

## LUTZ TWIST AND CONTACT SURGERY\*

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**Abstract.** For any knot  $T$  transverse to a given contact structure on a 3-manifold, we exhibit a Legendrian two-component link  $\mathbb{L} = L_1 \sqcup L_2$  such that  $T$  equals the transverse push-off of  $L_1$  and contact (+1)-surgery on  $\mathbb{L}$  has the same effect as a Lutz twist along  $T$ .

**Key words.** contact structure, Lutz twist, contact  $r$ -surgery, surgery diagram, Legendrian and transverse knots

**AMS subject classifications.** 53D10, 57R65, 58A30

**1. Introduction.** The theorem of Lutz and Martinet [12] asserts that any closed, oriented 3-manifold  $Y$  admits a contact structure in each homotopy class of tangent 2-plane fields. Here 2-plane fields are understood to be cooriented; a *contact structure* is understood to be cooriented and positive, that is, a 2-plane field  $\xi$  defined as the kernel of a global 1-form  $\alpha$  on  $Y$  such that  $\alpha \wedge d\alpha$  is a positive volume form.

The cited paper by Martinet only covers the existence of *some* contact structure on a given  $Y$ ; for a proof of the existence of such a structure in every homotopy class of 2-plane fields see [9], which gives a proof of that result along the lines of the original (and never fully published) argument by Lutz.

The key to that second step is what is nowadays known as a *Lutz twist*, a surgery on a knot  $T$  in a given contact manifold  $(Y, \xi)$  — with  $T$  transverse to  $\xi$  — that is topologically trivial (i.e., does not change  $Y$ ), but transforms  $\xi$  to a contact structure  $\xi'$  in a different homotopy class of 2-plane fields.

In a series of papers [1, 2, 3] we described a notion of contact  $r$ -surgery on *Legendrian* knots in a contact manifold (that is, knots tangent to the given contact structure), where  $r \in \mathbb{Q}^* \cup \{\infty\}$  denotes the framing of the surgery relative to the natural contact framing of the Legendrian knot. This generalises the contact surgery introduced by Eliashberg [5] and Weinstein [14], which in our language is a contact (−1)-surgery. Amongst other things, we discussed explicit surgery diagrams for various contact manifolds and gave an alternative proof of the Lutz-Martinet theorem via such Legendrian surgeries. We did not, however, fully elucidate the relation between our surgery diagrams and the Lutz twist (although the principal connection was described in [2], cf. [8]). The intention of the present note is to give an explicit Legendrian surgery diagram for the Lutz twist. In particular, this yields surgery representations for all contact structures on  $S^3$  analogous to [3] and provides concrete realisations for the considerations in Section 6 of [2].

We shall henceforth assume that the reader is familiar with the basics of this notion of contact surgery; if not, the best place to start may well be [3], cf. also [13]. One fact from [1, 2] we should like to recall here is that contact (+1)-surgery is the inverse of contact (−1)-surgery.

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**2. The Lutz twist.** We briefly recall the definition of the Lutz twist, cf. [9]. Let  $T$  be a knot transverse to a contact structure  $\xi$  on a 3-manifold  $Y$ . Then there is a tubular neighbourhood  $\nu T$  of  $T$  that is contactomorphic to the solid torus  $S^1 \times D_\delta^2$  (with  $D_\delta^2$  denoting the 2-disc of radius  $\delta$ ) for some suitable  $\delta > 0$ , with contact structure  $\zeta = \ker(d\theta + r^2 d\varphi)$ , where  $\theta$  denotes the  $S^1$ -coordinate, and  $r, \varphi$  are polar coordinates on  $D_\delta^2$ . For ease of notation we identify  $(\nu T, \xi)$  with  $(S^1 \times D_\delta^2, \zeta)$ .

A *simple Lutz twist* along  $T$  is the operation that replaces the contact structure  $\xi$  on  $Y$  by the one that coincides with  $\xi$  outside  $\nu T$ , and on  $S^1 \times D_\delta^2$  is given by

$$\zeta' = \ker(h_1(r) d\theta + h_2(r) d\varphi),$$

where  $h_1, h_2: [0, \delta] \rightarrow \mathbb{R}$  are smooth functions satisfying the following conditions:

- (i)  $h_1(r) = -1$  and  $h_2(r) = -r^2$  for  $r$  near 0,
- (ii)  $h_1(r) = 1$  and  $h_2(r) = r^2$  for  $r$  near  $\delta$ ,
- (iii)  $(h_1(r), h_2(r))$  is never parallel to  $(h_1'(r), h_2'(r))$  — in particular, neither of them is ever equal to  $(0, 0)$  —,
- (iv)  $h_1$  has exactly one zero on the interval  $[0, \delta]$ .

