SUT Journal of Mathematics
Vol. 49, No. 2 (2013), 79–92

Even vertex odd mean labeling of graphs

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(Received June 14, 2012; Revised August 24, 2013)

Abstract. In this paper we introduce a new type of labeling known as even vertex odd mean labeling. A graph G with p vertices and q edges is said to have an even vertex odd mean labeling if there exists an injective function $f : V(G) \to \{0, 2, 4, \dots, 2q-2, 2q\}$ such that the induced map $f^* : E(G) \to \{1, 3, 5, \dots, 2q-1\}$ defined by $f^*(uv) = \frac{f(u)+f(v)}{2}$ is a bijection. A graph that admits an even vertex odd mean labeling is called an even vertex odd mean graph. Here we investigate the even vertex odd mean behaviour of some standard graphs.

AMS 2010 *Mathematics Subject Classification.* 05C.

Key words and phrases. Labeling, even vertex odd mean labeling, even vertex odd mean graph.

§1. Introduction

Throughout this paper, by a graph we mean a finite, undirected simple graph. Let $G(V, E)$ be a graph with p vertices and q edges. For notations and terminology we follow [4].

Path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. $K_{1,m}$ is called a star and it is denoted by S_m. The bistar $B_{m,n}$ is the graph obtained from K_2 by identifying the center vertices of $K_{1,m}$ and $K_{1,n}$ at the end vertices of K_2 respectively. $B_{m,m}$ is often denoted by $B(m)$. The union of two graphs G_1 and G_2 is a graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$. The union of m disjoint copies of a graph G is denoted by mG.

A quadrilateral snake G_n is obtained from a path $u_1, u_2, \dots, u_{n+1}$ by joining u_i and u_{i+1} to new vertices v_i and w_i respectively and joining v_i and w_i, that is, every edge of a path is replaced by a cycle C_4. The corona of a graph G on p vertices $v_1, v_2, \dots, v_p$ is the graph obtained from G by adding p new vertices $u_1, u_2, \dots, u_p$ and the new edges u_iv_i for $1 \le i \le p$. The corona of G is denoted by $G \odot K_1$. The graph $P_n \odot K_1$ is called a comb. The baloon of a graph G, $P_n(G)$ is the graph obtained from G by identifying an end vertex of P_n at a vertex of G. $P_n(C_m)$ is called a dragon.

Let G_1 and G_2 be any two graphs with p_1 and p_2 vertices respectively. Then the cartesian product $G_1 \times G_2$ has p_1p_2 vertices which are $\{(u,v)/u \in G_1, v \in G_2\}$. The edge set of $G_1 \times G_2$ is obtained as follows:(u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$ if either $u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent in G_1 and $v_1 = v_2$. The product $P_m \times P_n$ is called a planar grid and $P_n \times P_2$ is called a ladder, denoted by L_n. The product $C_m \times P_n$ is called a prism. The graph $P_2 \times P_2 \times P_2$ is called a cube and is denoted by Q_3. Let S_m be a star with central vertex v_0 and pendant vertices $v_1, v_2, \ldots, v_m$ and let $[P_n; S_m]$ be the graph obtained from n copies of S_m with vertices $v_{0_j}, v_{1_j}, \ldots, v_{m_j} (1 \le j \le n)$ and joining v_{0_j} and $v_{0_{j+1}}$ by means of an edge, $1 \le j \le n-1$.

The graceful labelings of graphs was first introduced by Rosa, in 1967 [1] and R. B. Gnanajothi introduced odd graceful graphs [3]. The concept of mean labeling was introduced and meanness of some standard graphs was studied by S. Somasundaram and R. Ponraj [9, 10, 6, 7]. Further some more results on mean graphs are discussed in [8, 11, 12]. A graph G is said to be a mean graph if there exists an injective function f from $V(G)$ to $\{0, 1, 2, \ldots, q\}$ such that the induced map f^* from $E(G)$ to $\{1, 2, 3, \ldots, q\}$ defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection.

In [5], K. Manickam and M.Marudai introduced odd mean labeling of a graph. A graph G is said to be odd mean if there exists an injective function f from $V(G)$ to $\{0, 1, 2, 3, \ldots, 2q-1\}$ such that the induced map f^* from $E(G)$ to $\{1, 3, 5, \ldots, 2q-1\}$ defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection. The concept of even mean labeling was introduced and studied by B. Gayathri and R. Gopi [2]. A function f is called an even mean labeling of a graph G with p vertices and q edges, if f is an injection from the vertices of G to the set $\{2, 4, 6, \ldots, 2q\}$ such that when each edge uv is assigned the label $\frac{f(u)+f(v)}{2}$, then the resulting edge labels are distinct. A graph which admits an even mean labeling is said to be even mean graph. It motivates us to define a new concept called even vertex odd mean labeling of graphs.

