tau(\sigma')\int^{\sigma'} d\sigma''\,(J^R)_\tau(\sigma'')$$
$$\Rightarrow \partial^\mu(\mathcal{J}_2)_\mu = 0.$$

It is straightforward to check $\partial_\tau \int(\mathcal{J}_2)_\tau = 0$ directly by (2.15) and (2.16).

In induction there exists a potential χ_n for a conserved current, $\partial^\mu(\mathcal{J}_n)_\mu = 0$,

$$(\mathcal{J}_n)_\mu = \epsilon_{\mu\nu}\partial^\nu\chi_n \quad (n \geq 0) \tag{2.19}$$

with $\partial^\mu\chi_n = -\epsilon^{\mu\nu}(\mathcal{J}_n)_\nu$. Acting $\mathcal{D}_\mu$ on χ_n and converting an anomalous term into a divergence of current give an infinite number of conserved currents as $\partial^\mu(\mathcal{J}_n)_\mu = 0$. Conserved currents can be constructed as

$$(\mathcal{J}_{n+1})_\mu = \mathcal{D}_\mu\chi_n + \frac{1}{2}\epsilon_{\mu\nu}q^\nu\sum_{l=0}^{[n/2]} a_{n-1-2l}\chi_{n-1-2l}, \tag{2.20}$$

with $a_{n-1} = 1$, $a_{n-3} = 1/4$, $a_{n-5} = 1/8$, $a_{n-7} = 5/64, \ldots,$ and a_{n-1-2l}'s are determined perturbatively. The obtained conserved nonlocal currents are given by

$$\begin{cases} (\mathcal{J}_0)_\mu(\sigma) = J^R_\mu(\sigma), \\ (\mathcal{J}_1)_\mu(\sigma) = \epsilon_{\mu\nu}\left(J^R - \frac{1}{4}q\right)^\nu(\sigma) + 2J^R_\mu(\sigma)\int^\sigma d\sigma'\,(J^R)^\tau(\sigma'), \\ (\mathcal{J}_2)_\mu(\sigma) = \left(J^R - \frac{1}{4}q\right)_\mu + 2\epsilon_{\mu\nu}\left(J^R - \frac{1}{4}q\right)^\nu\int^\sigma d\sigma'\,(J^R)^\tau(\sigma') \\ \qquad - 2J^R_\mu\int^\sigma d\sigma'\left(J^R - \frac{1}{4}q\right)_\sigma(\sigma') \\ \qquad + 4J^R_\mu\int^\sigma d\sigma'\,(J^R)_\tau(\sigma')\int d\sigma''\,(J^R)_\tau(\sigma'') \\ \qquad \vdots \end{cases} . \tag{2.21}$$

There exists infinite number of the conserved nonlocal charges $Q_n = \int d\sigma(\mathcal{J}_n)_\tau$. There is ambiguity of functions of Q_0; so we begin with

$$Q_1 = \int d\sigma\;(\mathcal{J}_1)_\tau \equiv \int d\sigma\left(J^R_\sigma - \frac{1}{4}q_\sigma\right)(\sigma)$$
$$- \int d\sigma\int^\sigma d\sigma'\;\left[J^R_\tau(\sigma),\, J^R_\tau(\sigma')\right]$$

$$= \int d\sigma \left(J^R_\sigma - \frac{1}{4} q_\sigma \right)(\sigma)$$
$$- \frac{1}{2} \int d\sigma \int d\sigma' \, \epsilon(\sigma - \sigma') \left[J^R_\tau(\sigma), \, J^R_\tau(\sigma') \right], \qquad (2.22)$$

with $\epsilon(\sigma - \sigma') = \theta(\sigma - \sigma') - \theta(\sigma' - \sigma)$.

It is noted that our Noether current and the first-level nonlocal charge are equal to ones obtained by Bena *et al.* [4] with use of constraints[4] . It is unclear whether all other nonlocal charges coincide.

3 Super Yangian algebra

In this section, we compute classical algebra among nonlocal charges obtained as super Yangian generators in the previous section. The Green–Schwarz-type superstring has fermionic second class constraints, which forces to use the Dirac bracket for the algebra computation. For an operator that commute with the second class constraints its Dirac bracket with any operator reduces to its Poisson bracket. At first we will find a nonlocal charge in such a way that it commutes with the fermionic constraints. Then algebra is calculated by the Poisson bracket. The gauge invariance of these generators is also confirmed as expected.

