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ARMAND BOREL

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Armand Borel, professor emeritus in the School of Mathematics at the Institute for Advanced Study since 1993, died at home in Princeton on August 11, 2003, only two months after the first symptoms of a terminal illness appeared. He had celebrated his eightieth birthday on May 21.

Borel was born in 1923 in the French-speaking city of La Chaux-de-Fonds in Switzerland. He soon distinguished himself as an exceptional student and graduated in 1947 from the Swiss Federal Institute of Technology[1] in Zürich, where he was introduced to the study of topology and Lie groups by the famous mathematicians Heinz Hopf and Eduard Stiefel. He immediately obtained a position as assistant at the same institution, which he held for two years, and then, with a research grant from the French CNRS,[2] he moved to Paris for the 1949–50 year. This was a turning point in his mathematical development. There he quickly got acquainted with senior members of the Bourbaki group—namely Henri Cartan, Jean Dieudonné, Laurent Schwartz—and with the younger members—notably Roger Godement, Pierre Samuel, Jacques Dixmier, and most importantly Jean-Pierre Serre, who became a close friend and collaborator of Borel. The discussions with these mathematicians had a lasting influence on Borel and completed his preparation. He joined the Bourbaki group in the same year.

Borel returned to Switzerland with a position as adjunct professor of algebra at the University of Geneva from 1950 to 1952. In these years he completed the write-up of his thesis for a Doctorat d'État and defended it at the Sorbonne in Paris. His thesis, of fundamental importance in the theory of Lie groups, was published without delay in the prestigious journal *Annals of Mathematics*.

The same year, with his thesis as his entry card, Borel arrived with his young bride, Gaby, at the Institute as a member of the School of Mathematics. His membership in the School was renewed for a second year (at that time renewal of membership was done almost automatically, Borel told me, adding that he thought it was a very good thing). Then he spent a year in Chicago, where he profited highly from the presence of André Weil, thus adding algebraic geometry and number theory to his already vast knowledge of algebra and topology.

In 1957 he joined the School of Mathematics at the Institute as a professor, remaining until his retirement in 1993. At the time of his death he had authored or edited 16 books and over 180 papers and was working on a major monograph in collaboration with Lizhen Ji of the University of Michigan at Ann Arbor on the subject of compactifications of homogeneous spaces. He became a U.S. citizen on February 18, 1986.

He was a member of the National Academy of Sciences of the USA and of the American Academy of Arts and Sciences; a foreign member of the Finnish Academy

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[1]Eidgenössische Technische Hochschule.

[2]Centre National de la Recherche Scientifique.

of Sciences and Letters, of the American Philosophical Society, and of the Academia Europæa; a foreign associate of the French Academy of Sciences; an honorary fellow of the Tata Institute in Bombay, India; and the laureate of an honorary doctorate from the University of Geneva. He was a recipient of the Brouwer Medal of the Dutch Mathematical Society, of the Steele Prize of the American Mathematical Society, and of the Balzan Prize of the Italian-Swiss International Balzan Foundation.

His scientific activities, besides research, involved participation in the Consultative Committees of the International Congresses of Mathematicians in 1966 and 1978, participation in the editorial boards of the most prestigious mathematical journals in a span of over thirty years, and also teaching (I can mention various summer schools on mathematical topics at a high level and a three-year program in Hong Kong in his last years).

Less obvious, but not less important, was his presence in the Bourbaki group. The Bourbaki group was founded in 1934 by a small group of young French mathematicians, with the purpose of writing *ex novo* the foundations of modern mathematics in a correct and coherent fashion. Among these "young Turks" was André Weil, later mentor of Borel in Chicago and professor at the Institute. Their work was published anonymously under the pseudonym of "Bourbaki", a name borrowed from the French general with the Army of Napoleon III operating in Italy. The Italian school of algebraic geometry had produced a great body of fundamental work, but its foundations were indeed quite shaky and in need of drastic revision, so the comparison of the fictitious mathematician Bourbaki with the real general Bourbaki at war with Italy was not inappropriate. The influence of Bourbaki in the development of twentieth-century mathematics cannot be overestimated: its axiomatic approach, its quest for general statements, and its absolute mathematical rigor have been a model for many decades, and its texts are basic references. Borel was a member of this group from 1949 to 1973 (fifty is the mandatory retirement age for membership in Bourbaki).