A graph G with p vertices and q edges is said to have an even vertex odd mean labeling if there exists an injective function $f : V(G) \to \{0, 2, 4, \ldots, 2q-2, 2q\}$ such that the induced map $f^* : E(G) \to \{1, 3, 5, \ldots, 2q-1\}$ defined by $f^*(uv) = \frac{f(u)+f(v)}{2}$ is a bijection. A graph that admits an even vertex odd mean labeling is called an even vertex odd mean graph.

An even vertex odd mean labeling of the cube Q_3 is given in Figure 1.

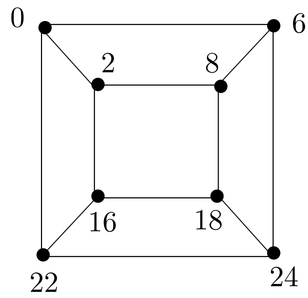


Figure 1: An even vertex odd mean labeling of Q_3

K_3 is a mean graph but not an even vertex odd mean graph. The graph shown in Figure 2, is an odd mean graph but not an even vertex odd mean graph. Every star graph is an even mean graph but $K_{1,n}(n \geq 3)$ is not an even vertex odd mean graph. These examples show that the notion of even vertex odd mean graph is independent of mean graph, odd mean graph and even mean graph.

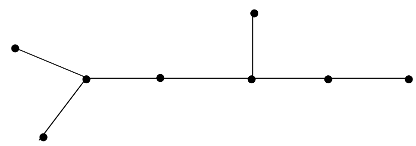


Figure 2: An odd mean graph but not an even vertex odd mean graph

In this paper, we prove that the path P_n, the cycle C_n for $n \equiv 0 \pmod 4$, $K_{1,n}$ for $n \leq 2$, $K_{2,n}$ for all n, the bistar $B_{m,n}$ for $n = m, m+1$, the quadrilateral snake, the comb $P_n \odot K_1, [P_n; S_2], [P_{2n}; S_m]$, the planar grid $P_m \times P_n$, the prism $C_m \times P_n$ for $m \equiv 0 \pmod 4), n \geq 1, Q_3 \times P_n$, the Ladder L_n, $L_n \odot K_1$ and the dragon are even vertex odd mean graphs.

Also, we prove that $K_{1,n}(n \geq 3)$ is not an even vertex odd mean graph.

§2. Even vertex odd mean graphs

Theorem 2.1. *If G is an even vertex odd mean graph, then G is a bipartite graph.*

Proof. Let uv be an edge of G. If vertex u is labeled by $0 \pmod 4$, then vertex v is labeled by $2 \pmod 4$ because $\frac{f(u)+f(v)}{2}$ is odd. Let V_1 be a set of vertices labeled by $0 \pmod 4$, and $V_2 = V(G) - V_1$. Then G is bipartite graph with partite sets V_1 and V_2. □

Theorem 2.2. *Any path is an even vertex odd mean graph.*

Proof. Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n. Define $f : V(P_n) \to \{0, 2, 4, \ldots, 2q-2, 2q = 2n-2\}$ by $f(u_i) = 2i-2, 1 \leq i \leq n$. The label of the edge $u_{i-1}u_i$ is $2i-3, 2 \leq i \leq n$. Hence, P_n is an even vertex odd mean graph. For example, an even vertex odd mean labeling of P_8 is shown in Figure 3. □

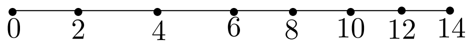


Figure 3: An even vertex odd mean labeling of P_8

Theorem 2.3. *Cycle C_n is an even vertex odd mean graph if and only if $n \equiv 0 \pmod 4$.*

Proof. If C_n is a cycle of odd length, then at least one edge uv on the cycle in which both $f(u)$ and $f(v)$ are congruent to either $0 \pmod 4$ or $2 \pmod 4$ and hence its induced edge label $f^*(uv)$ is even.

Suppose $n = 2m, m \geq 2$ and C_n admits an even vertex odd mean labeling. Then $\sum\limits_{uv \in E(G)} f^*(uv) = \sum\limits_{uv \in E(G)} \left(\frac{f(u)+f(v)}{2}\right)$. This implies that $1+3+5+\cdots+4m-1 = (0+2+4+6+\cdots+4m) - 2i$, where $2i$ is not a vertex label of C_n. From this, $i = m$. If m is odd, then the number of values congruent to $0 \pmod 4$ is in excess of 2 that of the number of values congruent to $2 \pmod 4$ and they are to be assigned as vertex labels in C_n. Thus m should be even if C_n admits an even vertex odd mean labeling.