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[4]Correspondence with their notation is the following; For example, Noether current in their notation is given by

$$\left(p + \frac{1}{2} q'\right)_\mu = \begin{cases} Z \begin{pmatrix} (J_\tau)_{\langle ab \rangle} & -\frac{1}{2}(J_\sigma)_{\bar{b}a} \\ -\frac{1}{2}(J_\sigma)_{b\bar{a}} & (J_\tau)_{\langle \bar{a}\bar{b} \rangle} \end{pmatrix} Z^{-1} \\ Z \begin{pmatrix} (J_\sigma)_{\langle ab \rangle} & -\frac{1}{2}(J_\tau)_{\bar{b}a} \\ -\frac{1}{2}(J_\tau)_{b\bar{a}} & (J_\sigma)_{\langle \bar{a}\bar{b} \rangle} \end{pmatrix} Z^{-1} \end{cases} \Leftrightarrow J^R_\mu = \begin{cases} ZDZ^{-1} = Z\Pi \\ Z \begin{pmatrix} \langle \mathbf{J}_\sigma \rangle_{\langle ab \rangle} & \frac{1}{2} j_{\sigma a \bar{b}} \\ \frac{1}{2} \bar{j}_{\sigma \bar{a} b} & \langle \bar{\mathbf{J}}_\sigma \rangle_{\langle \bar{a}\bar{b} \rangle} \end{pmatrix} Z^{-1}. \end{cases}$$

In our notation τ-derivative is determined by (2.7) as

$$(J_\tau)_{\langle ab \rangle} = \mathbf{D}_{ab}, \quad (J_\tau)_{\langle \bar{a}\bar{b} \rangle} = \bar{\mathbf{D}}_{\bar{a}\bar{b}}, \quad (J_\sigma)_{\bar{b}a} = -j_{\sigma a \bar{b}} = 2D_{a\bar{b}}, \quad (J_\sigma)_{b\bar{a}} = -\bar{j}_{\sigma \bar{a} b} = 2\bar{D}_{\bar{a}b}$$

with use of H-gauge constraints and fermionic constraints in (2.5).

3.1 Invariance of super Yangian generators

Let us examine invariance of super Yangian generators:

$$Q_0 = \int d\sigma\ J_\tau^R(\sigma), \tag{3.1}$$
$$Q_1 = \int d\sigma \left(J_\sigma^R - \frac{1}{4}q_\sigma\right)(\sigma) - \frac{1}{2}\int d\sigma \int d\sigma'\, \epsilon(\sigma-\sigma')\left[J_\tau^R(\sigma), J_\tau^R(\sigma')\right].$$

At first let us confirm the H-gauge invariance of the super Yangian charges. The H-gauge constraints in the first line of (2.5) generating two Sp(4)'s and two GL(1)'s transformations are

$$\phi_i = \{(\mathbf{D}_{(ab)}),\ (\bar{\mathbf{D}}_{(\bar{a}\bar{b})}),\ \mathrm{tr}\mathbf{D},\ \mathrm{tr}\bar{\mathbf{D}}\}. \tag{3.2}$$

It is easy to confirm that H-invariance of Q's

$$[Q_0, \phi_i\}_{\mathrm{P}} = [Q_1, \phi_i\}_{\mathrm{P}} = 0. \tag{3.3}$$

Next let us examine the fermionic constraints in the second and third lines of (2.5) whose half is first class generating the κ-symmetry and another half is second class. It is obvious that $[Q_0, F\}_{\mathrm{P}} = [Q_0, \bar{F}\}_{\mathrm{P}} = 0$ since $F, \bar{F}$ are made of LI currents. Then the Dirac bracket between Q_0 with any operator $\mathcal{O}$ is equal to its Poisson bracket, $[Q_0, \mathcal{O}\}_{\mathrm{Dirac}} = [Q_0, \mathcal{O}\}_{\mathrm{P}}$.

