

ON QUOTIENT HYPERMODULES

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(Communicated by Mamadou Mboup)

Abstract. A hypermodule is a multivalued algebraic system satisfying the module like axioms. In this paper, we construct quotient hypermodule. Let M be a hypermodule, N be a subhypermodule of M and I be a hyperideal of R . Then, $[M : N^*]$ is R -hypermodule and $[R : I^*]$ -hypermodule, and prove that when N is normal subhypermodule, $[M : N^*]$ is a $[R : I^*]$ -module. Hence, the quotient hypermodules considered by Anvarieh and Davvaz are modules.

AMS Subject Classification: 20N20, 16Y99.

Keywords: hypermodule, strong regular relation, multiplicative hypermodule.

1 Introduction and Basic Definitions

The hypergroup notion was introduced in 1934 by a French mathematician F. Marty [14], at the 8th Congress of Scandinavian Mathematicians. He published some notes on hypergroups, using them in different contexts: algebraic functions, rational fractions, non commutative groups. Algebraic hyperstructures are a suitable generalization of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Since then, hundreds of papers and several books have been written on this topic, see [4, 5, 9, 6, 15]. In [8], Davvaz et al. provided, for the first time, a physical example of hyperstructures associated with the elementary particle physics, Leptons. They have considered this important group of the elementary particles and shown that this set along with the interactions between its members can be described by the algebraic hyperstructures. Mendel, the father of genetics took the first steps in defining “contrasting characters, genotypes in F_1 and $F_2 \dots$ and setting different laws”. The genotypes of F_2 is dependent on the type of its parents genotype

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and it follows certain roles. In [10], Ghadiri, Davvaz and Nekouian analyzed the second generation genotypes of monohybrid and a dihybrid with a mathematical structure. They used the concept of H_v -semigroup structure in the F_2 -genotypes with cross operation and proved that this is an H_v -semigroup. They determined the kinds of number of the H_v -subsemigroups of F_2 -genotypes. Also, in [7], inheritance issue based on genetic information is looked at carefully via a new hyperalgebraic approach. Several examples are provided from different biology points of view, and it is shown that the theory of hyperstructures exactly fits the inheritance issue. Also, see [3, 11]. The notion of hypermodules is studied by many authors, for example see [1, 2, 16, 17, 18, 19, 20].