Let $u_1, u_2, \ldots, u_n$ be the vertices of the cycle C_n where $n \equiv 0 \pmod 4$. We define $f : V(G) \to \{0, 2, 4, \ldots, 2q-2, 2q = 2n\}$ as follows:

$$f(u_i) = \begin{cases} 2i-2, & 1 \leq i \leq \frac{n}{2} \\ n+4+2\left(i - \left(\frac{n}{2}+1\right)\right) & \text{if } i \text{ is odd and } \frac{n}{2}+1 \leq i \leq n-1 \\ n+2+2\left(i - \left(\frac{n}{2}+2\right)\right) & \text{if } i \text{ is even and } \frac{n}{2}+2 \leq i \leq n. \end{cases}$$

The induced edge labels are given by

$$f^*(u_iu_{i+1}) = \begin{cases} 2i-1, & 1 \leq i \leq \frac{n}{2}-1 \\ n+1+2\left(i-\frac{n}{2}\right), & \frac{n}{2} \leq i \leq n-1 \text{ and} \end{cases}$$

$$f^*(u_nu_1) = 3$$

Hence, C_n is an even vertex odd mean graph if $n \equiv 0 \pmod 4$. For example, an even vertex odd mean labeling of C_{12} is shown in Figure 4. □

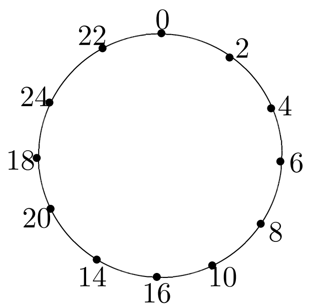


Figure 4: An even vertex odd mean labeling of C_{12}

The star $K_{1,1}$ is P_2 and $K_{1,2}$ is P_3. Thus, $K_{1,1}$ and $K_{1,2}$ are even vertex odd mean graphs by Theorem 2.2.

Theorem 2.4. *If $n \geq 3$, $K_{1,n}$ is not an even vertex odd mean graph.*

Proof. Let $\{V_1, V_2\}$ be the bipartition of $K_{1,n}$ with $V_1 = \{u\}$. To get the edge label $2q-1$, we must have $2q$ and $2q-2$ as the labels of adjacent vertices. Thus either $2q$ or $2q-2$ must be a label of u. In both cases, since $n \geq 3$, there will be no edge whose label is 1. This contradiction proves that $K_{1,n}$ is not an even vertex odd mean graph. □

Theorem 2.5. *$K_{2,n}$ is an even vertex odd mean graph for all n.*

Proof. Let $\{V_1, V_2\}$ be the bipartition of $K_{2,n}$ with $V_1 = \{u, v\}, V_2 = \{u_1, u_2, \ldots, u_n\}$. We define $f : V(K_{2,n}) \to \{0, 2, 4, \ldots, 2q-2, 2q = 4n\}$ as follows:

$$f(u) = 0,$$
$$f(v) = 4n \text{ and}$$
$$f(u_i) = 4i - 2, 1 \leq i \leq n.$$

The label of the edge uu_i is $2i-1, 1 \leq i \leq n$. The label of the edge vu_i is $2n + 2i - 1, 1 \leq i \leq n$. Hence, $K_{2,n}$ is an even vertex odd mean graph for all n.

For example, an even vertex odd mean labeling of $K_{2,8}$ is shown in Figure 5. □

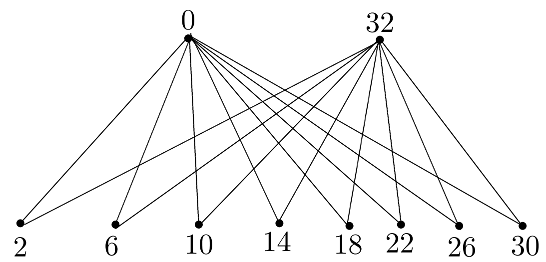


Figure 5: An even vertex odd mean labeling of $K_{2,8}$

Theorem 2.6. *The Bistar $B_{m,n}$ is an even vertex odd mean graph for $n = m, m+1$.*

Proof. Let $V(K_2) = \{u.v\}$ and $u_i (1 \leq i \leq m), v_j (1 \leq j \leq n)$ be the vertices adjacent to u and v respectively. Define $f : V(B_{m,n}) \to \{0, 2, 4, \ldots, 2q-2, 2q\}$ by

$$f(u) = 0, f(v) = \begin{cases} 4n+2 & \text{if } n = m \\ 4n-2 & \text{if } n = m+1, \end{cases}$$

$$f(u_i) = 4i - 2, 1 \leq i \leq m \text{ and}$$
$$f(v_j) = 4j, 1 \leq j \leq n (= m, m+1).$$