However the F-invariance of Q_1 is not realized by itself. It turns out that fermionic constraints must be added to the nonlocal charge Q_1 in such a way that a Dirac bracket of $\hat{Q}_1$ with any operator are equal to its Poisson bracket as

$$\begin{aligned}
\hat{Q}_1 &= Q_1 + \int d\sigma\ Z \begin{pmatrix} & \bar{F}^T \\ F^T & \end{pmatrix} Z^{-1}(\sigma) \\
&= \int d\sigma \left\{ Z'Z^{-1} + Z \begin{pmatrix} \mathbf{A} & -\frac{1}{4}j + \bar{D}^T \\ -\frac{1}{4}\bar{j} + D^T & \bar{\mathbf{A}} \end{pmatrix} Z^{-1}(\sigma) \right\} \\
&\quad - \frac{1}{2}\int d\sigma \int d\sigma'\, \epsilon(\sigma-\sigma')[J_\tau^R(\sigma), J_\tau^R(\sigma')]. \qquad (3.4)\\
&\Rightarrow [\hat{Q}_1, F\}_{\mathrm{P}} = [\hat{Q}_1, \bar{F}\}_{\mathrm{P}} = 0 \Rightarrow [\hat{Q}_1, \mathcal{O}\}_{\mathrm{Dirac}} = [\hat{Q}_1, \mathcal{O}\}_{\mathrm{P}}.
\end{aligned}$$

3.2 Super Yangian algebra

Now let us calculate the super Yangian algebra. From now on we denote $\hat{Q}_1$ by Q_1 for simpler notational although, fermionic constraints in (3.4) must be taken into account for evaluation of brackets.

We obtain the classical super Yangian algebra:

$$\begin{aligned}
[Q_{0M}{}^N, Q_{0L}{}^K\}_\mathrm{P} &= (-)^N[s\delta_M^K Q_{0L}{}^N - \delta_L^N Q_{0M}{}^K], \\
[Q_{0M}{}^N, Q_{1L}{}^K\}_\mathrm{P} &= (-)^N[s\delta_M^K Q_{1L}{}^N - \delta_L^N Q_{1M}{}^K], \\
[Q_{1M}{}^N, Q_{1L}{}^K\}_\mathrm{P} &= (-)^N[s\delta_M^K Q_{2L}{}^N - \delta_L^N Q_{2M}{}^K \\
&\quad + 4s(Q_{0L}{}^P Q_{0P}{}^N Q_{0M}{}^K - Q_{0L}{}^N Q_{0M}{}^{OP} Q_{0P}{}^K)],
\end{aligned} \tag{3.5}$$

where

$$Q_{2M}{}^N = 3Q_{0M}{}^N + \int (\mathcal{J}_2)_{\tau M}{}^N \tag{3.6}$$

with Grassmann sign factor $s = (-)^{(N+L)(1+M+L)}$. The resultant algebra is the same as [8] but expression of the charge in (3.6) and (2.21) is different. Details of the computation are given in the appendix.

The Serre relation is followed from (3.5); so we showed that the nonlocal charge in (3.4) together with the Noether charge in (3.1) satisfy the super Yangian algebra.

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Appendix

Appendix A Nonlocal currents

The second-level conserved current includes $\mathcal{D}_\mu \chi_1$ with $(\mathcal{J}_1)_\mu = \epsilon_{\mu\nu}\partial^\nu \chi_1$ which is not conserved

$$\begin{aligned}
\partial^\mu\left(\mathcal{D}_\mu \chi_1\right) &= -\epsilon^{\mu\nu}\mathcal{D}_\mu\left(\mathcal{D}_\nu \chi_0 - \mathrm{i}\Delta J_\nu \chi_{-1}\right) \\
&= 2\Xi\chi_0 + \mathrm{i}\epsilon^{\mu\nu}\mathcal{D}_\mu\left(\Delta J_\nu \chi_{-1}\right) \\
&= \frac{2}{4i}[\partial^\mu, \tilde{\mathcal{D}}_\mu]\chi_0 - \mathrm{i}\epsilon^{\mu\nu}\Delta J_\mu \mathcal{D}_\nu \chi_{-1} \\
&= \frac{2}{4i}\left\{\partial^\mu\left(-2\Delta J_\mu \chi_0\right) + 2\Delta J_\mu \partial^\mu \chi_0\right\} - \mathrm{i}\epsilon^{\mu\nu}\Delta J_\mu (J_0)_\nu \\
&= \frac{2}{4i}\left\{\partial^\mu\left(-2\Delta J_\mu \chi_0\right) - 2\epsilon^{\mu\nu}\Delta J_\mu J^R_\nu\right\} - \mathrm{i}\epsilon^{\mu\nu}\Delta J_\mu J^R_\nu \\
&= \mathrm{i}\partial^\mu\left(\Delta J_\mu \chi_0\right).
\end{aligned}$$

It is denoted by $\Delta J_\mu = \frac{i}{2}\epsilon_{\mu\nu}q^\nu$.