The boundary conditions (i) and (ii) ensure that  $\zeta'$  is defined around  $r = 0$  and coincides with  $\zeta$  near  $r = \delta$ ; (iii) is the condition for  $\zeta'$  to be a contact structure; condition (iv) fixes the homotopy class (as 2-plane field) of the new contact structure.

The contact structure  $\zeta'$  is a so-called *overtwisted* contact structure in the sense of Eliashberg [4], and as shown in that paper (specifically, Theorem 3.1.1), the classification of such overtwisted contact structures up to isotopy fixed near the boundary coincides with the classification of 2-plane fields up to homotopy rel boundary. An immediate consequence of that classification is that the contact structure on  $Y$  obtained from  $\xi$  by a Lutz twist along  $T$  is (up to isotopy) independent of any of the choices in the construction described above.

**3. The surgery diagram for a Lutz twist.** Let  $(Y, \xi)$  be a given contact 3-manifold and  $T$  a knot in  $Y$  transverse to  $\xi$ . We want to describe a Legendrian link  $\mathbb{L}$  in  $Y$  such that  $(+1)$ -contact surgery on  $\mathbb{L}$  has the same effect as a Lutz twist along  $T$ . By [2] we may assume that  $(Y, \xi)$  has been obtained from  $S^3$  with its standard contact structure  $\xi_{st}$  by contact  $(\pm 1)$ -surgery on a Legendrian link in  $(S^3, \xi_{st})$ . Thus,  $(Y, \xi)$  can be represented by the front projection (to the  $yz$ -plane) of this Legendrian link, considered as a link in  $\mathbb{R}^3$  with its standard contact structure  $\xi_{st} = \ker(dz + x dy)$ , which is contactomorphic to  $(S^3, \xi_{st})$  with a point removed. Then  $\mathbb{L}$  can be represented in the same front projection diagram.

For the representation of Legendrian and transverse knots via their front projection we refer to [6, 7, 10]. Beware that these three papers use three different conventions for writing the standard contact structure on  $\mathbb{R}^3$ . We follow the one from [10] (which is also that of [9]). The positive transversality condition  $\dot{z} + x\dot{y} > 0$  for a curve  $t \mapsto (x(t), y(t), z(t))$  implies that in the front projection of a positively transverse knot there can be no vertical tangencies going downwards ( $\dot{y} = 0, \dot{z} < 0$ ), and all but the crossing shown in Figure 1 are possible.

From these front projections it is easy to describe the positive transverse push-off of an oriented Legendrian knot: smooth the up-cusps and replace the down-cusps by kinks (there is only one possibility for the sign of the crossing in this kink). Similarly, one can easily describe an oriented Legendrian knot whose positive transverse push-off is a given transverse knot, cf. [6]:

- (i) In the front projection of the given transverse knot (oriented positively), replace vertical (upwards) tangencies by cusps.

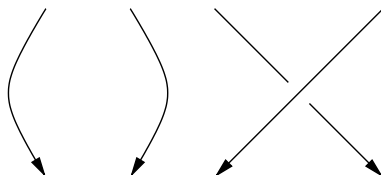


FIG. 1. Impossible front projections of positively transverse curve.

- (ii) By Figure 1, in those crossings of a positively transverse knot that cannot be interpreted as the front projection of a Legendrian knot, at least one of the strands is pointing up ( $\dot{z} > 0$ ). If one adds a zigzag to that strand (if both are going up, either can be chosen), it is possible to realise the given crossing by the front projection of a Legendrian curve.

Therefore, the following theorem gives a complete surgery description of Lutz twists.

