

## **PROFESSOR BOGDAN BOJARSKI, MY MEMORABLE ADVISER**

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**1.** Good mathematical societies celebrate and cherish their distinguished scholars, academic leaders, accomplished administrators and teachers. It gives me a special pleasure to write a few words in recollection of my mentor Professor Bogdan Bojarski. I consider myself lucky and proud to be his former student. He never tutored me to become routine mathematician, but prepared me to become independent scholar. More importantly, he always believed in me and made me feel successful. I thank him for this.

**2.** Bogdan Bojarski was a brilliant research associate in the team of the Georgian academician I.N. Vekua. His close ties with Russian mathematical schools (Steklov's Institute in Moscow) and subsequent contacts with European and American mathematicians helped him establish and run his own extremely effective program in Poland. It originated with his early work on elliptic partial differential equations, boundary value problems by methods of complex function theory, singular integrals and quasiconformal mappings. His program also included some general aspects of so-called global analysis (index theory, Fredholm pairs, etc.). In the last two decades, Bojarski extended this program by successfully attracting new PhD students (P. Hajłasz, P. Strzelecki, A. Kałamajska) to work with him on geometric theory of Sobolev spaces. Their studies and results became crucial parts of the flourishing and internationally acclaimed theory called analysis on metric spaces.

**3.** After habilitation in Steklov's Institute (1959) and a year later in the Jagiellonian University in Cracow, Bogdan Bojarski came to Warsaw as a docent (1960-1968) and then was appointed to a professorship. By that time he had already done an impressive amount of research. Three years later Professor Bojarski offered me an assistant-mastership at the University of Warsaw under his supervision. It was fascinating opportunity for me to learn about his work and to develop it later as his PhD student. I accepted his offer.

**4.** However, Bojarski's goal was not only to do research but also to serve the Department of Mathematics and the greater mathematical community. He became

visible rather early as a gifted leader of the mathematical life at the University of Warsaw and Polish Academy of Sciences. Leadership was required from him and yet, in my personal experience, his passion and commitment to working with young scholars did not end. Passing knowledge and helping the young to achieve their own career is a thing that I have learnt from Professor Bojarski; nowadays it turns out well in my own interaction with young scholars from the United States, Italy, Finland and other countries. I think that unselfish devotion to the next generation is the noble duty and the highest responsibility every scientist should accept.

**5.** In the 1950's, when complex analysis regained its status in partial differential equations, Bojarski's contribution to these new advances was phenomenal. He earned fame by inaugurating and developing the  $L^p$ -theory of the first order elliptic systems with measurable coefficients. These systems were already known to C.B. Morrey (1938) and M.M. Lavrentiev (1948-1949), because they emerged from a profound interplay between PDEs and quasiconformal mappings. However, the history of such equations actually goes back as far as the celebrated works by d'Alembert on the Cauchy-Riemann systems in hydrodynamics (1746), by Gauss on geometry of surfaces (1827), by Łobaczewski on non-Euclidean geometry, and by the paper of E. Beltrami "Saggio di interpretazione della geometria non euclidea" (1867). The freshness of Bojarski's approach was his application of the newly grown theory of singular integrals. Nowadays, his  $L^p$ -estimates are the analytical foundation of quasiconformal theory. In particular, his proof and qualitative ingredients to Morrey's Measurable Riemann Mapping Theorem have found applications to modern analysis (complex dynamics for instance). Yet from a practical point of view Bojarski's results, and parallel ones by L. Bers (1957-61), were a resumption of Lavrentiev's pioneering work in nonlinear hydrodynamics and gas dynamics. The new complex variable approach with quasiconformal mappings proved very useful and effective.

**6.** Our engagement in research was tremendously inspiring to me. It begun in early 1970's with a series of joint papers on nonlinear elliptic equations, which concluded with "Quasiconformal Mapping Theorem for General Nonlinear Beltrami Systems" (1976). Among my numerous joint papers with Bojarski is the "Analytical Foundations of the Theory of Quasiconformal Mappings in  $\mathbb{R}^m$ " (1983). In a way this paper was a synthesis of our joint work on nonlinear Beltrami type systems in higher dimensions. Shortly after this publication our interaction discontinued. Fred Gehring invited me to the United States and later I accepted an offer of permanent position with full professorship at Syracuse University. Nonetheless, Bojarski's mathematical influence on me never stopped. Recently (2007) we wrote a joint paper with two Italian scholars C. Sbordone and L. D'Onofrio on  $G$ -compactnes of the general complex Beltrami operators.

**7.** In 1960's Bojarski introduced homotopy classification of complex elliptic systems, both first and second order. Without getting into detail, he showed that the elliptic systems in the same homotopy class as the Cauchy-Riemann system or the

complex second order Laplace equation enjoy better properties than the general ones. He then turned into a study of the Fredholm index theory. Here one must mention his effective computation of the index in the Riemann-Hilbert problem for systems of first order. Perhaps that was the reason he extended his interests to global analysis, just to mention his joint seminars with Professor Gęba (alternating between Warsaw and Sopot) and later joint work with K. Wojciechowski (his former student). In my opinion it was a successful realization of the relevance of complex analysis to PDEs and topology of manifolds.

**8.** Finding universal common features of seemingly unrelated concepts is a characteristic of Bojarski's research sometimes not recognized by others. As an example let me mention his joint work with P. Hajłasz (former PhD student, currently at the University of Pittsburgh) concerning definition of the Sobolev functions (1996). With the aid of maximal operators they found inequalities characterizing Sobolev functions without having first order derivatives involved. It is fair to say that his work with P. Hajłasz triggered intense programs for analysis in metric spaces with measure; J. Cheeger, J. Heinonen, J. Tyson.

Today Bojarski's numerous successful students are the living messages he sent to the present and future time.

**9.** When I contemplate his encouragement and devotion to the mathematical community in Poland during struggling economy and political challenge I am more and more convinced that he did things right. This gratified many scholars like him and astonished some disbelievers. He was extremely tough when he had to be. The best example is the restoration of the Mathematical Institute on Śniadeckich 8. He aimed for success, if not perfection. For him, perhaps, doing what could not be done was the triumph of his energy and position of trust. Bojarski is an amendable man by all means, with a remarkable ability to move forward with right projects. He believes deeply in patience to succeed in risky ventures. The outcome was what mattered most. The Będlewo Mathematical Research and Conference Center is the result of Bojarski's great courage, enthusiasm and dedication. It will remain permanently on the stage of his legacy.

**10.** The input of Professor Bojarski to mathematics goes far beyond my ability to fully describe. His perceptions and results are outstanding, rich in concepts and technical novelties. The Banach Center in Będlewo is his self-portrait. He built it for the Polish Mathematical Community. I can only admire the total record of his lifetime accomplishments.

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