In induction there exists a potential χ_n for a conserved current, $\partial^\mu(\mathcal{J}_n)_\mu = 0$,

$$(\mathcal{J}_n)_\mu = \epsilon_{\mu\nu}\partial^\nu \chi_n \quad (n \geq 0) \tag{A.1}$$

with $\partial^\mu \chi_n = -\epsilon^{\mu\nu}(\mathcal{J}_n)_\nu$ because of notation $\epsilon^{\mu\nu}\epsilon_{\mu\rho} = \delta^\nu_\rho$. Acting $\mathcal{D}_\mu$ on χ_n and canceling the anomalies by the anomalous term ΔJ_μ as (1.4) give an infinite number of conserved currents as $\partial^\mu(\mathcal{J}_n)_\mu = 0$

$$\begin{aligned}
(\mathcal{J}_3)_\mu &= \mathcal{D}_\mu \chi_2 - \mathrm{i}\Delta J_\mu(\chi_1 + \tfrac{1}{4}\chi_{-1}), \\
(\mathcal{J}_4)_\mu &= \mathcal{D}_\mu \chi_3 - \mathrm{i}\Delta J_\mu(\chi_2 + \tfrac{1}{4}\chi_0), \\
(\mathcal{J}_5)_\mu &= \mathcal{D}_\mu \chi_4 - \mathrm{i}\Delta J_\mu(\chi_3 + \tfrac{1}{4}\chi_1 + \tfrac{1}{8}\chi_{-1}), \\
(\mathcal{J}_6)_\mu &= \mathcal{D}_\mu \chi_5 - \mathrm{i}\Delta J_\mu(\chi_4 + \tfrac{1}{4}\chi_2 + \tfrac{1}{8}\chi_0), \\
&\cdots.
\end{aligned}$$

In this way, conserved currents can be constructed as

$$(\mathcal{J}_{n+1})_\mu = \mathcal{D}_\mu \chi_n - \mathrm{i}\Delta J_\mu \sum_{l=0}^{[n/2]} a_{n-1-2l}\chi_{n-1-2l}, \tag{A.2}$$

with $a_{n-1} = 1,\ a_{n-3} = 1/4,\ a_{n-5} = 1/8,\ a_{n-7} = 5/64, \ldots$ and a_{n-1-2l}'s are determined perturbatively.

Appendix B Fermionic constraint invariance of nonlocal charge

The definition of the Poisson bracket in the footnote 2 gives convenient formula:

$$\left[\int \mathrm{str}\Pi\Psi_1, \int \mathrm{str}Z\Psi_2\right]_{\mathrm{P}} = \int \mathrm{str}\Psi_1\Psi_2.$$

In order to compute the Poisson bracket between the nonlocal charge Q_1 in (2.22) and fermionic constraints $F, \bar{F}$ in (2.5), we take supertrace with some parameters, a constant parameter Λ for $\hat{Q}_1$ and a local parameter $\lambda(\sigma)$ for $F, \bar{F}$, as

$$\begin{aligned}
&\left[\mathrm{str}\hat{Q}_1\Lambda, \int d\sigma\ \mathrm{Str}F(\sigma)\lambda(\sigma)\right]_{\mathrm{P}},\\
&\mathrm{Str}F(\sigma)\lambda(\sigma) = \mathrm{Str}\begin{pmatrix} & F(\sigma)\\ \bar{F}(\sigma) & \end{pmatrix}\begin{pmatrix} & \lambda(\sigma)\\ \bar{\lambda}(\sigma) & \end{pmatrix}.
\end{aligned}$$

