THE ANNALS of MATHEMATICAL STATISTICS

FOUNDED AND EDITED BY H. C. CARVER, 1930-1938
EDITED BY S. S. WILKS, 1938-1949

THE OFFICIAL JOURNAL OF THE INSTITUTE OF MATHEMATICAL STATISTICS

Contents

PAGE

Curve estimates ..... MURRAY ROSENBLATT 1815

Branching processes with random environments, II: Limit theorems
KRISHNA B. ATHREYA AND SAMUEL KARLIN 1843

Exponentially bounded stopping time of sequential probability ratio tests for composite hypotheses ..... R. A. WIJSMAN 1859

A polynomial algorithm for density estimation ..... GRACE WAHBA 1870

A general qualitative definition of robustness ..... FRANK R. HAMPEL 1887

Procedures for reacting to a change in distribution ..... G. LORDEN 1897

A note on the estimation of the mode ..... EDWARD J. WEGMAN 1909

Proper scores for probability forecasters
ARLO D. HENDRICKSON AND ROBERT J. BUEHLER 1916

Random variables with independent binary digits ..... GEORGE MARSAGLIA 1922

Noncooperative stochastic games ..... MATTHEW J. SOBEL 1930

On the existence of the optimal stopping rule in the S_n/n problem when the second moment is infinite ..... M. E. THOMPSON, A. K. BASU AND W. L. OWEN 1936

A note on distribution-free statistical inference with upper and lower probabilities
R. J. BERAN 1943

A locally most powerful tied rank test in a Wilcoxon situation ..... J. KRAUTH 1949

A new proof of the Bahadur representation of quantiles and an application
J. K. GHOSH 1957

Tests of coordinate independence for a bivariate sample on a torus
EDWARD D. ROTHMAN 1962

On the Wald-optimality of rank-order tests for paired comparisons .. MALAY GHOSH 1970

Asymptotic properties of maximum likelihood estimators for the independent not identically distributed case ..... BRUCE HOADLEY 1977

Asymptotically optimal tests for finite Markov chains ..... LUIS B. BOZA 1992

Transformations of Gaussian processes to the Brownian motion
P. K. BHATTACHARYA 2008

On the asymptotic distribution of the sequences of random variables with random indices ..... SILVIU GUIAŞU 2018

Continued on back cover

Vol. 42, No. 6—December 1971

Institute of Mathematical Statistics is collaborating with JSTOR to digitize, preserve, and extend access to *The Annals of Mathematical Statistics.*
www.jstor.org

THE INSTITUTE OF MATHEMATICAL STATISTICS

(Organized September 12, 1935)

The purpose of the Institute of Mathematical Statistics is to encourage the development, dissemination, and application of mathematical statistics.

OFFICERS

President:
R. C. Bose, Department of Mathematics and Statistics, Colorado State University, Fort Collins, Colorado 80521

President-Elect:
Lucien LeCam, Department of Statistics, University of California, Berkeley, California 94720

Executive Secretary:
Leo Katz, Statistical Laboratory, Michigan State University, East Lansing, Michigan 48823

Program Secretary:
R. V. Hogg, Department of Statistics, University of Iowa, Iowa City, Iowa 52240

Treasurer:
George J. Resnikoff, Department of Statistics, California State College, Hayward, California 94542

Editor: *Annals of Statistics*
Ingram Olkin, Editor, Annals of Statistics, Educational Testing Service, Princeton, New Jersey 08540. *Address after June* 15, 1972: Ingram Olkin, Department of Statistics, Stanford University, Stanford, California 94305

Editor: *Annals of Probability*
Ronald Pyke, Department of Mathematics, University of Washington, Seattle, Washington 98105

Managing Editor:
K. J. C. Smith, Department of Statistics, University of North Carolina, Chapel Hill, North Carolina 27514

Membership. Membership dues including a subscription to the ANNALS OF MATHEMATICAL STATISTICS are $20.00 per year for residents of the United States or Canada and $12.00 per year for residents of other countries. Special rates of $10.00 per year are available to students. Inquiries regarding membership in the Institute should be sent to the Treasurer of the Institute.