The more general structure that satisfies the ring-like axioms is the hyperring in the general sense: $(R, +, \cdot)$ is a hyperring if $+$ and \cdot are two hyperoperations such that $(R, +)$ is a hypergroup and \cdot is an associative hyperoperation, which is distributive with respect to $+$. There are different notions of hyperrings. If only the addition $+$ is a hyperoperation and the multiplication \cdot is a usual operation, then we say that R is an *additive hyperring*. A special case of this type is the hyperring introduced by Krasner[12]. The concept of hypermodule over a hyperring has been investigated by many authors, for example, see [1, 2, 17]. In this section, we present some notions. These definitions and results are necessary for the next section. Let H be a non-empty set and $\circ : H \times H \rightarrow \mathcal{P}^*(H)$ be a hyperoperation, where $\mathcal{P}^*(H)$ is the family of non-empty subsets of H . The couple (H, \circ) is called a *hypergroupoid*. For any two non-empty subsets A and B of H and $x \in H$, we define $A \circ B = \bigcup_{a \in A, b \in B} a \circ b$, $A \circ \{x\} = A \circ x$ and $\{x\} \circ B = x \circ B$. A hypergroupoid (H, \circ) is called a *semihypergroup* if for all a, b, c of H we have $(a \circ b) \circ c = a \circ (b \circ c)$. A hypergroupoid (H, \circ) is called a *quasihypergroup* if for all a of H we have $a \circ H = H \circ a = H$. This condition is also called the *reproduction axiom*. A hypergroupoid (H, \circ) which is both a semihypergroup and a quasihypergroup is called a *hypergroup*. A special case of this type is the *hyperring* introduced by Krasner [12]. Also, Krasner introduced a new class of hyperrings and hyperfields: the quotient hyperrings and hyperfields. A Krasner hyperring is an algebraic structure $(R, +, \cdot)$ which satisfies the following axioms: (1) $(R, +)$ is a canonical hypergroup, i.e., (i) for every $x, y, z \in R$; $(x + y) + z = x + (y + z)$, (ii) for every $x, y \in R$; $x + y = y + x$, (iii) there exists $0 \in R$ such that $x = x + 0$, for all $x \in R$, (iv) for every $x \in R$ there exists a unique element x' such that $0 \in x + x'$ (we shall write $-x$ for x' and we call it the opposite of x), (v) $z \in x + y$ implies that $y \in -x + z$ and $x \in z - y$; (2) Relating to the multiplication, (R, \cdot) is a semigroup having zero as a bilaterally absorbing element; (3) The multiplication is distributive with respect to the hyperoperation $+$. Let $(R, +, \cdot)$ be a hyperring and I be a non-empty subset of R . Then, I is said to be a *subhyperring* of R if $(I, +, \cdot)$ is itself a hyperring. A subhyperring I of a hyperring R is a *left (right) hyperideal* of R if $r \cdot a \subseteq I$ ($a \cdot r \subseteq I$) for all $r \in R$, $a \in I$. I is called a *hyperideal* if I is both a left and a right hyperideal. An ideal I of hyperring R is called *normal* if $x + I - x \subseteq I$, for every $x \in R$. Let (H, \circ) be a semihypergroup and ρ be an equivalence relation on H . If A and B are non-empty subsets of H , then $A \bar{\rho} B$ means that for every $a \in A$, there is $b \in B$ such that $\rho(a) = \rho(b)$ and for every $b \in B$ there is $a \in A$ such that $\rho(a) = \rho(b)$, and $A \bar{\bar{\rho}} B$, means that for every $a \in A$ and $b \in B$, we have $\rho(a) = \rho(b)$. The equivalence relation ρ is called *regular on the right* (on the left) if for all x of H , from apb , it follows that $(a \circ x) \bar{\rho} (b \circ x)$ ($(x \circ a) \bar{\rho} (x \circ b)$) respectively) and ρ is called *strongly regular on the right* (on the left) if for all a, b of H , from apb , it follows that $(a \circ x) \bar{\bar{\rho}} (b \circ x)$ ($(x \circ a) \bar{\bar{\rho}} (x \circ b)$) respectively), and ρ is called *regular (strongly regular)* if it is regular (strongly regular) on the right and on the left. Let (H, \circ) be a semihypergroup and ρ be an equivalence relation on H . If ρ is regular, then $H/\rho = \{\rho(a) : a \in H\}$ is a semihypergroup, with respect to the hyperoperation $\rho(a) \circ \rho(b) = \{\rho(c) : c \in a \circ b\}$ and if this hyperoperation is well defined on H/ρ , then ρ is regular (see Theorem 2.5.2 in [9]). Moreover, if (H, \circ) is a hypergroup and ρ is an equivalence relation on H , then ρ is strongly regular if and only if $(H/\rho, \circ)$, is a group (see Corollary 2.5.6 in [9]).

In this paper, the notion of quotient hypermodules are studied. Let M be a R -hypermodule, I be

a hyperideal of R and N be a subhypermodule of M . Then, the quotient $[M : N^*]$ is also $[R : I^*]$ -hypermodule. But if N is a normal subhypermodule and I is a normal hyperideal of R , then the quotient $[M : N^*]$ is a $[R : I^*]$ -module.

2 Quotient Hypermodule

Let N be a subhypermodule of a hypermodule M . In this section, we construct quotient canonical hypergroup $[M : N^*]$ and prove that when N is normal, $[M : N^*]$ is an abelian group. Let $(R, +, \cdot)$ be a hyperring and $(M, +)$ be a hypergroup. We say that M is a *hypermodule* over a hyperring R , if there exists an external hyperoperation $\cdot : R \times M \rightarrow \mathcal{P}^*(M)$ with $(r, m) \mapsto r \cdot m$ such that (i) $r \cdot (m_1 + m_2) = r \cdot m_1 + r \cdot m_2$, (ii) $(r_1 + r_2) \cdot m = r_1 \cdot m + r_2 \cdot m$, (iii) $(r_1 \cdot r_2) \cdot m = r_1 \cdot (r_2 \cdot m)$, for every $r_1, r_2, r \in R$ and $m_1, m_2, m \in M$. Let M be a hypermodule and N be a non-empty subset of M . Then, N is called a *subhypermodule* of M if $(N, +)$ is a canonical subhypergroup of $(M, +)$ and for every $r \in R$ and $n \in N$, $r \cdot n \subseteq N$. A subhypermodule N is called *normal* if for every $m \in M$, $m + N - m \subseteq N$. Let X be a subset of a hypermodule of M and $\{M_i : i \in I\}$ be the family of all subhypermodule of M which contain X . Then, $\bigcap_{i \in I} M_i$ is called the hypermodule generated by X . This hypermodule is denoted by $\langle X \rangle$. If $X = \{m_1, m_2, \dots, m_n\}$, then the hypermodule $\langle X \rangle$ is denoted by $\langle m_1, m_2, \dots, m_n \rangle$. Let M be an R -hypermodule, R_1 and M_1, M_2 be nonempty subsets of R and M , respectively. We define

