

# THE ANNALS of MATHEMATICAL STATISTICS

FOUNDED AND EDITED BY H. C. CARVER, 1930-1938

EDITED BY S. S. WILKS, 1938-1949

THE OFFICIAL JOURNAL OF THE INSTITUTE  
OF MATHEMATICAL STATISTICS

## Contents

	PAGE
Our Silver Anniversary . . . . .	ALLEN T. CRAIG 835
Simplex-Sum Designs: A Class of Second Order Rotatable Designs Derivable from those of First Order . . . . .	G. E. P. BOX AND D. W. BEHNKEN 838
Third Order Rotatable Designs in Three Dimensions . . . . .	NORMAN R. DRAPER 865
A Third Order Rotatable Design in Four Dimensions . . . . .	NORMAN R. DRAPER 875
Some Aspects of Weighing Designs . . . . .	DAMARAJU RAGHAVARAO 878
Random Allocation Designs I: On General Classes of Estimation Methods . . . . .	A. P. DEMPSTER 885
A Mixed Model for the Complete Three-Way Layout with Two Random-Effects Factors . . . . .	J. P. IMHOF 906
Tests for Regression Coefficients when Errors are Correlated . . . . .	M. M. SIDDIQUI 926
Mixed Model Variance Analysis with Normal Error and Possibly Non-Normal Other Random Effects, Part I: The Univariate Case . . . . .	S. N. ROY AND WHITFIELD COBB 939
Mixed Model Variance Analysis with Normal Error and Possibly Non-Normal Other Random Effects, Part II: The Multivariate Case . . . . .	S. N. ROY AND WHITFIELD COBB 958
An Associated Polynomial for Least Squares Approximations . . . . .	RICHARD WARREN 969
Asymptotic Rate of Discrimination for Markov Processes . . . . .	LAMBERT H. KOOPMANS 982
Equalities for Stationary Processes Similar to an Equality of Wald . . . . .	SHU-TEH CHEN MOY 995
Multivariate Chebyshev Inequalities . . . . .	ALBERT W. MARSHALL AND INGRAM OLKIN 1001
On Deviations of the Sample Mean . . . . .	R. R. BAHADUR AND R. RANGA RAO 1015
On the Independence of a Sample Central Moment and the Sample Mean . . . . .	R. G. LAHA, E. LUKACS AND M. NEWMAN 1028
Order Statistics of Partial Sums . . . . .	J. G. WENDEL 1034
Probability Distributions Related to Random Mappings . . . . .	BERNARD HARRIS 1045
Some Asymptotic Results for a Coverage Problem . . . . .	MAX HALPERIN 1063
Statistical Programming . . . . .	D. F. VOTAW, JR. 1077
The Life Distribution and Reliability of a System with Spare Components . . . . .	DONALD F. MORRISON AND H. A. DAVID 1084
Maximizing the Probability that Adjacent Order Statistics of Samples from Several Populations Form Overlapping Intervals . . . . .	RICHARD COHN, FREDERICK MOSTELLER, JOHN W. PRATT, AND MAURICE TATSUOKA 1095
Normalizing the Noncentral $t$ and $F'$ Distributions . . . . .	NICO F. LAUBSCHER 1105
Probability Content of Regions Under Spherical Normal Distributions, II: The Distribution of the Mean in Normal Samples . . . . .	HAROLD RUBEN 1113
Tables of Range and Studentized Range . . . . .	H. LEON HARTER 1122
Asymptotic Formulae for the Distribution of Hotelling's Generalized $T^2$ Statistic, II . . . . .	KOICHI ITO 1148
On Unbiased Estimation . . . . .	L. SCHMETTERER 1154
Two-Stage Experiments for Estimating a Common Mean . . . . .	DONALD RICHTER 1164
Rank-Sum Tests for Dispersions . . . . .	A. R. ANSARI AND R. A. BRADLEY 1174
A Relationship between Hodges' Bivariate Sign Test and a Non-Parametric Test of Daniels' . . . . .	BRUCE M. HILL 1190