I have mentioned topology and Lie groups (the name is from the Norwegian mathematician Sophus Lie) as the main subjects of research by Borel. Topology at its simplest is the study of geometric shapes under continuous deformations, namely without jumps or breaks; a Lie group (and I hope not to raise the disapproval of my mathematical colleagues in my gross oversimplification) can be seen as formed by continuous transformations of a highly symmetric object. An example is formed by the rotations of a sphere. Topology and Lie theory are a big part of the backbone of mathematics (Borel would say they *are* the backbone of mathematics). The contributions of Borel in the field will remain in the history of the subject. The citation of the Balzan Prize could not be more appropriate: "For his fundamental contributions to the theory of Lie groups, algebraic groups and arithmetic groups, and for his indefatigable action in favour of high quality in mathematical research and of the propagation of new ideas."

Borel's view of mathematics is very interesting, and I will spend a few words by reading an excerpt from his response[3] in receiving the Balzan Prize.

"Mathematics is a gigantic intellectual construction, very difficult, if not impossible, to view in its entirety. Sometimes I like to compare it to an iceberg, because it has a small visible part and a large invisible part. By visible part I mean the mathematics useful in the external world, in technology, physics, natural sciences, astronomy, computers, and so on, whose usefulness and social justification cannot be doubted. Indeed, it is certain that practical problems in ancient times were at the very

[3]Translated here from the French [Œ], IV, 375-6.

origin of mathematics. However, with the development of mathematics the subject acquired a life of its own and mathematicians became more and more interested in purely mathematical problems, not necessarily paying attention to applications outside mathematics itself. This forms the invisible part of the iceberg; I mean invisible or at least very difficult to grasp for the nonmathematician, the part that is pure mathematics.

"This does not mean at all that these researches will never find applications, that the invisible will never become visible. Experience shows the opposite; even the most abstract parts of mathematics sooner or later can find practical applications, often in the most unlikely ways. However, this point usually has no importance for the pure mathematician, who works in a world of intellectual forms with its own laws and its internal motivations, and he is often guided by æsthetic considerations. In the present environment dominated by competition for funding, it is easy for the agencies in charge of financing research to ignore or pay little attention to this intellectual speculation that apparently has no motivation and that seems to be an intellectual luxury item, thereby giving priority only to the visible part, from which one may expect a concrete practical return in a short period of time."

He returned to this last point in other writings, and he lamented the shortsightedness of such an attitude and its potential danger for the future development of mathematics. He continues:

"Mathematics has been for me a profession but also my preferred hobby. The course my investigations have taken, the choice of arguments to study, have been influenced by both points of view, which often are not quite distinct. Again and again I have been guided by a sense of the architecture of this building to which we continue to add new wings and new floors, while renovating the parts already built, by the feeling that certain problems had priority over others, so to open new perspectives or to establish a new foundation for future constructions. This is the professional point of view, but happily these problems were those that attracted me the most. In other instances I was not guided by such motives, being attracted only by curiosity, by the need to know the answer to an enigma, without reference to its importance in a general context."

We see here Borel as architect and planner of mathematics, a builder of magnificent constructions and of foundations for other buildings. Like all great architects, his constructions are tempered with the touch of the artist, following what he calls "æsthetic considerations". However, æsthetic considerations were for him always an aid and did not take over his overall view and philosophy of unity in mathematics.

I recall a conversation I had with him last June when I asked him about the origin of his well-known paper with Jean-Pierre Serre on Grothendieck's sweeping generalization of the Riemann-Roch theorem. He smiled and explained to me that there were no written notes by Grothendieck and they felt that what he had achieved was so important that it had to be written up in absolutely perfect form to make it accessible to everyone. I asked him why Grothendieck was delaying publication of his work. He explained to me that Grothendieck wanted the whole thing to be kind of automatic, a consequence of his constructions in algebra and his view of geometry. Indeed this was so for the first half of the proof, which dealt with embeddings. However, the second half of the proof, dealing with projections, needed a trick, technically known as a blow-up along a subvariety, which did not fit with his philosophy. It was a trick, a special tool, and there had to be something else more intrinsic that would fit better with the rest. Borel and Serre were more pragmatic and certainly had no

qualms about using a well-known tool in the course of a proof in order to complete an argument. The paper they wrote is a real gem, a model for clarity, and, to a mathematician, it is very beautiful indeed.