$$\begin{aligned} R_1 \cdot M_1 &= \left\{ x \in M : x \in \sum_{i=1}^n r_i \cdot m_i, r_i \in R_1, m_i \in M_1, n \in \mathbb{N} \right\}, \\ M_1 + M_2 &= \{x \in M : x \in m_1 + m_2, m_1 \in M_1, m_2 \in M_2\}, \\ \mathbb{Z}X &= \{m \in M : m \in \sum_{i=1}^n n_i x_i, n_i \in \mathbb{Z}, x_i \in X\}. \end{aligned}$$

Proposition 2.1. *Let M be an R -hypermodule and $X \subseteq M$. Then, $\langle X \rangle = \mathbb{Z}X + R \cdot X$.*

Definition 2.2. Let M be an R -hypermodule such that $(M, +)$ be an abelian group. Then, M is called *multiplicative hypermodule*.

If N is a subhypermodule of a hypermodule M , then we define the relation $m_1 \equiv m_2$ if and only if $m_1 \in m_2 + N$, for every $m_1, m_2 \in M$. This relation is denoted by $m_1 N^* m_2$.

Proposition 2.3. *Let N be a subhypermodule of hypermodule M . Then, N^* is an equivalence relation.*

Proof. Suppose that $m \in M$. Since 0 is neutral element and $0 \in N$, it follows that $m = m + 0 \in m + N$, the relation N^* is reflexive. Let $m_1, m_2 \in M$ and $m_1 N^* m_2$. Then, $m_1 \in m_2 + n$, for some $n \in N$. Hence, $m_2 \in m_1 - n \in m_1 + N$. So this relation is symmetric. Let $m_1, m_2, m_3 \in M$ such that $m_1 N^* m_2$ and $m_2 N^* m_3$. Then, for some $n_1, n_2 \in N$, $m_1 \in m_2 + n_1, m_2 \in m_3 + n_2$. So, $m_1 \in m_2 + n_1 \subseteq m_3 + n_1 + n_2 \subseteq m_3 + N$. Therefore, $m_1 N^* m_3$. This completes the proof. \square

If I is a hyperideal of a hyperring R , then we define the relation with the following hyperoperations: $x \equiv y$ if and only if $x \in y + I$. This relation is denoted by $x I^* y$.

Proposition 2.4. *Let I be a hyperideal of R . Then, $[R : I^*]$ is a hyperring with the following hyperoperations:*

$$\begin{aligned} I^*(x) \oplus I^*(y) &= \{I^*(z) : z \in I^*(x) + I^*(y)\}, \\ I^*(x) \odot I^*(y) &= \{I^*(z) : z \in I^*(x) \cdot I^*(y)\}. \end{aligned}$$

Proof. The proof is straightforward. \square

Theorem 2.5. *Let M be an R -hypermodule, I be an ideal of R and N be a subhypermodule of M . Then, $[M : N^*]$ is a $[R : I^*]$ hypermodule with the following hyperoperations:*

$$\begin{aligned} N^*(m_1) \oplus N^*(m_2) &= \{N^*(m) : m \in N^*(m_1) + N^*(m_2)\}, \\ I^*(r) \odot N^*(m) &= \{N^*(m) : m \in I^*(r) \cdot N^*(m)\}, \end{aligned}$$

and $[M : N^*]$ is R -hypermodule with the following hyperoperations:

$$\begin{aligned} N^*(m_1) \oplus N^*(m_2) &= \{N^*(m) : m \in N^*(m_1) + N^*(m_2)\}, \\ r \odot N^*(m) &= \{N^*(m) : m \in r \cdot N^*(m)\}. \end{aligned}$$

Proof. The proof is straightforward. □

Let M be an R -hypermodule and N be a subhypermodule of M . Then, the zero element of $[M : N^*]$ is $\{N\}$ and $|\langle\{N\}\rangle| = 1$.