**THEOREM.** *Let  $L_1$  be an oriented Legendrian knot in  $(Y, \xi)$ , represented by the front projection of a Legendrian knot in  $(\mathbb{R}^3, \xi_{st})$  disjoint from the link describing  $(Y, \xi)$ . Let  $L_2$  be the Legendrian push-off of  $L_1$  with two additional up-zigzags (see Figure 2). Let  $\xi'$  be the contact structure on  $Y$  obtained from  $\xi$  by contact  $(+1)$ -surgery on both  $L_1$  and  $L_2$ , and  $\xi''$  the contact structure obtained from  $\xi$  by a simple Lutz twist along the positive transverse push-off  $T$  of  $L_1$ . Then  $\xi'$  and  $\xi''$  are isotopic via an isotopy fixed outside a tubular neighbourhood of  $L_1$ .*

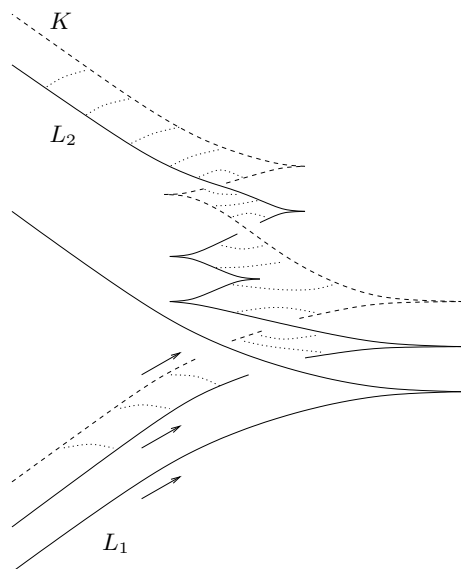


FIG. 2. Surgery diagram for Lutz twist.

The proof of this theorem proceeds as follows: First of all, we verify that the described surgeries on  $L_1$  and  $L_2$  taken together do not change the manifold  $Y$ . Secondly, we check that the resulting contact structure is overtwisted by exhibiting an explicit overtwisted disc. Then, again by Eliashberg's classification of overtwisted

contact structures, and thanks to the fact that the two surgeries only change the contact structure in a tubular neighbourhood of  $L_1$  (which contains the overtwisted disc just mentioned), it suffices to show that the described surgeries and the corresponding Lutz twist have the same effect on the homotopy class of the contact structure, regarded as a mere plane field.

(1) Recall that contact  $r$ -surgery on a Legendrian knot  $L$  means that topologically we perform surgery with coefficient  $r \in \mathbb{Q}^* \cup \{\infty\}$  relative to the contact framing of  $L$ , which is determined by a vector field along  $L$  transverse to the contact structure. In the front projection picture this corresponds to pushing  $L$  in the  $z$ -direction, and it is this what we mean by *the* Legendrian push-off of  $L$ . As shown in [1], contact  $(+1)$ -surgery along  $L$  and contact  $(-1)$ -surgery along its Legendrian push-off cancel each other, and in particular do not change the underlying manifold. Since adding two zigzags to a Legendrian knot adds two negative twists to its contact framing, we see that topologically the two contact  $(+1)$ -surgeries on  $L_1$  and  $L_2$  are the same as a contact  $(+1)$ -surgery along  $L_1$  and a  $(-1)$ -surgery along its Legendrian push-off, and hence topologically trivial.

We can easily see this directly: Write  $t$  for the Thurston-Bennequin invariant of  $L_1$ , so that the linking number between  $L_1$  and its Legendrian push-off (or with  $L_2$ ) is given by  $\ell k(L_1, L_2) = t$ . Then the topological framings (i.e., framings relative to the surface framing) of the surgeries are  $n_1 = t + 1$  and  $n_2 = t - 1$ . After a handle slide (cf. [11, Chapter 5]) we may replace the link  $(L_1, L_2)$  by  $(L_1, L_2 - L_1)$ , with linking number  $\ell k(L_1, L_2 - L_1) = t - n_1 = -1$  and framing of  $L_2 - L_1$  equal to

$$(L_2 - L_1)^2 = L_2^2 + L_1^2 - 2\ell k(L_2, L_1) = n_2 + n_1 - 2t = 0.$$

By construction,  $L_2 - L_1$  is an unknot, and the computation above shows that it is a 0-framed meridian of  $L_1$ , which proves the claim that the composition of the two surgeries is topologically trivial.

(2) We next exhibit the overtwisted disc in the manifold obtained by the contact surgeries along  $L_1$  and  $L_2$ . Let  $K$  be the knot indicated in Figure 2, i.e., the Legendrian push-off of  $L_1$  with one additional zigzag and with one extra negative linking with  $L_2$ . Alternatively,  $L_2$  may be regarded as the Legendrian push-off of  $K$  with one additional zigzag. The surface framing of  $L_2$  determined by the Seifert surface of the oriented link  $(-L_2) \sqcup K$  indicated in Figure 2 is equal to  $t - 1$  (the contact framing of  $K$ ), hence equal to the topological framing used for the surgery on  $L_2$ . So that Seifert surface glued to the meridional disc used for the surgery on  $L_2$  defines a disc with boundary  $K$  in the surgered manifold. The surface framing of  $K$  determined by that disc equals the contact framing  $t - 1$ , which is exactly the condition for an overtwisted disc.

