*Functiones et Approximatio*
*58.1 (2018), 79–87*
*doi: 10.7169/facm/1646*

# REPRESENTATION OF A RATIONAL NUMBER AS A SUM OF NINTH OR HIGHER ODD POWERS

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**Abstract:** In the present paper, we substantially generalize one of the results obtained in our earlier paper [RM]. We present a solution of a problem of Waring type: if $F(x_1, \ldots, x_N)$ is a symmetric form of odd degree $n \geq 9$ in $N = 16 \cdot 2^{n-9}$ variables, then for any $q \in \mathbb{Q}$, $q \neq 0$, the equation $F(x_i) = q$ has rational parametric solutions, that depend on $n - 8$ parameters.

**Keywords:** Diophantine equations, parametric solutions.

## 1. Introduction

For any form $F(x_1, \ldots, x_N)$ with rational coefficients, one can look for conditions under which the equation $F(x_1, \ldots, x_N) = q$ has a rational solution for any $q \in \mathbb{Q}$, $q \neq 0$. In the present paper, we examine *symmetric* forms $F(x_1, \ldots, x_N)$, i.e., forms invariant under the natural action of the symmetric group $S_N$ on the variables, and the equation of Waring type: $F(x_1, \ldots, x_N) = q$, for any $q \in \mathbb{Q}$, $q \neq 0$. For such an equation, we are interested in deriving of *parametric* rational solutions and estimating of number of parameters.

We will begin with the case of the equation $x_1^9 + \ldots + x_{16}^9 = q$ (see Section 2). We apply to this equation the same transformation that was applied in paper [M] on the equation $x_1^7 + \ldots + x_{12}^7 = q$.Thus we reduce the solution of our equation to the system of three equations of the sixth, fourth and second degree:$y_1^r - y_2^r + y_3^r - y_4^r + y_5^r - y_6^r + y_7^r - y_8^r = 0, r = 2, 4, 6$ The parametric solutions of this system given by Tarry [D]. We will find two new parametric solutions of this system of equations, and consequently two parametric solutions of the equation $x_1^9 + \ldots + x_{16}^9 = q$. Further, we will generalize these results to the equation $F(x_1, \ldots, x_{16}) = q$ for any $q \in \mathbb{Q}$, $q \neq 0$, where the $F(x_1 \ldots, x_{16})$ is a *symmetric* form in 16 variables of degree 9. Note that to the best of our knowledge, such a result has not appeared in the literature yet. Among the works on this subject, we can mention [A]. Our method significantly differs both from the elementary

**2010 Mathematics Subject Classification:** primary: 11D72; secondary: 11P05 approach in [R], [C] and computer investigation of [LP], [LPS], [E]. It might be compared with the geometric construction in [Br], which led to some new insight into parametric solutions originally obtained in [SwD].

Next, in Section 3, we extend our method to include symmetric forms of arbitrary odd degree. Our main result (Theorem 3.1) states that for any $q \in \mathbb{Q}$, $q \neq 0$, the equation $F(x_1, \ldots, x_N) = q$, where $F(x_1, \ldots, x_N)$ is an arbitrary symmetric form of odd degree $n \geq 9$ in $16 \cdot 2^{n-9}$ variables, has rational parametric solutions, that depend on $n - 8$ parameters.

## 2. Equation $x_1^9 + \ldots + x_{16}^9 = q$, for any $q \in \mathbb{Q}$

We begin with a simple general observation:

**Lemma 2.1.** *Using any solution of the system of equations:*

$$y_1^r - y_2^r + y_3^r - y_4^r + y_5^r - y_6^r + y_7^r - y_8^r = 0, \qquad r = 2, 4, 6$$

*one can construct a solution of the equation of Waring type:*

$$x_1^9 + \ldots + x_{16}^9 = q,$$

*for any $q \in \mathbb{Q}$, $q \neq 0$, where the $x_i$ are rational functions in the $y_i$.*

