

# THE ANNALS *of* PROBABILITY

AN OFFICIAL JOURNAL OF  
THE INSTITUTE OF MATHEMATICAL STATISTICS

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