Proposition 2.6. *Let N be a normal subhypermodule of hypermodule M . Then, for every $m_1, m_2 \in M$ the following are equivalent:*

- (i) $m_2 \in m_1 + N$,
- (ii) $m_1 - m_2 \subseteq N$,
- (iii) $(m_1 - m_2) \cap N \neq \emptyset$.

Proof. Suppose that $(m_1 - m_2) \cap N \neq \emptyset$. Then there exists $m \in (m_1 - m_2) \cap N$. So $-m_2 + m_1 \subseteq -m_2 + m + m_2 \subseteq N$. If $x \in -m_2 + m_1$, then $x \in N$. Hence $-m_2 \in x - m_1$ and $m_2 \in m_1 - x \subseteq m_1 + N$. Therefore, (iii) implies (i). It is easy to see that (i) implies (ii) and (ii) implies (iii). □

Definition 2.7. Let M be an R -hypermodule and N be a subhypermodule of M . We denote $\Omega(N) = \{m \in M : m - m \subseteq N\}$.

Proposition 2.8. *Let M be an R -hypermodule and N be a subhypermodule of M . Then, $\Omega(N)$ is a subhypermodule of M and $N \subseteq \Omega(N)$.*

Proof. Since $N \neq \emptyset$, the set $\Omega(N)$ is non-empty. Let $m_1, m_2, m \in \Omega(N)$, $r \in R$, $x \in m_1 - m_2$ and $y \in r \cdot m$. Then,

$$\begin{aligned} x - x &\subseteq (m_1 - m_2) - (m_1 - m_2) = (m_1 - m_1) + (m_2 - m_2) \subseteq N + N = N, \\ y - y &\subseteq r \cdot m - r \cdot m = r \cdot (m - m) \subseteq N. \end{aligned}$$

Hence, $m_1 - m_2 \subseteq \Omega(N)$ and $r \cdot m \subseteq \Omega(N)$. Moreover, for every $n \in N$, since N is a subhypermodule of M , $n - n \subseteq N$. Therefore, $\Omega(N)$ is a subhypermodule of M containing N . □

Proposition 2.9. *Let M be an R -hypermodule and $m_1, m_2 \in \Omega(\{0\})$. Then, $m_1 + m_2$ is a singleton set.*

Proof. The proof is straightforward. □

Proposition 2.10. *Let M be an hypermodule. Then, $\Omega(\{0\})$ is an abelian group and for every submodule M_1 of M , $M_1 \subseteq \Omega(\{0\})$.*

Proof. Suppose that $m_1, m_2 \in \Omega(\{0\})$ and $x, y \in m_1 + m_2$. Then

$$x - y \subseteq (m_1 + m_2) - (m_1 + m_2) = (m_1 - m_2) - (m_1 - m_2) = 0,$$

This implies that $m_1 + m_2$ is a singleton and $\Omega(\{0\})$ is a subgroup. Let M_1 be any subgroup of M and $x \in M_1$. Then, $x - x = \{0\}$. Hence $x \in \Omega(\{0\})$ and $M_1 \subseteq \Omega(\{0\})$. This completes the proof. \square

Corollary 2.11. *Let M be an R -hypermodule and N be a subhypermodule of M . Then, N is normal if and only if $\Omega(N) = M$. Moreover, $(M, +)$ is an abelian group if and only if $\Omega(\{0\}) = M$.*

Let $H(M) = \{x \mid x \in m - m, \forall m \in M\}$.

Proposition 2.12. *Let M be an R -hypermodule and N be a subhypermodule of M . Then, N is normal if and only if $H(M) \subseteq N$.*

Proof. Suppose that N be a subhypermodule and $H(M) \subseteq N$. Then for every $m \in M$ and $n \in N$ we have $m + n - m = m - m + n \subseteq H(M) + n \subseteq N + N = N$. Hence, N is normal. Let N be a normal subhypermodule and $m \in M$. This implies that $m + 0 - m \subseteq m + N - m \subseteq N$. Hence $m - m \subseteq N$, for every $m \in M$. Therefore, $H(M) \subseteq N$. This completes the proof. \square

Corollary 2.13. *Let M be an R -hypermodule. Then, $H(M)$ is the smallest normal subhypermodule of M .*

Corollary 2.14. *Let N_1 and N_2 be subhypermodules of M such that $N_1 \subseteq N_2$ and N_1 be normal subhypermodule. Then, N_2 is also normal.*