**Proof.** Consider the identity:

$$\begin{aligned}
&(y_1 + c)^9 + (-y_1 + c)^9 + (y_2 - c)^9 + (-y_2 - c)^9 + (y_3 + c)^9 + (-y_3 + c)^9 \\
&+ (y_4 - c)^9 + (-y_4 - c)^9 + (y_5 + c)^9 + (-y_5 + c)^9 + (y_6 - c)^9 + (-y_6 - c)^9 \\
&+ (y_7 + c)^9 + (-y_7 + c)^9 + (y_8 - c)^9 + (-y_8 - c)^9 \\
= {}& 18 \cdot c \cdot (y_1^8 - y_2^8 + y_3^8 - y_4^8 + y_5^8 - y_6^8 + y_7^8 - y_8^8) \\
&+ 168 \cdot c^3 \cdot (y_1^6 - y_2^6 + y_3^6 - y_4^6 + y_5^6 - y_6^6 + y_7^6 - y_8^6) \\
&+ 252 \cdot c^5 \cdot (y_1^4 - y_2^4 + y_3^4 - y_4^4 + y_5^4 - y_6^4 + y_7^4 - y_8^4) \\
&+ 72 \cdot c^7 \cdot (y_1^2 - y_2^2 + y_3^2 - y_4^2 + y_5^2 - y_6^2 + y_7^2 - y_8^2)
\end{aligned}$$

Now, let $y_i$ be a rational solution of the system:

$$y_1^r - y_2^r + y_3^r - y_4^r + y_5^r - y_6^r + y_7^r - y_8^r = 0, \qquad r = 2, 4, 6$$

We introduce a new variable $q$ such that

$$q = 18 \cdot c \cdot (y_1^8 - y_2^8 + y_3^8 - y_4^8 + y_5^8 - y_6^8 + y_7^8 - y_8^8)$$

and get the identity

$$\begin{aligned}
&(y_1 + c)^9 + (-y_1 + c)^9 + (y_2 - c)^9 + (-y_2 - c)^9 + (y_3 + c)^9 + (-y_3 + c)^9 \\
&+ (y_4 - c)^9 + (-y_4 - c)^9 + (y_5 + c)^9 + (-y_5 + c)^9 + (y_6 - c)^9 + (-y_6 - c)^9 \\
&+ (y_7 + c)^9 + (-y_7 + c)^9 + (y_8 - c)^9 + (-y_8 - c)^9 = q
\end{aligned}$$

where $c = q/18 \cdot (y_1^8 - y_2^8 + y_3^8 - y_4^8 + y_5^8 - y_6^8 + y_7^8 - y_8^8)$. ■

Now we will look for the parametric solutions of the system

$$y_1^r - y_2^r + y_3^r - y_4^r + y_5^r - y_6^r + y_7^r - y_8^r = 0, \qquad r = 2, 4, 6$$

**Proposition 2.2.** *The system* $(*)$ *has two parametric solutions where* $y_i$ *are polynomials depending on one parameter.*

**Proof.** To solve this system we shall assume that:

$$\begin{array}{llll} y_1 = a+m+z, & y_2 = a-m+z, & y_3 = a+m-z, & y_4 = a-m-z \\ y_5 = b+n+y, & y_6 = b-n+y, & y_7 = b+n-y, & y_8 = b-n-y \end{array}$$

If we substitute these expressions in the system (*), the first equation will be transformed into:

$$8 \cdot a \cdot m + 8 \cdot b \cdot n = 0$$

If we solve this equation and substitute the value: $a = -(b \cdot n)/m)$ in the other two equations of the system (*), after reducing we obtain the new system of the next two equations :

$$(**) \begin{cases} b \cdot n \cdot (b^2 \cdot (m^2 - n^2) + m^2 \cdot (-m^2 + n^2 + 3y^2 - 3 \cdot z^2)) = 0 \\ b \cdot n \cdot (b^4 \cdot (m^4 - n^4) + 10 \cdot b^2 \cdot (m^4 \cdot y^2 - m^2 \cdot n^2 \cdot z^2) \\ + m^4 \cdot (-m^4 + n^4 + 10 \cdot n^2 \cdot y^2 + 5 \cdot y^4 \cdot -10 \cdot m^2 \cdot z^2 - 5 \cdot z^4)) = 0 \end{cases},$$

Now we introduce the new variable $k$: $k = y - z$. If we substitute $y = z + k$ into the first equation of the system $(**)$. Solving this equation we obtain the expression for $z$:

$$z = (-b^2 \cdot m^2 - 3 \cdot k^2 \cdot m^2 + m^4 + b^2 \cdot n^2 - m^2 \cdot n^2)/(6 \cdot k \cdot m^2).$$