A charge has an ambiguity of the fermionic constraints so we examine the following candidate:

$$\begin{aligned}
\hat{Q}_1 = &\int d\sigma\,(J^R_\sigma - \tfrac{1}{4}q_\sigma)(\sigma) - \frac{1}{2}\int d\sigma \int d\sigma'\, \epsilon(\sigma-\sigma')[J^R_\tau(\sigma), J^R_\tau(\sigma')]\\
&+ c\int d\sigma\, Z\begin{pmatrix} & \bar{F}^T\\ F^T & \end{pmatrix} Z^{-1}.
\end{aligned}$$

In appendices, we use $\hat{Q}_1$ and Q_1 separately in order to stress a role of the fermionic constraints. The Poisson bracket between the local term in $\hat{Q}_1$ and F is computed as

$$\begin{aligned}
&\left[\int d\sigma\,(J^R_\sigma - \tfrac{1}{4}q_\sigma)(\sigma), \int d\sigma\ \mathrm{Str}F\lambda\right]_{\mathrm{P}}\\
&= \int Z\left([\langle\langle J^L\rangle\rangle, \lambda] - \langle\langle [J^L, \lambda]\rangle\rangle - \langle\langle \partial_\sigma\lambda\rangle\rangle\right) Z^{-1} \qquad \text{(B.1)}
\end{aligned}$$

with

$$J^R_\sigma - \frac{1}{4}q_\sigma = Z\begin{pmatrix} \langle\mathbf{J}\rangle & j/4\\ \bar{j}/4 & \langle\bar{\mathbf{J}}\rangle \end{pmatrix} Z^{-1} \equiv Z\langle\langle J^L\rangle\rangle Z^{-1}.$$

The Poisson bracket of the nonlocal term and F is computed as

$$\left[-\frac{1}{2}\int d\sigma \int d\sigma' \,\epsilon(\sigma-\sigma')\left[J_\tau^R(\sigma), J_\tau^R(\sigma')\right], \int d\sigma \;\mathrm{Str} F\lambda\right]_{\mathrm{P}}$$
$$= -\int d\sigma \; Z[D, \lambda^T]Z^{-1}, \tag{B.2}$$
$$\lambda^T = \begin{pmatrix} & \bar{\lambda}^T(\sigma) \\ \lambda^T(\sigma) & \end{pmatrix}. \tag{B.3}$$

These terms are canceled by the fermionic constraint as the third term in $\hat{Q}_1$

$$\left[\int Z\begin{pmatrix} & \bar{F}^T \\ F^T & \end{pmatrix}Z^{-1}, \int d\sigma \;\mathrm{Str} F\lambda\right]_{\mathrm{P}}$$
$$= Z\left([\langle \mathbf{D}\rangle, \lambda^T] - [\langle \mathbf{J}\rangle, \lambda] + \left[\begin{pmatrix} & \bar{F}^T \\ F^T & \end{pmatrix}, \lambda\right]\right)Z^{-1}. \tag{B.4}$$

As a result the Poisson bracket is given by

$$\left[\hat{Q}_1, \int d\sigma \;\mathrm{Str} F(\sigma)\lambda(\sigma)\right]_{\mathrm{P}} = Z\Big((c-1)\left[\langle \mathbf{D}\rangle, \lambda^T\right] + (1-c)\left[\langle \mathbf{J}\rangle, \lambda\right]$$
$$- \left[D\,|_{\mathrm{fermi}} + \frac{1}{2}J^{L;T}\,|_{\mathrm{fermi}}, \lambda^T\right]\Big)Z^{-1},$$
$$= 0 \text{ for } c = 1, \tag{B.5}$$

where both H-gauge and fermionic constraints are used. The Dirac bracket of $\hat{Q}_1$ is equal to the Poisson bracket

$$[\hat{Q}_1, \mathcal{O}\}_{\mathrm{Dirac}} = [\hat{Q}_1, \mathcal{O}\}_{\mathrm{P}}. \tag{B.6}$$

Appendix C Derivation of super Yangian algebra

Analogous to the previous computation it is convenient to multiply parameters as

$$[\mathrm{str}\;\hat{Q}_1\Lambda \;,\; \mathrm{str}\;\hat{Q}_1\Sigma]_{\mathrm{P}}.$$