(Continued on back cover)

Vol. 31, No. 4 — December, 1960

# THE ANNALS OF MATHEMATICAL STATISTICS

**Subscription Rates.** Current issues are \$15 per volume (four issues of one calendar year) in the U. S. and Canada, \$10 per volume elsewhere. Single issues are \$4. Back numbers for all issues up to and including 1956 (Vol. XXVII) are \$12 per volume, \$3.50 per issue, \$200 for the first 25 volumes, \$10 per additional volume purchased at the same time as Volume I through XXV.

Rates to members of the Institute of Mathematical Statistics are lower (see inside back cover).

Communications concerning **subscriptions, back numbers, payment of dues, etc.**, should be addressed to Gerald J. Lieberman, Treasurer, Institute of Mathematical Statistics, Department of Statistics, Stanford University, Stanford, Calif.

Communications concerning **membership, changes of address, etc.**, should be addressed to George E. Nicholson, Jr., Secretary, Department of Statistics, University of North Carolina, Chapel Hill, N. C. Changes of address which are to become effective for a given issue of the *Annals* should be reported to the Secretary on or before the 10th of the month preceding the month of issue.

**Editorial Office**, Department of Statistics, Eckhart Hall, University of Chicago, Chicago 37, Illinois. William Kruskal, Editor.

**Preparation of manuscripts.** Manuscripts should be submitted to the editorial office. Each manuscript should be typewritten, double spaced, with wide margins at sides, top, and bottom, and the original should be submitted with one additional copy, on paper that will take corrections. Dittoed or mimeographed papers are acceptable only if completely legible. Footnotes should be reduced to a minimum, and where possible replaced by remarks in the text, or a bibliography at the end of the paper; formulae in footnotes should be avoided. References should follow current *Annals* style, and should be numbered alphabetically according to authors' names.

Figures, charts, and diagrams should be professionally drawn on plain white paper or tracing cloth in black India ink twice the size they are to be printed.

Authors are asked to keep in mind the typographical difficulties of complicated mathematical formulae. The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between such pairs as zero and the letter O, the numeral 1 and the letter l, numeral 1 used as superscript and prime ('), alpha and a, kappa and k, mu and u, nu and v, eta and n, etc. Subscripts or superscripts should be clearly below or above the line. Bars above groups of letters (e.g.,  $\overline{\log x}$ ) and underlined letters (e.g.,  $\underline{x}$ ) are difficult to print and should be avoided. Symbols are automatically italicized by the printer and should not be underlined on manuscripts. Boldface letters may be indicated by underlining with a wavy line on the manuscript; boldface subscripts and superscripts are not available. Complicated exponentials should be represented with the symbol exp. In writing square roots the fractional exponent is preferable to the radical sign. Fractions *in the body of the text* (as opposed to displayed expressions) and fractions occurring in the numerators or denominators of fractions are preferably written with the solidus; thus  $(a + b)/(c + d)$  rather than  $\frac{a + b}{c + d}$ .

Authors will ordinarily receive only galley proofs. Fifty reprints without covers will be furnished free. Additional reprints and covers will be furnished at cost.

---

Mail to the *Annals of Mathematical Statistics* should be addressed to either the Editor or the Treasurer, as described above. It should *not* be addressed to Waverly Press.

---

COMPOSED AND PRINTED AT THE  
WAVERLY PRESS, INC., BALTIMORE, MARYLAND, U. S. A.

Second-class postage paid at Baltimore, Maryland

## The Twenty-Fifth Anniversary of the Founding of the Institute of Mathematical Statistics

The Institute of Mathematical Statistics was established and organized on September 12, 1935, in a classroom at the University of Michigan in Ann Arbor. In 1960, the Institute has completed twenty-five years of existence and reached its Silver Anniversary.