The induced edge labels are given as follows:

$$f^*(uv) = \begin{cases} 2n+1 & \text{if } n = m \\ 2n-1 & \text{if } n = m+1, \end{cases}$$

$$f^*(uu_i) = 2i-1, 1 \leq i \leq m \text{ and}$$

for $1 \leq j \leq n$, $f^*(vv_j) = \begin{cases} 2n+2j+1 & \text{if } n = m \\ 2n+2j-1 & \text{if } n = m+1. \end{cases}$

Hence, f is an even vertex odd mean labeling and hence $B_{m,n}$ is an even vertex odd mean graph. An even vertex odd mean labeling of $B_{5,5}$ and $B_{6,7}$ are shown in Figure 6. □

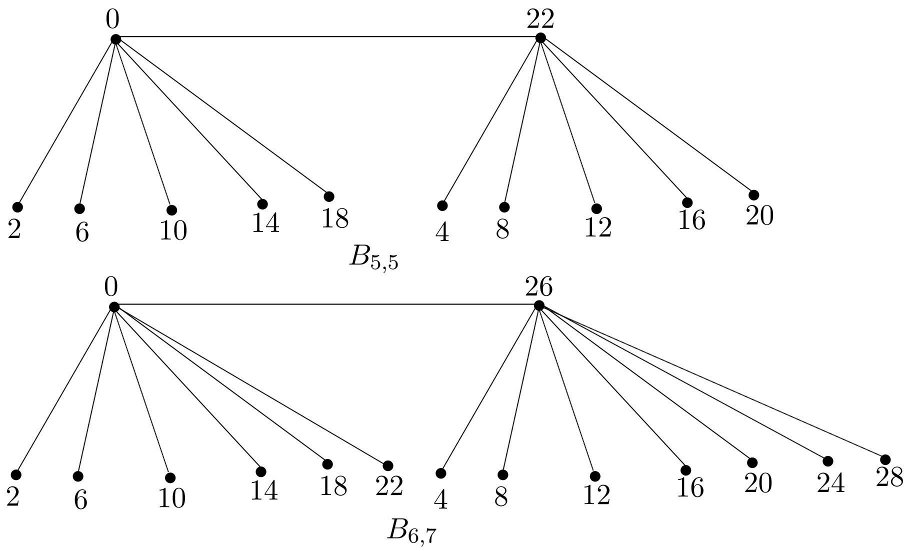


Figure 6: An even vertex odd mean labeling of $B_{5,5}$ and $B_{6,7}$

Theorem 2.7. *A quadrilateral snake is an even vertex odd mean graph.*

Proof. Let G_n denote the quadrilateral snake obtained from $u_1, u_2, \ldots, u_{n+1}$ by joining u_i, u_{i+1} to new vertices v_i, w_i respectively and joining v_i and $w_i, 1 \leq i \leq n$.

We define $f : V(G_n) \to \{0, 2, 4, \ldots, 2q-2, 2q = 8n\}$ as follows:

For $1 \leq i \leq n+1$,

$f(u_i) = \begin{cases} 8i-8 & \text{if } i \text{ is odd} \\ 8i-10 & \text{if } i \text{ is even.} \end{cases}$

For $1 \leq i \leq n$, $f(v_i) = \begin{cases} 8i-6 & \text{if } i \text{ is odd} \\ 8i-4 & \text{if } i \text{ is even and} \end{cases}$

$f(w_i) = \begin{cases} 8i & \text{if } i \text{ is odd} \\ 8i-2 & \text{if } i \text{ is even.} \end{cases}$

The induced edge labels are given by

$$f^*(u_iu_{i+1}) = 8i - 5, 1 \leq i \leq n,$$
$$f^*(v_iw_i) = 8i - 3, 1 \leq i \leq n,$$
$$f^*(u_iv_i) = 8i - 7, 1 \leq i \leq n \text{ and}$$
$$f^*(u_iw_{i-1}) = 8i - 9, 2 \leq i \leq n + 1.$$

Thus, f is an even vertex odd mean labeling and hence G_n is an even vertex odd mean graph.