Now let us substitute the expressions for $y$ and $z$ into the second equation of the system $(**)$. Having reduced it we obtain the next equation:

$$((b^2 - m^2) \cdot (m^2 - n^2) \cdot (45 \cdot k^4 \cdot m^4 + 5 \cdot (m^2 - n^2)^2 \cdot (m^2 - b^2)^2 \\ - 18 \cdot k^2 \cdot m^2 \cdot (m^2 + n^2) \cdot (m^2 + b^2)))/(27 \cdot k^2 \cdot m^2) = 0.$$

Now, if we assume that: $b^2 \neq m^2, m^2 \neq n^2, k \neq 0, m \neq 0$, we will obtain the next equation:

$$(***)\ 45 \cdot k^4 \cdot m^4 + 5 \cdot (m^2 - n^2)^2 \cdot (m^2 - b^2)^2 - 18 \cdot k^2 \cdot m^2 \cdot (m^2 + n^2) \cdot (m^2 + b^2) = 0.$$

We will find two various parametric solutions of this equation.

One can easily see that the substitution : $n = m + k, b = m \cdot k$ transforms the given equation into the next one :

$$k^2 \cdot (-4 + k^2) \cdot m^4 \cdot (5 \cdot k^4 + 4 \cdot k \cdot m + 20 \cdot k^3 \cdot m + 4 \cdot m^2 + 4 \cdot k^2 \cdot (-2 + 5 \cdot m^2)) = 0.$$

Now, assuming that $k \neq 0$, $k^2 \neq 4$, $m \neq 0$, we obtain the equation of the second degree for the variable $m$, where the coefficients are the polynomials of the variable $k$:

$$(5 \cdot k^4 + 4 \cdot k \cdot m + 20 \cdot k^3 \cdot m + 4 \cdot m^2 + 4 \cdot k^2 \cdot (-2 + 5 \cdot m^2)) = 0.$$

The discriminant of this equation is:

$$k^2 \cdot (-4 + k^2)^2 \cdot (1 + 5 \cdot k^2).$$

Obviously, the discriminant of the equation is a full square, if and only if the next expression is true :

$$(1 + 5 \cdot k^2) = R^2.$$

This conic has the rational point $P(k = 1/2, R = 3/2)$. Therefore, according to the famous theorem, this conic has the parametric solution depending on one parameter. Using this solution we can obtain the parametric solution of the system $(*)$ depending on one parameter.

To find the second parametric solution we use the substitution $n = m + 2 \cdot k$, $b = m \cdot k$ into the equation $(***)$. Similarly, we obtain the conic:

$$(-11 + 5k^2) = R^2.$$

This conic has the rational point $Q(k = 2, R = 3)$. Therefore, according to the famous theorem, this conic has the parametric solution depending on one parameter. Using this solution we can obtain the parametric solution of the system $(*)$ depending on one parameter.

Now, according to Lemma 2.1, we get the two parametric solutions of the equation $x_1^9 + \ldots + x_{16}^9 = q$, for any $q \in \mathbb{Q}$, $q \neq 0$. ■

This solution represents sixteen rational functions of degree 36 of one parameter.

This method can be extended onto more complicated cases.

**Theorem 2.3.** *Every equation $F(x_1, \ldots, x_{16}) = q$, where $F$ is a symmetric form of ninth degree, has a parametric solution, where $x_i$ are polynomials of one parameter and $x_1 + \cdots + x_{16} = 0$.*

**Proof.** Assuming that $x_1 + \cdots + x_{16} = 0$, every symmetric form of ninth degree can be represented as

$$\begin{aligned} &A_1 \cdot (x_1^9 + \cdots + x_{16}^9) + A_2 \cdot (x_1^7 + \cdots + x_{16}^7) \cdot (x_1^2 + \cdots + x_{16}^2) \\ &+ A_3 \cdot (x_1^6 + \cdots + x_{16}^6) \cdot (x_1^3 + \cdots + x_{16}^3) + A_4 \cdot (x_1^5 + \cdots + x_{12}^5) \cdot (x_1^4 + \cdots + x_{16}^4) \\ &+ A_5 \cdot (x_1^5 + \cdots + x_{16}^5) \cdot (x_1^2 + \cdots + x_{16}^2)^2 \\ &+ A_6 \cdot (x_1^3 + \cdots + x_{16}^3) \cdot (x_1^4 + \cdots + x_{16}^4) \cdot (x_1^2 + \cdots + x_{16}^2) \\ &+ A_7 \cdot (x_1^3 + \cdots + x_{16}^3) \cdot (x_1^2 + \cdots + x_{16}^2)^3. \end{aligned}$$

Indeed, let us denote $s_k = x_1^k + \cdots + x_{16}^k$. Then any symmetric polynomial of $x_1, \ldots, x_{16}$ is a polynomial of $s_1, s_2, \ldots$ (see [Wa, §33]). In our case it is enough to take linear combinations of monomials $s_1, \ldots, s_9$ of degree 9 of $x_1, \ldots, x_{16}$.