The above verification that  $K$  is the boundary of an overtwisted disc in the surgered manifold is completely straightforward. Nonetheless, it may be instructive to see that  $K$  is not found by accident. Start with a meridian  $K'$  to both  $L_1$  and  $L_2$ , that is, an unknot with  $\ell k(K', L_1) = \ell k(K', L_2) = 1$ . If surgery along  $L_1$  and  $L_2$  has any chance of being a Lutz twist, we expect  $K'$  to be isotopic to the boundary of an overtwisted disc.

There is an obvious pair of pants with boundary the oriented link  $L_1 \sqcup (-K') \sqcup (-L_2)$  that gives  $K'$  the surface framing  $n_{K'} = 0$  and  $L_1, L_2$  the framing  $n_1 = t + 1, n_2 = t - 1$ , respectively. Now perform a handle slide of  $-K'$  over the 2-handle attached to  $L_1$  (corresponding to the surgery) to form, in the surgered manifold, the

knot  $L_1 - K'$ , which is the knot  $K$  from above. We compute the linking numbers

$$\begin{aligned}\ell k(L_1 - K', L_1) &= n_1 - 1 = t, \\ \ell k(L_1 - K', L_2) &= \ell k(L_1, L_2) - 1 = t - 1,\end{aligned}$$

and the surface framing of  $L_1 - K'$ , now with respect to the annulus with boundary  $(L_1 - K') \sqcup (-L_2)$ :

$$\begin{aligned}(L_1 - K')^2 &= n_1 + n_{K'} - 2 \ell k(L_1, K') \\ &= (t + 1) + 0 - 2 \cdot 1 \\ &= t - 1,\end{aligned}$$

which is exactly what we had found for  $K$  before.

(3) It remains to be shown that the topological effect of the surgeries described in the theorem has the same effect on the homotopy class of the contact structure (regarded merely as a plane field) as a Lutz twist. By the neighbourhood theorem for Legendrian submanifolds (cf. [9]), the particular nature of  $L_1$  is irrelevant for this consideration. Therefore we can localise the problem in a tubular neighbourhood of  $L_1$ , and so it suffices to consider specific examples for  $L_1$ , where the effect of the surgeries on the obstruction classes determining the homotopy type of the plane field can be computed explicitly.

For the following considerations cf. [9]. The tangent bundle of the solid torus  $S^1 \times D^2$  being trivial, (cooriented) tangent 2-plane fields on  $S^1 \times D^2$  can be identified with maps  $S^1 \times D^2 \rightarrow S^2$ . Thus, the obstructions to homotopy of 2-plane fields on  $S^1 \times D^2$  rel boundary  $T^2$  are in

$$H^2(S^1 \times D^2, T^2; \pi_2(S^2)) \cong \mathbb{Z}$$

and

$$H^3(S^1 \times D^2, T^2; \pi_3(S^2)) \cong \mathbb{Z}.$$

The first obstruction corresponds to the extension of a given 2-plane field along  $T^2$  over a meridional disc of the solid torus and is detected by the (relative) first Chern class of the plane field (here the absence of 2-torsion is crucial). The second obstruction relates to the extension of the plane field over the 3-cell one needs to attach to  $T^2 \cup$  (meridional disc) to form the solid torus. This obstruction is captured by the Hopf invariant.

(3a) In order to deal with the first obstruction, we consider  $Y = S^1 \times S^2 \subset S^1 \times \mathbb{R}^3$  with its standard tight contact structure  $\xi = \ker(x d\theta + y dz - z dy)$ , in obvious notation, and take  $L_1$  to be an oriented Legendrian knot in the homology class of  $S^1 \times \{pt.\}$ . The contact manifold  $(S^1 \times S^2, \xi)$  can be represented by contact (+1)-surgery on a Legendrian unknot  $L_0$  with only two cusps, see [3]. For  $L_1$  we may then take another such unknot linked once with  $L_0$ , and for  $L_2$  its Legendrian push-off with additional zigzags as in the theorem. Write  $\xi'$  for the contact structure on  $Y$  obtained by performing contact (+1)-surgery on  $L_1$  and  $L_2$ .