An even vertex odd mean labeling of G_5 is shown in Figure 7. □

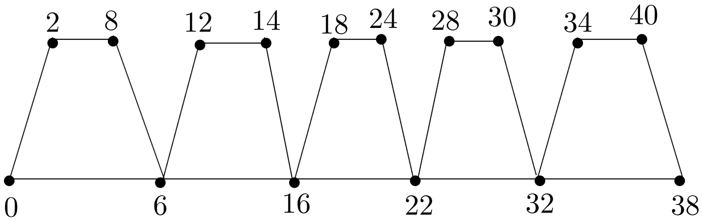


Figure 7: An even vertex odd mean labeling of G_5

Theorem 2.8. *Any comb is an even vertex odd mean graph.*

Proof. Let G be the comb obtained from a path $P_n : v_1, v_2, \ldots, v_n$ by joining a vertex u_i to $v_i (1 \leq i \leq n)$. Define $f : V(G = P_n \odot K_1) \to \{0, 2, 4, \ldots, 2q - 2, 2q = 4n - 2\}$ as follows:

For $1 \leq i \leq n$,

$$f(v_i) = \begin{cases} 4i - 2 & \text{if } i \text{ is odd} \\ 4i - 4 & \text{if } i \text{ is even and} \end{cases}$$

$$f(u_i) = \begin{cases} 4i - 4 & \text{if } i \text{ is odd} \\ 4i - 2 & \text{if } i \text{ is even.} \end{cases}$$

The induced edge labels are obtained as follows:

$f^*(v_iv_{i+1}) = 4i - 1, 1 \leq i \leq n - 1$ and

$f^*(u_iv_i) = 4i - 3, 1 \leq i \leq n$.

Hence, f is an even vertex odd mean labeling of $P_n \odot K_1$ and hence comb is an even vertex odd mean graph.

An even vertex odd mean labeling of $P_7 \odot K_1$ is shown in Figure 8. □

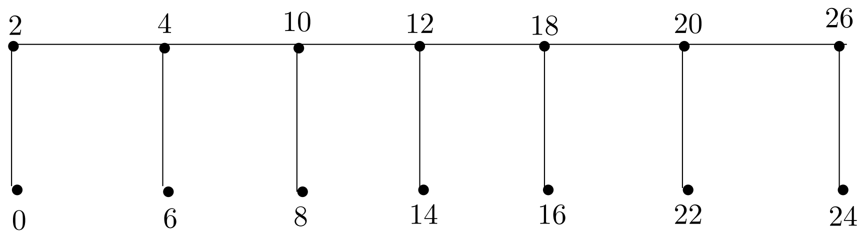


Figure 8: An even vertex odd mean labeling of $P_7 \odot K_1$

Theorem 2.9. *$[P_n; S_2]$ is an even vertex odd mean graph.*

Proof. Let $u_i, 1 \leq i \leq n$ be the vertices of the path P_n and $v_i, w_i, 1 \leq i \leq n$ be the vertices which are made adjacent with u_i.

We define $f : V[P_n; S_2] \to \{0, 2, 4, \ldots, 2q-2, 2q = 6n-2\}$ as follows:

$$f(u_i) = 6i - 4, 1 \leq i \leq n,$$
$$f(v_i) = 6i - 6, 1 \leq i \leq n \text{ and}$$
$$f(w_i) = 6i - 2, 1 \leq i \leq n.$$

The induced edge labels are given by

$$f^*(u_i u_{i+1}) = 6i - 1, 1 \leq i \leq n-1,$$
$$f^*(u_i v_i) = 6i - 5, 1 \leq i \leq n \text{ and}$$
$$f^*(u_i w_i) = 6i - 3, 1 \leq i \leq n.$$

Thus, f is an even vertex odd mean labeling and hence $[P_n; S_2]$ is an even vertex odd mean graph.

For example, an even vertex odd mean labeling of $[P_6; S_2]$ is shown in Figure 9. □

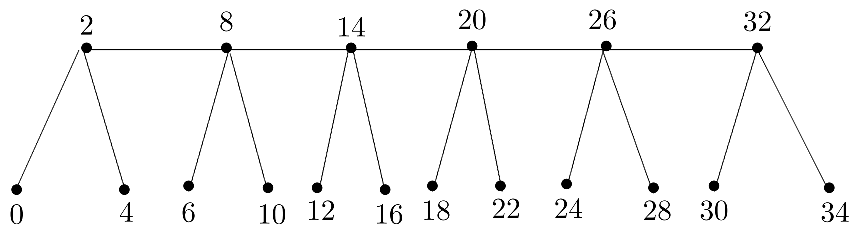


Figure 9: An even vertex odd mean labeling of $[P_6; S_2]$

Theorem 2.10. *$[P_{2n}; S_m]$ is an even vertex odd mean graph for $m \geq 3, n \geq 1$.*