Corollary 2.15. *Let M be an R -hypermodule such that $\{0\}$ is normal. Then, all subhypermodules of M are normal.*

Theorem 2.16. *Let M be an R -hypermodule. Then, $(M, +)$ is abelian group if and only if $H(M) = \{0\}$.*

Proof. We know that $(M, +)$ is abelian group if and only if $\Omega(\{0\}) = M$. Moreover, $\Omega(\{0\}) = M$ if and only if $\{0\}$ is a normal subhypermodule. Hence, $(M, +)$ is an abelian group if and only if $\{0\}$ is a normal subhypermodule of M . Since $H(M)$ is a smallest subhypermodule of M , then $\{0\}$ is normal if and only if $H(M) = \{0\}$. This completes the proof. \square

Corollary 2.17. *Since $H(M)$ is the smallest normal subhypermodule of M , it follows that M is a module if and only if all subhypermodules of M are normal.*

Corollary 2.18. *Let N be a normal subhypermodule of hypermodule M . Then, the equivalence relation defined in Proposition 2.3, is a strongly regular relation. Hence $[M : N^*]$ is abelian group.*

Theorem 2.19. *Let M be an R -hypermodule and N be a normal subhypermodule of M . Then, $[M : N^*]$ is a multiplicative hypermodule.*

Proof. Suppose that N is a normal subhypermodule of M . The zero element of this quotient hypermodule is $\{N\}$. Moreover, $\{N\}$ is normal. By Theorem 2.16, $[M : N^*]$ is a multiplicative hypermodule. \square

Theorem 2.20. *Let M be a multiplicative hypermodule. Then, the following statements are equivalent:*

- (i) *there exists $m \in M$ such that $|0 \cdot m| = 1$,*

(ii) there exists $r \in R$, such that $|r \cdot 0| = 1$,

(iii) $|0 \cdot 0| = 1$,

(iv) $\forall r \in R, m \in M$, we have $|r \cdot m| = 1$.

Proof. (ii) \Rightarrow (iii). Suppose that $r \in R$. We have $0 \cdot 0 = (r - r) \cdot 0 = r \cdot 0 - r \cdot 0$ and by (ii), it follows that $0 \cdot 0 = \{0\}$, whence we obtain (iii).

(iii) \Rightarrow (iv). Let $r \neq 0$ be an element of R . We have $0 \cdot 0 = (r - r) \cdot 0 = r \cdot 0 - r \cdot 0$. If there exists $x \neq y$ elements of $r \cdot 0$, then $0 \cdot 0$ would contain $x - y \neq 0$ and 0 , and it is a contradiction. On the other hand, for every $r \in R$ and $m \in M$, $r \cdot (m - m) = r \cdot m - r \cdot m$, whence it follows that $r \cdot m$ contains only an element. The other implications (iv) \Rightarrow (i) are immediate. Similarly, the condition (i) is equivalent to (iii), (iv). \square

Proposition 2.21. *Let M be a multiplicative hypermodule. Then,*

(i) $0 \in r \cdot 0$, for every $r \in R$,

(ii) $0 \in 0 \cdot m$, for every $m \in M$,

(iii) if there exist $r_0 \in R$ and $m_0 \in M$ such that $|r_0 \cdot m_0| = 1$, then $|0 \cdot 0| = 1$,

(iv) if N is a subhypermodule of M , then for any element $N^*(m) \in [M : N^*]$, we have $|N^*(m) \odot N^*(0)| = 1$.

Proof. By Theorem 2.20, we obtain (i), (ii), (iii) and (iv). \square

Proposition 2.22. *Let M be a multiplicative hypermodule over hyperring R . Then, the external hyperoperation $R \times M \rightarrow \mathcal{P}^*(M)$ is operation if and only if there exist $r_0 \in R$ and $m_0 \in M$ such that $|r_0 \cdot m_0| = 1$.*

Proof. By Theorem 2.20, it is sufficient to check that $|r_0 \cdot 0| = 1$. We have

$$r_0 \cdot 0 = r_0 \cdot (m_0 - m_0) = r_0 \cdot m_0 - r_0 \cdot m_0,$$

whence we obtain that $r_0 \cdot 0$ contain only 0 . \square

Corollary 2.23. *Let M be an R -hypermodule and N be a subhypermodule of M . Then, $[M : N^*]$ is also an R -hypermodule. Moreover, if N is a normal subhypermodule of M , then by the Corollary 2.18, $[M : N^*]$ is a multiplicative hypermodule and by Theorem 2.20, the external hyperoperation in this quotient is operation.*