It is easy to see that all terms, except for:

$$s_9, \quad s_7 \cdot s_2, \quad s_6 \cdot s_3, \quad s_5 \cdot s_4, \quad s_5 \cdot s_2^2, \quad s_4 \cdot s_3 \cdot s_2, \quad s_3 \cdot s_2^3$$

vanish if $s_1 = 0$

To find a parametric solution of the resulting form of ninth degree, we use exactly the same method as for the diagonal form of ninth degree in Proposition. Let us represent the symmetric form of ninth degree as follows:

$$\begin{gathered} A_1 \cdot (x_1^9 + \cdots + x_{16}^9) + B_1(x_i) \cdot (x_1^7 + \cdots + x_{16}^7) \\ + C_1(x_i) \cdot (x_1^5 + \cdots + x_{16}^5) + D_1(x_i) \cdot (x_1^3 + \cdots + x_{16}^3) \end{gathered}$$

where $B_1(x_i)$, $C_1(x_i)$, $D_1(x_i)$ are polynomials of 2, 4, 6 degree, respectively.

Let us consider the three identities:

$$\begin{aligned} &(y_1+c)^9 + (-y_1+c)^9 + (y_2-c)^9 + (-y_2-c)^9 + (y_3+c)^9 + (-y_3+c)^9 \\ &+ (y_4-c)^9 + (-y_4-c)^9 + (y_5+c)^9 + (-y_5+c)^9 + (y_6-c)^9 + (-y_6-c)^9 \\ &+ (y_7+c)^9 + (-y_7+c)^9 + (y_8-c)^9 + (-y_8-c)^9 \\ = {}& 18 \cdot c \cdot (y_1^8 - y_2^8 + y_3^8 - y_4^8 + y_5^8 - y_6^8 + y_7^8 - y_8^8) \\ &+ 168 \cdot c^3 \cdot (y_1^6 - y_2^6 + y_3^6 - y_4^6 + y_5^6 - y_6^6 + y_7^6 - y_8^6) \\ &+ 252 \cdot c^5 \cdot (y_1^4 - y_2^4 + y_3^4 - y_4^4 + y_5^4 - y_6^4 + y_7^4 - y_8^4) \\ &+ 72 \cdot c^7 \cdot (y_1^2 - y_2^2 + y_3^2 - y_4^2 + y_5^2 - y_6^2 + y_7^2 - y_8^2). \end{aligned}$$

$$\begin{aligned} &(y_1+c)^7 + (-y_1+c)^7 + (y_2-c)^7 + (-y_2-c)^7 + (y_3+c)^7 + (-y_3+c)^7 \\ &+ (y_4-c)^7 + (-y_4-c)^7 + (y_5+c)^7 + (-y_5+c)^7 + (y_6-c)^7 + (-y_6-c)^7 \\ &+ (y_7+c)^7 + (-y_7+c)^7 + (y_8-c)^7 + (-y_8-c)^7 \\ = {}& 14 \cdot c \cdot (y_1^6 - y_2^6 + y_3^6 - y_4^6 + y_5^6 - y_6^6 + y_7^8 - y_8^8) \\ &+ 70 \cdot c^3 \cdot (y_1^4 - y_2^4 + y_3^4 - y_4^4 + y_5^4 - y_6^4 + y_7^4 - y_8^4) \\ &+ 42 \cdot c^5 \cdot (y_1^2 - y_2^2 + y_3^2 - y_4^2 + y_5^2 - y_6^2 + y_7^2 - y_8^2), \end{aligned}$$

$$\begin{aligned} &(y_1+c)^5 + (-y_1+c)^5 + (y_2-c)^5 + (-y_2-c)^5 + (y_3+c)^5 + (-y_3+c)^5 \\ &+ (y_4-c)^5 + (-y_4-c)^5 + (y_5+c)^5 + (-y_5+c)^5 + (y_6-c)^5 + (-y_6-c)^5 \\ &+ (y_7+c)^5 + (-y_7+c)^5 + (y_8-c)^5 + (-y_8-c)^5 \\ = {}& 10 \cdot c \cdot (y_1^4 - y_2^4 + y_3^4 - y_4^4 + y_5^4 - y_6^4 + y_7^4 - y_8^4) \\ &+ 2 \cdot c^2 \cdot (y_1^2 - y_2^2 + y_3^2 - y_4^2 + y_5^2 - y_6^2 + y_7^2 - y_8^2)), \end{aligned}$$