Proof. Let $v_{0_j}, v_{1_j}, v_{2_j}, \ldots, v_{m_j}$ be the vertices and $e_{1_j}, e_{2_j}, \ldots, e_{m_j}$ be the edges in the j^{th} copy of $S_m, 1 \leq j \leq 2n$ and joining v_{0_j} and $v_{0_{j+1}}$ by means of an edge, $1 \leq j \leq 2n-1$. We define $f : V[P_{2n}; S_m] \to \{0, 2, 4, \ldots, 2q-2, 2q\}$ as follows:

$$f(v_{0_{2j+1}}) = (4m+4)j, 0 \leq j \leq n-1,$$
$$f(v_{0_{2j}}) = (4m+4)j - 2, 1 \leq j \leq n,$$
$$f(v_{i_{2j+1}}) = (4m+4)j + 4i - 2, 0 \leq j \leq n-1, 1 \leq i \leq m \text{ and}$$
$$f(v_{i_{2j}}) = (4m+4)(j-1) + 4i, 1 \leq j \leq n, 1 \leq i \leq m.$$

The induced edge labels are given by

$f^*(v_{0_j}v_{0_{j+1}}) = (2m+2)(j-1)+11, 1 \le j \le 2n-1$ and
$f^*(e_{i_j}) = (2m+2)(j-1)+2i-1, 1 \le j \le 2n, 1 \le i \le m.$

Thus, f is an even vertex odd mean labeling of $[P_{2n}; S_m]$. Hence, $[P_{2n}; S_m]$ is an even vertex odd mean graph.

For example, an even vertex odd mean labeling of $[P_6; S_5]$ is shown in Figure 10. □

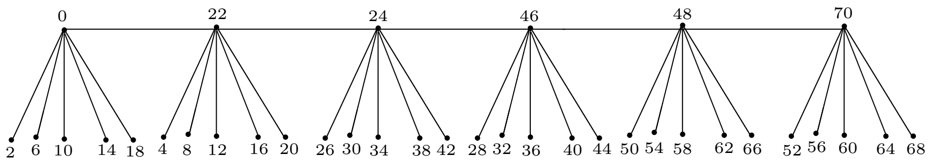


Figure 10: An even vertex odd mean labeling of $[P_6; S_5]$

Theorem 2.11. *The planar grid $P_m \times P_n$ is an even vertex odd mean graph for $m \ge 2, n \ge 2$.*

Proof. Let $V(P_m \times P_n) = \{a_{i_j} : 1 \le i \le m, 1 \le j \le n\}$ and $E(P_m \times P_n) = \{a_{i_{j-1}}a_{i_j} : 1 \le i \le m, 2 \le j \le n\} \cup \{a_{(i-1)_j}a_{i_j} : 2 \le i \le m, 1 \le j \le n\}$.

Define $f : V(P_m \times P_n) \to \{0, 2, 4, \ldots, 2q-2, 2q\}$ by
$f(a_{1_j}) = 2(j-1), 1 \le j \le n$ and
$f(a_{i_j}) = f(a_{(i-1)_n}) + 2(n-1) + 2j, 2 \le i \le m, 1 \le j \le n.$

The edge labels are given as follows:
$f^*(a_{i_j}a_{i_{j+1}}) = (i-1)(4n-3)+i+2j-2, 1 \le i \le m, 1 \le j \le n-1$ and
$f^*(a_{i_j}a_{(i+1)_j}) = (2i-2)(2n-1)+2n-3+2j, 1 \le i \le m-1, 1 \le j \le n.$

Then, $P_m \times P_n$ has an even vertex odd mean labeling and hence $P_m \times P_n$ is an even vertex odd mean graph for $m \ge 2, n \ge 2$.

For example, an even vertex odd mean labeling of $P_5 \times P_6$ is shown in Figure 11. □

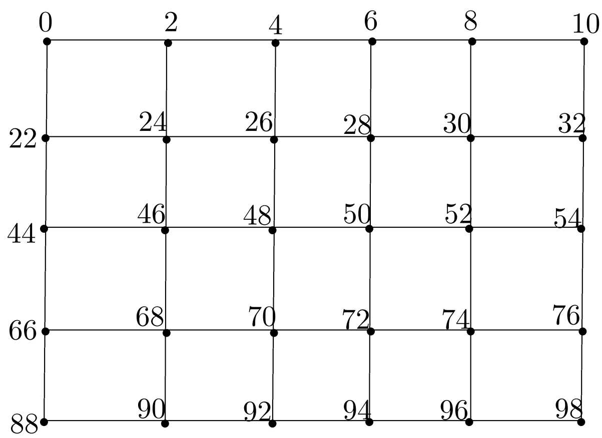


Figure 11: An even vertex odd mean labeling of $P_5 \times P_6$

Corollary 2.12. *L_n is an even vertex odd mean graph for all n.*

Theorem 2.13. *$L_n \odot K_1$ is an even vertex odd mean graph.*

Proof. Let L_n be the ladder. Let G be the graph obtained by joining a pendant edge to each vertex of the ladder. Let u_i and v_i be the vertices of the ladder. For $1 \le i \le n$, let u'_i and v'_i be the new vertices made adjacent with u_i and v_i respectively.