The Institute and the School of Mathematics were of primary importance to Borel. In the School of Mathematics he was always paying a lot of attention to the selection of visitors, and quite often I saw him in his office late at night reading carefully the material presented by applicants for membership; not limiting himself to a cursory reading of letters of recommendation, he read the research papers. Often we discussed candidly and openly the relative merits of the candidates, during long walks on the Institute lawns and, weather permitting, in the Institute woods.

There is one point that should be mentioned here in which his contribution to the Institute turned out to have lasting effects. In the mid 1970s a serious controversy started at the Institute about the appointment of a professor in the School of Social Science. The Director approved it, but the faculty was split, and there was strong opposition to this appointment. Things got ugly. Faculty members ended up by not talking to each other, by making statements to the newspapers, and matters eventually ended with the resignation of the Director. Clearly something had to be done by defining precisely the relative role of the trustees, the Director, and the faculty, and a special committee, chaired by trustee Marty Segal, was appointed to this task. Borel was the faculty representative, and he played a very big role in the formulation of the new Rules of Governance of the Institute, which have served us well since then.

He was also always very involved with our School, beyond the daily running of academic affairs. I will recall one amusing story, related to Simonyi Hall, the new mathematics building. The well-known architect Cesar Pelli had been selected for the task of designing the building and the auditorium, Wolfensohn Hall. Borel was very involved in the project. He was not at all intimidated by having to deal with a famous architect, and beyond the appearance Borel also wanted the building and the auditorium to be very functional. When the discussion came to the rather mundane topic of the heating and air-conditioning system, the architect proposed fixed windows and a forced-air system. Borel was adamant; he wanted windows that could be opened, at least in spring and the early autumn, when the weather in Princeton is really beautiful. The architect did not want such a change: it would change the visual aspect of the façade of the building. The administration did not want it: it could mean loss of heat in winter and loss of cooling in summer, with higher electricity bills. However, Borel persevered, and at last the architect decided to consult the Swedish firm that was going to supply the special windows, asking for a solution. The answer came as a surprise. Yes, it could be done by dividing the window into four horizontal sections, the lowest of which could be opened by pushing it forward. Pelli not only agreed to this but also found that the horizontal subdivision of the windows into four sections was visually much more appealing than the subdivision into two parts he had originally planned. There was one more problem: with an open window, screens are needed to keep out insects. The difficulty was the handle for opening the window. When the screen was mounted, a person could not reach the handle and had to remove the screen to open or close the window, hardly a practical solution. We all thought about how to solve the problem, but no satisfactory solution was found by us mathematicians nor by the architects. The solution was instantly found by a clever employee, sent to measure the windows by the firm chosen to build the screens: split the screen with an additional movable small screen in the center. In this way one

could slide the small screen sideways, creating an opening so as to reach the window handle, and then slide the small screen back in place. In the end, everyone was happy.

Mathematics and the Institute were not Borel's only interests. He loved music, especially jazz and Indian music, and he timed his professional trips to India with major music festivals, which he attended on a regular basis. He was instrumental in initiating a concert series at the Institute, which he directed until 1992 with a varied choice of performances ranging from early and baroque music, classical and contemporary, to jazz and Indian music. On a lighter side, he organized informal jazz concerts by members proficient in playing the piano or the saxophone, and he helped also in selecting good bands for playing in our traditional midwinter ball. He loved nature, and quite often I walked with him in the Institute woods, talking about the future of mathematics and of our School of Mathematics. He was very active and fit until his illness, and he loved hiking and swimming. He even took scuba diving certification when he was already over sixty. At some point scuba diving became too strenuous an exercise for him to do, but he continued to do snorkeling, the last time in Belize in winter 2003. He liked the Institute woods, and he was very relieved when eventually they did not fall to a developer and were preserved as a park. The last time I saw him I mentioned that the same afternoon I was planning to go in the Institute woods to visit my secret chanterelles patch, maybe I would find a few, would he like to have some too? He had a big smile and just said, "Oh yes!" I found quite a few chanterelles, and I suspect that they were much better for him than those one can buy in a store, not just because they were very fresh, but especially because they came from the Institute woods. He loved nature and was concerned with preserving the environment and with the future of our country, and he contributed generously to charities.

He always set very high standards for himself in his dedication to tasks, in honesty and integrity at work, in relationships with others, and he expected the same from other people too. He had a very reserved personality, and a first meeting with him was quite intimidating. However, people who knew him a little beyond a casual or purely business acquaintance soon found a good sense of humor, a warm human being, and a real friend under the surface.

He was a great scientist, a giant in the mathematical world, and a great colleague. We all miss him.