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ERRATUM TO "CUBIC EQUATIONS FOR THE HYPERELLIPTIC LOCUS", ASIAN J. MATH., VOL. 8, NO. 1, 161–172, 2004*

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The mistake in our paper [1] is at the end of the proof of lemma 4, and the correction is to replace lemma 4 by a general position assumption in theorem 3.

Theorem 3. (italics indicates the change made) Let X be an irreducible principally polarized abelian variety of dimension g, and let $A_0, \ldots, A_{g+1}$ be distinct points of X. Suppose that $\forall z \in X$ the $g+2$ points $K(A_i+z)$ in $\mathbb{C}^{2^g}$ are linearly dependent. *Suppose moreover that there exist some k and l such that for $y := -\frac{A_k+A_l}{2}$ the linear span of the points $K(A_i+y)$ is of dimension precisely $g+1$, and not less.* Then X is the Jacobian of some curve C, and all $A_i \in A(C)$.

Proof. Indeed, we know that for all $z \in X$ there must exist some numbers $c_i(z)$ such that $\sum_{i=0}^{g+1} c_i(z)K(A_i+z) = 0$. By the new assumption the rank of the $(g+2)\times 2^g$ matrix $K(A_i+y)$ is equal to $g+1$. Thus by continuity the rank of $K(A_i+z)$ is also equal to $g+1$ for all z sufficiently close to y. Thus locally near y the functions $c_i(z)$ are unique, up to a common factor, and we can eliminate Lemma 4 and follow the rest of the proof of Theorem 3 from the top of page 166 in [1]. ∎

For all the other results in [1], the extra condition above also needs to be added for the results to hold. Moreover, for proposition 8 and sections 4 and 5 of [1], where we use the results of [BK2] to write down the coefficients $c_i(z)$ explicitly, one should also assume that not all coefficients $c_i(z)$ are identically zero in z — otherwise we do not have any collinearity to start with. This means that the results hold unless $\theta(Q+R) = \theta(R) = 0$ in the non-hyperelliptic case, and unless $\theta(R) = \theta(Q+R) = \theta(Q+A_k+R) = 0$ for all k in the hyperelliptic case (in this case $\theta(R)$ cancels with the $\theta(2A_k+R)$ in the denominator in formula (4) in [1]). Since $R = K - A_1 - \ldots - A_g$, where K is the Riemann's constant and thus in particular $\theta(A_k+R) = 0$ for all $k = 1\ldots g$; this is very similar, but not quite the same, as the θ-general position condition of [2], see below.

We thank Mihnea Popa and Giuseppe Pareschi for pointing out to us that the hypothesis of theorem 3 in our published paper [1] is not strong enough. Their Castelnuovo-Schottky lemma in their July 2004 preprint [2] is equivalent to our theorem 3, but with a different general position assumption.

REFERENCES

[1] Grushevsky, S, *Cubic equations for the hyperelliptic locus*, special issue dedicated to Yum-Tong Siu on his 60th birthday, Asian Journal of Mathematics, 8:1 (2004), pp. 161–172.
[2] Pareschi, G., and Popa, M., *Castelnuovo theory and the geometric Schottky problem*, preprint math.AG/0407370.

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