Define $f : V(G) \to \{0, 2, 4, \ldots, 2q-2, 2q\}$ by

$$\begin{aligned} f(u_i) &= 10i - 8, 1 \le i \le n,\\ f(v_i) &= 10i - 6, 1 \le i \le n,\\ f(u'_i) &= 10i - 10, 1 \le i \le n \text{ and}\\ f(v'_i) &= 10i - 4, 1 \le i \le n. \end{aligned}$$

The edge labels are given as follows:

$$\begin{aligned} f^*(u_iu_{i+1}) &= 10i - 3, 1 \le i \le n-1,\\ f^*(v_iv_{i+1}) &= 10i - 1, 1 \le i \le n-1,\\ f^*(u_iu'_i) &= 10i - 9, 1 \le i \le n \text{ and}\\ f^*(v_iv'_i) &= 10i - 5, 1 \le i \le n. \end{aligned}$$

Thus, $L_n \odot K_1$ has an even vertex odd mean labeling and hence $L_n \odot K_1$ is an even vertex odd mean graph.

For example, an even vertex odd mean labeling of $L_6 \odot K_1$ is shown in Figure 12. □

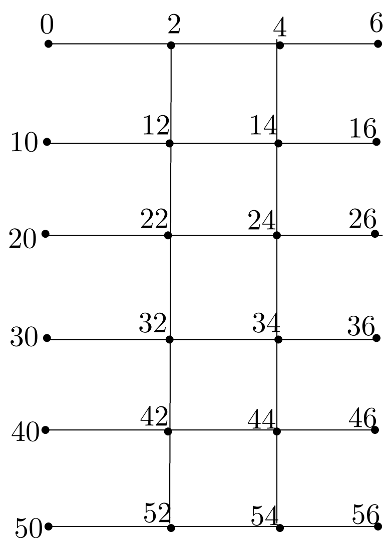


Figure 12: An even vertex odd mean labeling of $L_6 \odot K_1$

Theorem 2.14. *$C_m \times P_n$ is an even vertex odd mean graph for $n \geq 1$ and $m \equiv 0 \pmod 4$.*

Proof. Let $V(C_m \times P_n) = \{v_{i_j} : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(C_m \times P_n) = \{e_{i_j} : e_{i_j} = v_{i_j}v_{(i+1)_j}, 1 \leq j \leq n, 1 \leq i \leq m\} \cup \{E_{i_j} : E_{i_j} = v_{i_j}v_{i_{j+1}}, 1 \leq j \leq n-1, 1 \leq i \leq m\}$ where $i+1$ is taken modulo m. Let C_m^j denote the j^{th} copy of C_m in $C_m \times P_n$. Let the vertices of C_m^j be $v_{1_j}, v_{2_j}, \ldots, v_{m_j}$ for $1 \leq j \leq n$.

Label the vertices of $C_m^1, m \equiv 0 \pmod 4$ as follows:

$$f(v_{i_1}) = \begin{cases} 2i-2, & 1 \leq i \leq \frac{m}{2} \\ m+4+2\left(i-\left(\frac{m}{2}+1\right)\right) & \text{if } i \text{ is odd and } \frac{m}{2}+1 \leq i \leq m-1 \\ m+2+2\left(i-\left(\frac{m}{2}+2\right)\right) & \text{if } i \text{ is even and } \frac{m}{2}+2 \leq i \leq m. \end{cases}$$

If the vertices of C_m^{j-1} are labeled, then the vertices of C_m^j are labeled as follows:

$f(v_{i_j}) = f(v_{i-1_{j-1}}) + 4m$ where $i-1$ and $j-1$ are taken modulo m.

It can be verified that the label of the edges are $1, 3, 5, \ldots, 2q-1$.

Then, f is an even vertex odd mean labeling of $C_m \times P_n$ for $n \geq 1$ and $m \equiv 0 \pmod 4$.

Hence, $C_m \times P_n$ is an even vertex odd mean graph for $n \geq 1$ and $m \equiv 0 \pmod 4$.

