

# THE ANNALS of PROBABILITY

AN OFFICIAL JOURNAL OF  
THE INSTITUTE OF MATHEMATICAL STATISTICS

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A shortage of paper is the major cause of delays in the printing and delivery of the ANNALS. Also, there is a strong possibility of paper allotment soon.

We do not foresee alleviation in the near future, and we beg your indulgence in this situation beyond control.

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