The super Yangian generator $\hat{Q}_1$ in (3.4) has H-gauge symmetry, which allows a gauge $\mathbf{A} = \bar{\mathbf{A}} = 0$ for simpler computation as

$$\begin{aligned}
\hat{Q}_1 &= \hat{Q}_{1-1} + \hat{Q}_{1-2},\\
\hat{Q}_{1-1} &= \int d\sigma \left\{ Z'Z^{-1} + Z \begin{pmatrix} & -\frac{1}{4}j + \bar{D}^T \\ -\frac{1}{4}\bar{j} + D^T & \end{pmatrix} Z^{-1}(\sigma) \right\},\\
\hat{Q}_{1-2} &= -\frac{1}{2}\int d\sigma \int d\sigma'\, \epsilon(\sigma-\sigma')[J_\tau^R(\sigma), J_\tau^R(\sigma')].
\end{aligned} \tag{C.1}$$

The Poisson bracket between two $\hat{Q}_{1-1}$'s is

$$[\mathrm{str}\ \hat{Q}_{1-1}\Lambda, \mathrm{str}\ \hat{Q}_{1-1}\Sigma]_{\mathrm{P}} = \int \mathrm{str}\ D\mid_{\mathrm{bose}} [\lambda^T\mid_{\mathrm{fermi}}, \sigma^T\mid_{\mathrm{fermi}}]$$

with $\lambda = Z^{-1}\Lambda Z$ and $\sigma = Z^{-1}\Sigma Z$. The Poisson bracket between $\hat{Q}_{1-1}$ and $\hat{Q}_{1-2}$ is

$$\begin{aligned}
&[\mathrm{str}\ \hat{Q}_{1-1}\Lambda, \mathrm{str}\ \hat{Q}_{1-2}\Sigma]_{\mathrm{P}} + [\mathrm{str}\hat{Q}_{1-2}\Lambda, \mathrm{str}\hat{Q}_{1-1}\Sigma]_{\mathrm{P}}\\
&\quad = \int \mathrm{str}\ \left(\left(4J_\tau^R - \frac{1}{4}q_\tau\right)[\Sigma,\Lambda] - D\mid_{\mathrm{bose}} [\lambda^T\mid_{\mathrm{fermi}}, \sigma^T\mid_{\mathrm{fermi}}]\right)\\
&\qquad + 2\int d\sigma \int d\sigma'\ \mathrm{str}\ \left[\left(J_\sigma^R - \frac{1}{4}q_\sigma\right)(\sigma), J_\tau^R(\sigma')\right]\epsilon(\sigma-\sigma')[\Sigma,\Lambda],
\end{aligned} \tag{C.2}$$

where constraints are set to be zero on the right-hand side. Adding up these terms give

$$\begin{aligned}
&[\mathrm{str}\ \hat{Q}_{1-1}\Lambda, \mathrm{str}\ \hat{Q}_{1-1}\Sigma]_{\mathrm{P}} + [\mathrm{str}\hat{Q}_{1-1}\Lambda, \mathrm{str}\hat{Q}_{1-2}\Sigma]_{\mathrm{P}} + [\mathrm{str}\hat{Q}_{1-2}\Lambda, \mathrm{str}\hat{Q}_{1-1}\Sigma]_{\mathrm{P}}\\
&\quad = \int \mathrm{str}\ \left(4J_\tau^R - \frac{1}{4}q_\tau\right)[\Sigma,\Lambda]\\
&\qquad + 2\int d\sigma \int d\sigma'\ \mathrm{str}\ \left[\left(J_\sigma^R - \frac{1}{4}q_\sigma\right)(\sigma), J_\tau^R(\sigma')\right]\epsilon(\sigma-\sigma')[\Sigma,\Lambda].
\end{aligned} \tag{C.3}$$

The bracket between two $\hat{Q}_{1-2}$'s is the same as our previous result. Extracting parameters from the above we get the same final answer as before

$$\begin{aligned}
[Q_{1M}{}^N, Q_{1L}{}^K\}_{\mathrm{P}} &= (-)^N[s\delta_M^K Q_{2L}{}^N - \delta_L^N Q_{2M}{}^K\\
&\qquad + 4s(Q_{0L}{}^P Q_{0P}{}^N Q_{0M}{}^K - Q_{0L}{}^N Q_{0M}{}^{OP} Q_{0P}{}^K)],
\end{aligned}$$

where

$$Q_{2M}{}^N = 3Q_{0M}{}^N + \int (\mathcal{J}_2)_{\tau M}{}^N.$$

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