The contact structure  $\xi$  has first Chern class  $c_1(\xi) = 0$ . This follows from the observation that the vector field

$$(z - y) \partial_x + x \partial_y - x \partial_z + (y + z) \partial_\theta$$

defines a trivialisation of  $\xi$ . Alternatively, this is a consequence of the homological computations in Section 3 of [3], given the fact that the rotation number  $\text{rot}(L_0)$  (with any orientation on  $L_0$ ), which can be computed from the front projection as  $(\#(\text{down-cusps}) - \#(\text{up-cusps}))/2$ , is equal to 0.

In the sequel we assume that the reader is familiar with those homological computations. Write  $\mu_1, \mu_2$  for the meridional circles to  $L_1, L_2$ , respectively, as well as the homology classes they represent in the homology of the surgered manifold. Then, with  $PD$  denoting the Poincaré duality isomorphism from cohomology to homology,

$$\begin{aligned} c_1(\xi') &= \text{rot}(L_1)PD^{-1}(\mu_1) + \text{rot}(L_2)PD^{-1}(\mu_2) \\ &= -2PD^{-1}(\mu_2). \end{aligned}$$

(This would be true even if  $\text{rot}(L_1) \neq 0$ , since  $\mu_1 + \mu_2$  bounds a disc in  $Y$  also after the surgery.)

Let  $L'_1$  be a Legendrian push-off of  $L_1$ . Then the surgery along  $L_1$  and  $L_2$  may be assumed to occur in a tube containing  $L_1$  and  $L_2$ , but not  $L'_1$ . This implies that  $L'_1$  represents the same homology class in  $H_1(Y)$  both before and after the surgery. Since  $\ell k(L'_1, L_2) = t$  and along  $L_2$  we perform surgery with topological framing  $t - 1$ , we have that  $L'_1 - \mu_2$  is homologically trivial in the surgered manifold. (With the specific choice of  $L_1$  suggested above we would have  $t = -1$ .) Hence

$$\mu_2 = [L'_1] = [L_1] \in H_1(Y),$$

so that

$$c_1(\xi') = -2PD^{-1}([L_1]),$$

which is the same as for a Lutz twist along the positive transverse push-off of  $L_1$  (i.e., a transverse knot in the homology class of  $L_1$ ), see [9, Prop. 3.15].

Since  $[L_1]$  generates  $H_1(Y)$  in this example, this fully determines the effect of the surgery on the 2-dimensional obstruction class.

(3b) Finally, in order to see that the effect that the surgery on the link  $\mathbb{L} = L_1 \sqcup L_2$  has on the 3-dimensional obstruction is the same as that of a Lutz twist along a positive transverse push-off of  $L_1$ , it is sufficient to consider an arbitrary oriented Legendrian knot  $L_1$  in  $(S^3, \xi_{st})$ . Set  $r = \text{rot}(L_1)$ , so that  $\text{rot}(L_2) = r - 2$ . As before we write  $t$  for the Thurston-Bennequin invariant of  $L_1$ , so that the Thurston-Bennequin invariant of  $L_2$  equals  $t - 2$ . Let  $X$  be the handlebody obtained from  $D^4$  by attaching two 2-handles corresponding to the two surgeries. Let  $c \in H^2(X)$  be the cohomology class that evaluates to  $\text{rot}(L_i)$  on the surface in  $X$  given by gluing a Seifert surface (with induced orientation) of  $L_i$  in  $D^4$  with the core disc of the corresponding handle,  $i = 1, 2$ . Since we perform  $q = 2$  contact (+1)-surgeries, Corollary 3.6 of [3] tells us that the 3-dimensional invariant of the contact structure  $\xi'$  obtained by these surgeries is given by

$$\begin{aligned} d_3(\xi') &= \frac{1}{4}(c^2 - 3\sigma(X) - 2\chi(X)) + q \\ &= \frac{1}{4}c^2 - \frac{3}{4}\sigma(X) + \frac{1}{2}. \end{aligned}$$

The signature  $\sigma(X)$  is the signature of the matrix  $\begin{pmatrix} t+1 & t \\ t & t-1 \end{pmatrix}$ , hence equal to 0. Moreover, by that same formula we have  $d_3(\xi_{st}) = -1/2$ . So the change in the

$d_3$ -invariant caused by the surgery is

$$d_3(\xi') - d_3(\xi_{st}) = \frac{1}{4}c^2 + 1.$$

As shown in Section 3 of [3],  $c^2$  can be computed as  $ar + b(r - 2)$ , where  $(a, b)$  is the solution of

$$\begin{pmatrix} t+1 & t \\ t & t-1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} r \\ r-2 \end{pmatrix}.$$

This yields  $a = r - 2t$  and  $b = 2 - r + 2t$ , hence  $c^2 = 4r - 4t - 4$  and finally  $d_3(\xi') - d_3(\xi_{st}) = r - t$ . This is exactly minus the so-called self-linking number  $l(T)$  of the positive transverse push-off  $T$  of  $L_1$ , cf. [7].