$$\begin{aligned}
&(y_1+c)^3+(-y_1+c)^3+(y_2-c)^3+(-y_2-c)^3+(y_3+c)^3\\
&+(-y_3+c)^3+(y_4-c)^3+(-y_4-c)^3+(y_5+c)^3+(-y_5+c)^3\\
&+(y_6-c)^3+(-y_6-c)^3+(y_7+c)^3+(-y_7+c)^3+(y_8-c)^3+(-y_8-c)^3\\
&=6\cdot c\cdot(y_1^2-y_2^2+y_3^2-y_4^2+y_5^2-y_6^2+y_7^2-y_8^2)).
\end{aligned}$$

Thus we see that the obtained parametric solutions for the diagonal symmetric form are also solutions for any symmetric form. ■

In the next section, we will use the transformation mentioned in our previous paper [M].

## 3. General results

In this section we generalize Theorem 2.3 for the case of a form of an arbitrary odd degree.

**Theorem 3.1.** *Let $F$ be a symmetric form of $N$ variables of odd degree $n \geq 9$ with rational coefficients. If $N = 16 \cdot 2^{n-9}$, the equation $F(x_1,\ldots,x_N) = q$ has a parametric solution where $x_i$ are rational functions of $s = n-8$ parameters, and $x_1 + \cdots + x_N = 0$.*

**Proof.** Let $F$ be an arbitrary symmetric form of degree $n = 2k+1$ of $N = 4s$ variables. First of all, the variables are grouped in sets of four. For every quadruple of variables we use a transformation of the form

$$x_1 = y_1 + c_1, \qquad x_2 = -y_1 + c_1, \qquad x_3 = -y_2 - c_1, \qquad x_4 = y_2 - c_1. \tag{1}$$

An arbitrary symmetric equation takes the form:

$$y_1^{2k+1}+\cdots+y_N^{2k+1}+A_1(y_1^{2k-1}+\cdots+y_N^{2k-1})R_1(y_i)+\cdots+A_k(y_1+\cdots+y_N)R_k(y_i)=q,$$

where $R_j(y_i)$ are symmetric polynomials.

Thus we have obtained a form of degree $2k$, whose coefficients are functions of $c_1$. But we obtain a new construction of the form:

$$\begin{aligned}
&(y_1^{2k}-y_2^{2k}+y_3^{2k}-y_4^{2k}+\ldots)\cdot D_0(c_1)\\
&+(y_1^{2k-2}-y_2^{2k-2}+y_3^{2k-2}-\ldots)\cdot D_1(c_1,y_1,\ldots,y_{2s})\\
&+(y_1^{2k-4}-y_2^{2k-4}+y_3^{2k-4}-y_4^{2k-4}+\ldots)\cdot D_2(c_1,y_1,\ldots,y_{2s})\\
&+(y_1^{2k-6}+y_2^{2k-6}+\ldots)\cdot D_3(c_1,y_1,\ldots,y_{2s})\\
&+\cdots+(y_1^2-y_2^2+y_3^2-\ldots)\cdot D_{2k-1}(c_1,y_1,\ldots,y_{2s})+\cdots=q.
\end{aligned}$$

Once again, we split variables into quadruples and for each quadruple apply the transformation:

$$y_1 = z_1 + c_2, \qquad y_2 = -z_1 + c_2, \qquad y_3 = -z_2 - c_2, \qquad y_4 = z_2 - c_2. \tag{2}$$

We get a homogeneous symmetric equation of degree $2 \cdot k - 1$:

$$\begin{aligned} &(z_1^{2k-1} + z_2^{2k-1} + z_3^{2k-1} + \dots) \cdot D_0(c_1, c_2) \\ &+ (z_1^{2k-3} + z_2^{2k-3} + z_3^{2k-3} + \dots) \cdot D_1(c_1, c_2, z_1, \dots, z_s) \\ &+ \dots + (z_1 + z_2 + z_3 + \dots) \cdot D_{2k-2}(c_1, c_2, z_i) + \dots = q. \end{aligned} \tag{3}$$