To commemorate this Anniversary and to describe the Institute's origins to its present membership, Allen T. Craig, who was among those present at the Institute's founding and who served as the first Secretary-Treasurer of the Institute, was invited to prepare an account of the early days of the Institute and of *The Annals of Mathematical Statistics*. Mr. Craig's article appears on the following pages.

EDITOR

REFERENCES TO THE ESTABLISHMENT OF  
THE INSTITUTE OF MATHEMATICAL STATISTICS  
AND THE *ANNALS OF MATHEMATICAL STATISTICS*

*Annals of Mathematical Statistics*

- Vol. 1 (1930), pp. 1-2. Initial statement about publication of the *Annals*.  
Vol. 6 (1935), p. 227. Initial statement about formation of the Institute.  
Vol. 9 (1938), Second unnumbered page after p. 67.

*Journal of the American Statistical Association*

- Vol. 25 (1930), p. 184 (*Proceedings*).  
Vol. 30 (1935), pp. 710-11. Formation of the Institute and program at the organizing meeting.

## EDITORIAL STAFF

EDITOR

WILLIAM KRUSKAL

ASSOCIATE EDITORS

ALLAN BIRNBAUM  
DOUGLAS G. CHAPMAN  
W. S. CONNOR

DONALD A. DARLING  
WASSILY HOEFFDING  
N. L. JOHNSON

OSCAR KEMPTHORNE  
E. L. LEHMANN  
DAVID L. WALLACE

WITH THE COOPERATION OF

J. R. BLUM  
R. C. BOSE  
D. L. BURKHOLDER  
W. S. CONNOR  
D. R. COX

J. F. DALY  
CYRUS DERMAN  
J. L. DOOB  
MEYER DWASS  
D. A. S. FRASER  
SAMUEL KARLIN

HARRY KESTEN  
C. H. KRAFT  
SOLOMON KULLBACK  
EUGENE LUKACS  
G. E. NOETHER  
INGRAM OLKIN

J. W. PRATT  
HOWARD RAIFFA  
H. E. ROBBINS  
WALTER L. SMITH  
LIONEL WEISS

PAST EDITORS OF THE ANNALS

H. C. CARVER, 1930-1938      T. W. ANDERSON, 1950-1952  
S. S. WILKS, 1938-1949      E. L. LEHMANN, 1953-1955  
T. E. HARRIS, 1955-1958

Published quarterly by the Institute of Mathematical Statistics in March,  
June, September and December.