As shown in [9], the relative  $d_3$ -invariant  $d_3(\xi'', \xi_{st})$ , measuring the obstruction to homotopy over the 3-skeleton between  $\xi_{st}$  and the contact structure  $\xi''$  obtained by a Lutz twist along  $T$ , equals  $l(T)$ . Thus, to conclude the proof one would need to verify that the absolute  $d_3$ -invariant of [10] and the relative  $d_3$ -invariant of [9] are, in the case at hand, related by

$$d_3(\eta_1, \eta_2) = d_3(\eta_2) - d_3(\eta_1). \quad (3.1)$$

This can be done by looking at explicit geometric models, though, as always, it is difficult to keep track of signs. So here is a more roundabout algebraic argument. Let  $\xi_{\pm 1}$  be the contact structure obtained from  $\xi_{st}$  by a Lutz twist along a transverse knot  $T_{\mp 1}$  with self-linking number  $l(T_{\mp 1}) = \mp 1$  (this sign convention will be explained below); recall that the self-linking number is independent of the orientation of the transverse knot. For any natural number  $n$ , write  $\xi_{\pm n}$  for the contact structure on  $S^3$  given by taking the connected sum of  $n$  copies of  $(S^3, \xi_{\pm 1})$ . The additivity of the relative  $d_3$ -invariant implies  $d_3(\xi_{\pm n}, \xi_{st}) = \mp n$ , which means that we get a contact structure on  $S^3$  in each homotopy class of tangent 2-plane fields.

The absolute  $d_3$ -invariant — for 2-plane fields on  $S^3$  — takes all the values in  $\mathbb{Z} + 1/2$ , with  $d_3(\xi_{st}) = -1/2$ . By [3, Lemma 4.2], it satisfies the additivity rule

$$d_3(\eta_1 \# \eta_2) = d_3(\eta_1) + d_3(\eta_2) + \frac{1}{2}.$$

These observations imply that the absolute  $d_3$ -invariant takes the value  $1/2$  for one of the contact structures  $\xi_1$  and  $\xi_{-1}$ , and the value  $-3/2$  for the other. Equation (3.1) follows up to sign. We conclude

$$d_3(\xi'', \xi_{st}) = l(T) = d_3(\xi_{st}) - d_3(\xi') = \pm d_3(\xi', \xi_{st}).$$

By the considerations in (3a), we know that the extension of the contact structure over a meridional disc is the same, up to homotopy, for surgery on  $\mathbb{L}$  or Lutz twist along  $T$ . From the fact that there are standard models for the tubular neighbourhood of a Legendrian or transverse knot, respectively, we infer that  $d_3(\xi', \xi_{st})$  and  $d_3(\xi'', \xi_{st})$  can only differ by a constant term independent of the specific knot (corresponding to a different extension of the 2-plane field over the 3-cell attached to  $T^2 \cup$  (meridional disc)). Hence, the equation above can only hold if that constant is zero and the sign is the positive one. In turn, this yields equation (3.1) in full generality.

(Our definition of  $\xi_{\pm n}$  then entails  $d_3(\xi_1) = 1/2$  and  $d_3(\xi_{-1}) = -3/2$ , which accords with our labelling of these structures in [3].)

This concludes the proof of the theorem.

REMARK. If one defines  $L_2$  by adding two down-zigzags instead of up-zigzags, in (3a) one obtains  $c_1(\xi') = 2PD^{-1}([L_1])$ . This is the same as for a Lutz twist along the negative transverse push-off  $T_-$  of  $L_1$ , since  $T_-$  with the orientation that makes it positively transverse to  $\xi$  represents the class  $-[L_1]$ . Similarly, with this  $L_2$  we find in (3b) that  $d_3(\xi') - d_3(\xi_{st})$  is equal to minus the self-linking number  $t+r$  of the negative transverse push-off of  $L_1$ . Therefore, this choice of  $L_2$  amounts to performing a Lutz twist along  $T_-$ .

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