Proposition 2.24. *Let R be a Krasner hyperring and I be a normal ideal of R . Then, $[R : I^*]$ is a ring.*

Proof. Since R is an R -hypermodule, by Corollary 2.23, $[R : I^*]$ is a ring. \square

Corollary 2.25. *Let M be an R -hypermodule, I be a normal hyperideal and N be a normal subhypermodule of M . Then, the relation I^* and N^* are strong regular. This implies that the quotients $[R : I^*]$ and $[M : N^*]$ are ring and module, respectively. So, by the isomorphism theorems proved in [1], all the quotient hypermodules considered are modules.*

Acknowledgment. The authors are highly grateful to referees for their valuable comments and suggestions for improving the paper.

References

- [1] S. M. Anvariyehe and B. Davvaz, Strongly transitive geometric spaces associated to hypermodules, *J. Algebra* **322** (2009), 1340-1359.
- [2] S.M. Anvariyehe, S. Mirvakili and B. Davvaz, θ^* -Relation on hypermodules and fundamental modules over commutative fundamental rings, *Comm. Algebra* **36(2)** (2008), 622-631.
- [3] S.-C. Chung, K. M. Chun, N. J. Kim, S. Y. Jeong, H. Sim, J. Lee and H. Maeng, Chemical hyperalgebras for three consecutive oxidation states of elements, *MATCH Communications in Mathematical and in Computer Chemistry*, **72** (2014), 389-402.
- [4] P. Corsini and V. Leoreanu, *Applications of Hyperstructure Theory*, Advances in Mathematics vol.5 Kluwer Academic Publishers Dordrecht 2003.
- [5] P. Corsini, *Prolegomena of Hypergroup Theory*, Second Edition, Aviani Editore 1993.
- [6] B. Davvaz, *Polygroup Theory and Related Systems*, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2013.
- [7] B. Davvaz, A. Dehghan Nezhad and M. M. Heidari, Inheritance examples of algebraic hyperstructures, *Information Sciences* **224** (2013), 180-187.
- [8] A. Dehghan Nezhad, S.M. Moosavi Nejad, M. Nadjafikhah and B. Davvaz, A physical example of algebraic hyperstructures: Leptons, *Indian Journal of Physics* **86(11)** (2012), 1027-1032.
- [9] B. Davvaz and V. Leoreanu-Fotea, *Hyperring Theory and Applications*, International Academic Press USA, 2007.
- [10] M. Ghadiri, B. Davvaz and R. Nekouian, H_ν -Semigroup structure on F_2 -offspring of a gene pool, *International Journal of Biomathematics* **5(4)** (2012), 1250011 (13 pages).
- [11] Š. Hořková, J. Chvalina and P. Račková, Transposition hypergroups of Fredholm integral operators and related hyperstructures. Part II, *J. Basic Science* **4** (2008), 55-60.
- [12] M. Krasner, A class of hyperrings and hyperfields, *International J. Math. and Math. Sci.* **6** (1983), 307-312.
- [13] V. Leoreanu-Fotea, J. Zhan and L. Leoreanu, Fuzzy Γ -hyperrings and fuzzy Γ -hypermodules, *J. Intell. Fuzzy Systems* **24(3)** (2013), 647-655.
- [14] F. Marty, *Sur une generalization de la notion de group*, 8th Congres Math. Scandinaves (1934), 45-49.
- [15] T. Vougiouklis, *Hyperstructures and Their Representatitons*, Hadronic Press, Florida, 1994.
- [16] J. Wang, J. Zhan and W. Gu, Some new results on soft hypermodules, *Ars Combin.* **104** (2012), 289-306.
- [17] J. Zhan, B. Davvaz and K. P. Shum, Isomorphism theorems of hypermodules, *Acta Math. Sinica (Chinese Ser.)* **50** (2007), 909-914.

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- [18] J. Zhan, B. Davvaz and K. P. Shum, A new view on fuzzy hypermodules, *Acta Math. Sin. (Engl. Ser.)* **23(8)** (2007), 1345-1356.
- [19] J. Zhan, V. Leoreanu-Fotea and T. Vougiouklis, Fuzzy soft Γ -hypermodules, *Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys.* **73(3)** (2011), 13-28.
- [20] J. Zhan and I. Cristea, Γ -Hypermodules: isomorphisms and regular relations, *Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys.* **73(4)** (2011), 71-78.