Note that in the transformations (1) and (2) each time the number of variables is reduced by half. After repeating the transformations (1) and (2) in the alternating order $2k - 8$ times we get an equation of degree 9:

$$\begin{aligned} &(u_1^7 + \dots + u_{16}^9) \cdot D_0(c_i) + \\ &+ (u_1^5 + \dots + u_{16}^7) \cdot D_2(c_i, u_1, \dots, u_{16}) \\ &+ (u_1^5 + \dots + u_{16}^5) \cdot D_4(c_i, u_1, \dots, u_{16}) \\ &+ (u_1^3 + \dots + u_{16}^3) \cdot D_6(c_i, u_1, \dots, u_{16}) = q, \end{aligned} \tag{4}$$

where $D_2(u_1, \dots, u_{16})$ is a polynomial in $u_i$ of degree at most 2, $D_4(u_1, \dots, u_{16})$ is a polynomial of $u_i$ of degree at most 4, $D_6(u_1, \dots, u_{16})$ is a polynomial of $u_i$ degree at most 6.

One can easily see that a solution of the given equation can be obtained by the following transformation:

$$\begin{array}{llll} u_1 = r_1 + c, & u_2 = -r_1 + c, & u_3 = -r_2 - c, & u_4 = r_2 - c, \\ u_5 = r_3 + c, & u_6 = -r_3 + c, & u_7 = -r_4 - c, & u_8 = r_4 - c, \\ u_9 = r_5 + c, & u_{10} = -r_5 + c, & u_{11} = -r_6 - c, & u_{12} = r_6 - c, \\ u_{13} = r_7 + c, & u_{14} = -r_7 + c, & u_{15} = -r_8 - c, & u_{16} = r_8 - c. \end{array}$$

This transformation reduces the solution of the obtained equation to the solution of following system:

$$r_1^r - r_2^r + r_3^r - r_4^r + r_5^r - r_6^r + r_7^r - r_8^r = 0, \qquad r = 2, 4, 6.$$

We have found parametric solutions of the given system in the second section of the paper. ■

## 4. Numerical example

The computations in this section were made using Wolfram Mathematica.

We examine the equation $z_1^9 + \dots + z_{16}^9 = q$.

According to Lemma 2.1, the process of solving of the equation can be reduced to the following system:

$$y_1^r - y_2^r + y_3^r - y_4^r + y_5^r - y_6^r + y_7^r - y_8^r = 0, \qquad r = 2, 4, 6.$$

We have shown that the solution of the given system of the equations is reduced to the solution of the equation:

$$1 + 5 \cdot k^2 = R^2.$$

Now, we show that the solution of the equation $z_1^9 + \cdots + z_{16}^9 = q$ corresponds to the solution $(k = 4, R = 9)$ of the equation $1 + 5 \cdot k^2 = R^2$.

If $a_1 = 3177$, $a_2 = 3084$, $a_3 = 453$, $a_4 = 2490$, $b_1 = 2697$, $b_2 = 2790$, $b_3 = 3291$, $b_4 = 348$, we have identities:

$$a_1^r + a_2^r + a_3^r + a_4^r = b_1^r + b_2^r + b_3^r + b_4^r, \qquad r = 2, 4, 6.$$

Further on writing $t = -q/3447002593847409082117632000$, we have the following identities :

$$\begin{aligned}
&(t + a_1)^k + (t - a_1)^k + (-t + b_1)^k + (-t - b_1)^k + (t + a_2)^k + (t - a_2)^k \\
&+ (-t + b_2)^k + (-t - b_2)^k + (t + a_3)^k + (t - a_3)^k + (-t + b_3)^k \\
&+ (-t - b_3)^k + (t + a_4)^k + (t - a_4)^k + (-t + b_4)^k + (-t - b_4)^k = q
\end{aligned}$$

when $k = 9$ and

$$\begin{aligned}
&(t + a_1)^k + (t - a_1)^k + (-t + b_1)^k + (-t - b_1)^k + (t + a_2)^k + (t - a_2)^k \\
&+ (-t + b_2)^k + (-t - b_2)^k + (t + a_3)^k + (t - a_3)^k + (-t + b_3)^k \\
&+ (-t - b_3)^k + (t + a_4)^k + (t - a_4)^k + (-t + b_4)^k + (-t - b_4)^k = 0
\end{aligned}$$

when $k = 1, 3, 5, 7$.

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**Received:** 31 December 2016; **revised:** 2 March 2017