

# THE ANNALS of MATHEMATICAL STATISTICS

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EDITED BY S. S. WILKS, 1938-1949

THE OFFICIAL JOURNAL OF THE INSTITUTE  
OF MATHEMATICAL STATISTICS

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Meeting No.	Place	Date	Abstract Deadline*	Joint With
126	Laramie, Wyoming (Annual Meeting)	August 25-28, 1970	July 6, 1970	AMS
127	Hanover, West Germany (European Regional)	August 19-26, 1970	June 30, 1970	
	University Park, Penn. (Eastern Regional)	April 21-23, 1971		ASA, ENAR
	Columbia, Missouri (Central Regional)	Spring, 1971		.....
	Ames, Iowa (Central Regional)	Spring, 1972		ASA, ENAR

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