Subscription Rates. Current issues of the *Annals of Mathematical Statistics* are $30.00 per volume (six issues of one calendar year). Members of the Institute of Mathematical Statistics pay different rates (see above). Single issues are $5.75. Back numbers for all issues up to and including 1956 (Volume 27) are $16.50 per volume, $4.50 per issue; issues from 1957 through 1964 are $22.00 per volume, $5.75 per issue; from 1965 onward, $33.00 per volume, $5.75 per issue. Complete sets (Volumes 1 through 27) are $275.00. If additional volumes are ordered at the same time complete sets are ordered, the price is $16.50 per volume for Volumes 28 through 35, and $22.00 per volume for Volumes 36 through 40. Complete sets (Volumes 1 through 40) are $500.00.

Communications concerning **subscriptions, back numbers, payment of dues, membership, changes of address, etc.**, should be addressed to the Treasurer. **Advertising** correspondence should be directed to the Managing Editor.

Mail to the *Annals of Mathematical Statistics* should be addressed to either the Editor, Managing Editor or the Treasurer, as described above. It should *not* be addressed to Waverly Press.

PRINTED AT THE
WAVERLY PRESS, INC., BALTIMORE, MARYLAND 21202 U.S.A.
Second-class postage paid at Baltimore, Maryland 21202 and at additional mailing offices.

Copyright © 1971 by the Institute of Mathematical Statistics

EDITORIAL STAFF

EDITORS

Annals of Statistics
INGRAM OLKIN

Annals of Probability
RONALD PYKE

ASSOCIATE EDITORS

R. L. ANDERSON
R. R. BAHADUR
PETER J. BICKEL
DAVID R. BRILLINGER
L. BROWN
H. D. BRUNK
THOMAS S. FERGUSON
RONALD K. GETOOR
S. S. GHURYE
JAROSLAV HÁJEK
W. J. HALL
JAMES F. HANNAN
DAVID L. HANSON
PETER J. HUBER
DONALD L. IGLEHART
J. H. B. KEMPERMAN
J. F. C. KINGMAN
JOHN W. LAMPERTI
ALBERT W. MARSHALL
PETER NEY
JEROME SACKS
I. R. SAVAGE
J. SETHURAMAN
FRANK L. SPITZER
HENRY TEICHER

MANAGING EDITOR

K. J. C. SMITH

PAST EDITORS OF THE ANNALS

H. C. CARVER, 1930–1938
S. S. WILKS, 1938–1949
T. W. ANDERSON, 1950–1952
E. L. LEHMANN, 1953–1955
T. E. HARRIS, 1955–1958
WILLIAM KRUSKAL, 1958–1961
J. L. HODGES JR., 1961–1964
D. L. BURKHOLDER, 1964–1967
Z. W. BIRNBAUM, 1967–1970

Published bimonthly by the Institute of Mathematical Statistics in February, April, June, August, October and December.

EDITORIAL POLICY

The main aim of the *Annals of Mathematical Statistics* is to publish original contributions to theoretical statistics and to probability theory. The emphasis is on quality, not just on formal novelty and correctness of the contents of a paper. Expository papers are also welcome, especially if they provide authoritative surveys of areas that have been in vigorous development.

NOTICE

Manuscripts submitted for publication in the *Annals of Statistics* between August 15, 1971, and June 15, 1972, should be sent to the Editor at the following address: Ingram Olkin, Editor, Annals of Statistics, Educational Testing Service, Princeton, New Jersey 08540.

IMS INSTITUTIONAL MEMBERS

AEROSPACE CORPORATION
El Segundo, California

THE AMERICAN COLLEGE TESTING PROGRAM
Iowa City, Iowa

ARIZONA STATE UNIVERSITY
Tempe, Arizona

ARTHUR D. LITTLE, INC.
Cambridge, Massachusetts

BELL TELEPHONE LABORATORIES, TECHNICAL LIBRARY
New York, New York

BOEING SCIENTIFIC RESEARCH LABORATORIES
Seattle, Washington

CALIFORNIA STATE COLLEGE, HAYWARD, DEPARTMENT OF STATISTICS
Hayward, California

CASE WESTERN RESERVE UNIVERSITY, DEPARTMENT OF MATHEMATICS
Cleveland, Ohio

CATHOLIC UNIVERSITY OF AMERICA, STATISTICAL LABORATORY
Washington, D.C.

CHEVRON RESEARCH COMPANY
Richmond, California

CIBA PHARMACEUTICAL COMPANY
Summit, New Jersey

COLORADO STATE UNIVERSITY, STATISTICAL LABORATORY
Fort Collins, Colorado

CORNELL UNIVERSITY, DEPARTMENT OF MATHEMATICS
Ithaca, New York

FLORIDA STATE UNIVERSITY, DEPARTMENT OF STATISTICS
Tallahassee, Florida

FORD MOTOR COMPANY, SCIENTIFIC LABORATORIES
Dearborn, Michigan

GENERAL MOTORS CORPORATION, RESEARCH LABORATORIES
Warren, Michigan

GEORGE WASHINGTON UNIVERSITY, DEPARTMENT OF STATISTICS
Washington, D.C.