## IMS INSTITUTIONAL MEMBERS

ABERDEEN PROVING GROUNDS, BALLISTIC RESEARCH LABORATORIES, Aberdeen, Maryland  
AEROMET-GENERAL CORPORATION, P. O. Box 296, Azusa, California  
AMERICAN VISCOSE CORPORATION, MARCUS HOOK, Pennsylvania  
ATLANTIC REFINING COMPANY, 2700 Passyunk Avenue, Philadelphia, Pa.  
BELL TELEPHONE LABORATORIES, INC., TECHNICAL LIBRARY, 463 West Street, New York 14,  
New York  
BENDIX AVIATION CORPORATION, 1200 Fisher Bldg., Detroit, Michigan  
BOEING AIRPLANE COMPANY, Box 3707, Seattle, Washington  
CALIFORNIA RESEARCH CORPORATION, P. O. Box 1627, Richmond, California  
CASE INSTITUTE OF TECHNOLOGY, STATISTICAL LABORATORY, Cleveland 6, Ohio  
CATHOLIC UNIVERSITY OF AMERICA, STATISTICAL LABORATORY, MATHEMATICS DEPARTMENT,  
Washington, D. C.  
C-E-I-R, INC., 1200 Jefferson Davis Highway, Arlington 2, Virginia  
COLUMBIA UNIVERSITY, DEPARTMENT OF MATHEMATICAL STATISTICS, New York 27, N. Y.  
CORNELL UNIVERSITY, MATHEMATICS DEPARTMENT, Ithaca, New York  
FORD MOTOR COMPANY, P. O. Box 2053, Dearborn, Michigan  
GENERAL ELECTRIC COMPANY, Building C37, Room 248, Schenectady, New York  
INDIANA UNIVERSITY, THE LIBRARY, Bloomington, Indiana  
INTERNATIONAL BUSINESS MACHINES CORPORATION, MATHEMATICS AND APPLIED SCIENCE  
LIBRARY, 1271 Avenue of the Americas, New York 20, N. Y.  
IOWA STATE UNIVERSITY, STATISTICAL LABORATORY, Ames, Iowa  
LOCKHEED AIRCRAFT CORPORATION, ENGINEERING LIBRARY, Burbank, California  
MICHIGAN STATE UNIVERSITY, DEPARTMENT OF STATISTICS, East Lansing, Michigan  
MINNESOTA MINING AND MANUFACTURING COMPANY, APPLIED MATHEMATICS AND STA-  
TISTICS, St. Paul, Minnesota  
MONSANTO CHEMICAL COMPANY, 800 North Lindbergh Blvd., St. Louis 66, Missouri  
NATIONAL CASH REGISTER COMPANY, RESEARCH DEPARTMENT, Main and K Streets, Day-  
ton 9, Ohio  
NATIONAL SECURITY AGENCY, Fort George G. Meade, Maryland  
NORTHWESTERN UNIVERSITY, DEPARTMENT OF MATHEMATICS, Evanston, Illinois  
PRINCETON UNIVERSITY, DEPARTMENT OF MATHEMATICS, SECTION OF MATHEMATICAL  
STATISTICS, Princeton, New Jersey  
PURDUE UNIVERSITY LIBRARIES, Lafayette, Indiana  
RADIO CORPORATION OF AMERICA, R.C.A. LABORATORIES LIBRARY, Princeton, New Jersey  
RAMO-WOOLRIDGE CORPORATION, Los Angeles, California

*(Continued on next page)*

REMINGTON RAND—UNIVAC DIVISION, 315 Park Avenue South, New York 10, N. Y.  
SANDIA CORPORATION, Sandia Base, Albuquerque, New Mexico  
SOCONY MOBIL OIL COMPANY, INC., 150 E. 42nd Street, New York 17, New York  
SOUTHERN METHODIST UNIVERSITY, MATHEMATICS DEPARTMENT, Dallas 5, Texas  
SPACE TECHNOLOGY LABORATORIES, P. O. Box 95001, Los Angeles 45, California  
STANFORD UNIVERSITY, GIRSHICK MEMORIAL LIBRARY, Stanford, California  
STATE UNIVERSITY OF IOWA, Iowa City, Iowa  
UNION CARBIDE CORPORATION, 30 East 42nd Street, New York 17, New York  
UNION OIL COMPANY OF CALIFORNIA, UNION RESEARCH CENTER, Box 76, Brea, California  
UNITED STATES STEEL CORPORATION LIBRARY, Monroeville, Penna.  
UNIVERSITY OF CALIFORNIA, STATISTICAL LABORATORY, Berkeley, California  
UNIVERSITY OF ILLINOIS, SERIALS DEPARTMENT, Urbana, Illinois  
UNIVERSITY OF NORTH CAROLINA, DEPARTMENT OF STATISTICS, Chapel Hill, North Carolina  
UNIVERSITY OF PUERTO RICO, SCHOOL OF TROPICAL MEDICINE, San Juan, Puerto Rico  
UNIVERSITY OF WASHINGTON, LABORATORY OF STATISTICAL RESEARCH, Seattle, Washington  
W. R. GRACE AND COMPANY, RESEARCH DIVISION, Washington Research Center, Clarks-  
ville, Maryland  
W. R. GRACE AND COMPANY, DEWEY AND ALMY CHEMICAL DIVISION, 62 Whittemore Avenue,  
Cambridge 40, Massachusetts