INDIANA UNIVERSITY, THE LIBRARY
Bloomington, Indiana

INTERNATIONAL BUSINESS MACHINES CORPORATION
Armonk, New York

IOWA STATE UNIVERSITY, STATISTICAL LABORATORY
Ames, Iowa

JOHNS HOPKINS UNIVERSITY, DEPARTMENT OF BIOSTATISTICS, DEPARTMENT OF STATISTICS
Baltimore, Maryland

KANSAS STATE UNIVERSITY, DEPARTMENT OF STATISTICS
Manhattan, Kansas

KENT STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS
Kent, Ohio

KNOLLS ATOMIC POWER LABORATORY
Schenectady, New York

MIAMI UNIVERSITY, DEPARTMENT OF MATHEMATICS
Oxford, Ohio 45056

MICHIGAN STATE UNIVERSITY, DEPARTMENT OF STATISTICS
East Lansing, Michigan

MINNESOTA MINING AND MANUFACTURING COMPANY, APPLIED MATHEMATICS AND STATISTICS
Saint Paul, Minnesota

NATIONAL SECURITY AGENCY
Fort George G. Meade, Maryland

NEW MEXICO STATE UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES
Las Cruces, New Mexico

NORTHWESTERN UNIVERSITY, DEPARTMENT OF MATHEMATICS
Evanston, Illinois

OHIO STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS, STATISTICAL LABORATORY
Columbus, Ohio

OKLAHOMA STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS AND STATISTICS
Stillwater, Oklahoma

OREGON STATE UNIVERSITY, DEPARTMENT OF STATISTICS
Corvallis, Oregon

PENNSYLVANIA STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS LIBRARY
University Park, Pennsylvania

PRINCETON UNIVERSITY, DEPARTMENT OF STATISTICS
Princeton, New Jersey

PURDUE UNIVERSITY LIBRARIES
Lafayette, Indiana

THE RAND CORPORATION
Santa Monica, California

THE ROCKEFELLER UNIVERSITY
New York, New York

ROCKLAND STATE HOSPITAL, RESEARCH CENTER
Orangeburg, New York

SANDIA CORPORATION, SANDIA BASE
Albuquerque, New Mexico

SIMON FRASER UNIVERSITY, DEPARTMENT OF MATHEMATICS
Burnaby, British Columbia, Canada

SOUTHERN METHODIST UNIVERSITY, DEPARTMENT OF STATISTICS
Dallas, Texas

STANFORD UNIVERSITY, GIRSCHICK MEMORIAL LIBRARY
Stanford, California

STATE UNIVERSITY OF NEW YORK, BUFFALO, DEPARTMENT OF STATISTICS
Amherst, New York

TEMPLE UNIVERSITY, DEPARTMENT OF MATHEMATICS
Philadelphia, Pa.

THE TOBACCO INSTITUTE
Washington, D.C.

UNION OIL COMPANY OF CALIFORNIA, UNION RESEARCH CENTER
Brea, California

UNITED STATES ARMY RESEARCH AND DEVELOPMENT CENTER
Aberdeen Proving Ground, Maryland

UNIVERSITY OF ALBERTA, DEPARTMENT OF MATHEMATICS
Edmonton, Alberta, Canada

UNIVERSITY OF ARIZONA, DEPARTMENT OF MATHEMATICS
Tucson, Arizona

UNIVERSITY OF BRITISH COLUMBIA, DEPARTMENT OF MATHEMATICS
Vancouver, B.C., Canada

UNIVERSITY OF CALGARY, MATHEMATICS DEPARTMENT
Calgary 44, Alberta, Canada

UNIVERSITY OF CALIFORNIA, BERKELEY, STATISTICAL LABORATORY
Berkeley, California

UNIVERSITY OF CALIFORNIA, DEPARTMENT OF MATHEMATICS
Davis, California

UNIVERSITY OF CALIFORNIA, DEPARTMENT OF MATHEMATICS
Irvine, California

UNIVERSITY OF CALIFORNIA, DEPARTMENT OF MATHEMATICS
Riverside, California

UNIVERSITY OF CALIFORNIA, DEPARTMENT OF MATHEMATICS
Santa Barbara, California

UNIVERSITY OF GUELPH, LIBRARY, SERIALS DEPARTMENT
Guelph, Ontario, Canada

UNIVERSITY OF ILLINOIS AT CHICAGO CIRCLE, DEPARTMENT OF MATHEMATICS
Chicago, Illinois

UNIVERSITY OF ILLINOIS, LIBRARY, SERIALS DEPARTMENT
Urbana, Illinois

UNIVERSITY OF IOWA, DIVISION OF MATHEMATICAL SCIENCES
Iowa City, Iowa

UNIVERSITY OF MANITOBA, DEPARTMENT OF STATISTICS
Winnipeg-19, Manitoba, Canada

UNIVERSITY OF MARYLAND, DEPARTMENT OF MATHEMATICS
College Park, Maryland

UNIVERSITY OF MASSACHUSETTS, DEPARTMENT OF MATHEMATICS & STATISTICS
Amherst, Massachusetts

UNIVERSITY OF MICHIGAN, DEPARTMENT OF STATISTICS
Ann Arbor, Michigan

UNIVERSITY OF MINNESOTA, DEPARTMENT OF STATISTICS
Minneapolis, Minnesota

UNIVERSITY OF MISSOURI, DEPARTMENT OF STATISTICS
Columbia, Missouri

UNIVERSITY OF MISSOURI AT ROLLA, DEPARTMENT OF MATHEMATICS
Rolla, Missouri

UNIVERSITY OF MONTREAL, DEPARTMENT OF MATHEMATICS
Montreal, Quebec, Canada

UNIVERSITY OF NEW MEXICO, DEPARTMENT OF MATHEMATICS & STATISTICS
Albuquerque, New Mexico

UNIVERSITY OF NORTH CAROLINA, DEPARTMENT OF STATISTICS
Chapel Hill, North Carolina

UNIVERSITY OF OTTAWA, DEPARTMENT OF MATHEMATICS
Ottawa, Ontario, Canada

UNIVERSITY OF PENNSYLVANIA, THE LIBRARY
Philadelphia, Pennsylvania

UNIVERSITY OF PUERTO RICO, DEPARTMENT OF MATHEMATICS
Mayaguez, Puerto Rico

UNIVERSITY OF ROCHESTER
Rochester, New York

UNIVERSITY OF WASHINGTON, LABORATORY OF STATISTICAL RESEARCH
Seattle, Washington

UNIVERSITY OF WISCONSIN, MADISON, DEPARTMENT OF STATISTICS
Madison, Wisconsin

UNIVERSITY OF WISCONSIN, MILWAUKEE, DEPARTMENT OF MATHEMATICS
Milwaukee, Wisconsin

WAYNE STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS
Detroit, Michigan

WEST CHESTER STATE COLLEGE
West Chester, Pennsylvania

WESTAT RESEARCH, INC.
Bethesda, Maryland

WESTINGHOUSE ELECTRIC CORPORATION, RESEARCH LABORATORIES
Pittsburgh, Pennsylvania

MEETINGS OF THE INSTITUTE

NOTE: This calendar lists all of the meetings which have been approved by the IMS Council to date. Meeting dates which fall rather far in the future are subject to change.

Meeting No.	Place	Date	Abstract Deadline*	Joint With
132	Blacksburg, Virginia (Eastern Regional)	April 17–19, 1972	February 16, 1972	ASA
133	Ames, Iowa (Central Regional)	April 26–28, 1972	February 16, 1972	ASA, ENAR
134	Seattle, Washington (Western Regional)	June 19–21, 1972	April 19, 1972	
135	Montreal, Canada (Special Eastern Regional)	August 13–17, 1972	June 14, 1972	
136	Hanover, New Hampshire (Annual)	August 28–31, 1972	June 14, 1972	AMS
	Ithaca, New York (Eastern Regional)	Spring 1973		ASA, ENAR
	Albuquerque, New Mexico (Western Regional)	Spring 1973		

* Abstracts should be submitted in duplicate to the Executive Secretary on abstract blanks which may be obtained from him. Abstracts of papers to be presented *in person* at the meetings must be received on or before the corresponding deadline. Only one contributed paper may be given in person at any one meeting. Abstracts contributed by title may be submitted at any time and will be printed in the next issue to go to press. Abstracts should not exceed 200 words and should avoid displayed expressions and complicated formulae. They can be accepted from non-members of the IMS only if introduced by members. Abstracts must follow the stylistic requests on the abstract blank or they may be returned.