For example, an even vertex odd mean labeling of $C_8 \times P_4$ is shown in Figure 13. □

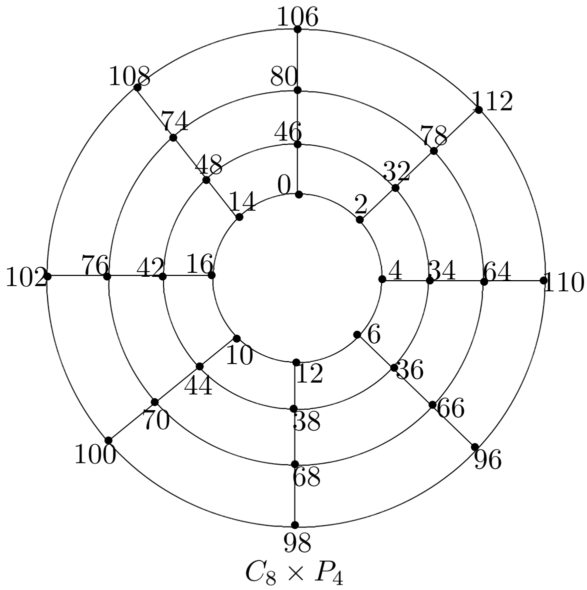


Figure 13: An even vertex odd mean labeling of $C_8 \times P_4$

Theorem 2.15. *$Q_3 \times P_n$ is an even vertex odd mean graph.*

Proof. Let Q_{3_j} denote the j^{th} copy of Q_3 in $Q_3 \times P_n$ and for $1 \leq i \leq 8$, let v_{i_j} denote the i^{th} vertex in Q_{3_j}, where $1 \leq j \leq n$.

The vertices and their labels of $Q_3 \times P_2$ are shown in Figure 14.

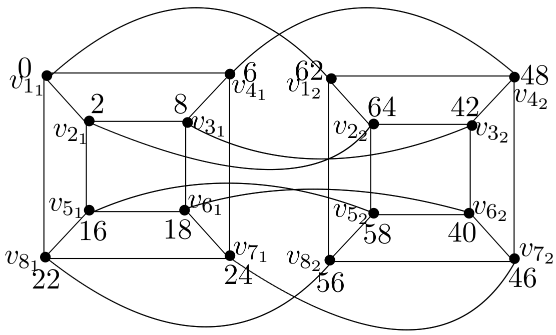


Figure 14: An even vertex odd mean labeling of $Q_3 \times P_2$

If the vertices of $Q_{3_{j-2}}$ are labeled by f, then the vertices of Q_{3_j} are labeled as follows:

$f(v_{i_j}) = f(v_{i_{j-2}}) + 80$, for $1 \leq i \leq 8$ and $3 \leq j \leq n$.

Let E_j be the set of all edges in Q_{3_j} and $E_{j_{j+1}}$ be the set of all edges having one end in Q_{3_j} and the other in $Q_{3_{j+1}}$.

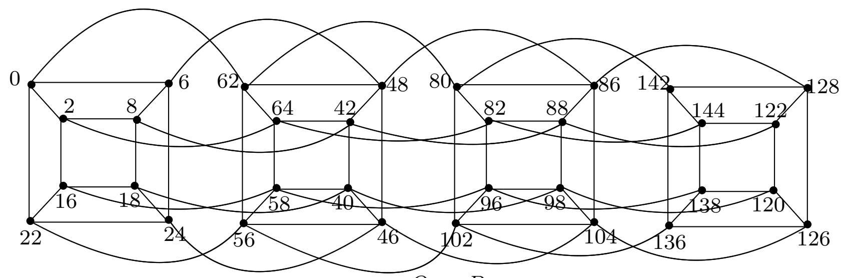
Denote the set of edge labels for the edges of E by $f^*(E)$.

Then, it is observed that

$f^*(E_j) = \{40 + f^*(e) : e \in E_{j-1}\}, 2 \leq j \leq n$ and

$f^*(E_{j_{j+1}}) = \{40 + f^*(e) : e \in E_{(j-1)_j}\}, 2 \leq j \leq n-1$.

Then, f is an even vertex odd mean labeling of $Q_3 \times P_n$. For example, an even vertex odd mean labeling of $Q_3 \times P_4$ is shown in Figure 15. $\square$



$Q_3 \times P_4$

Figure 15: An even vertex odd mean labeling of $Q_3 \times P_4$

Theorem 2.16. *If G is an even vertex odd mean graph, then $P_n(G)$ is also an even vertex odd mean graph.*

Proof. Let $v_1, v_2, \ldots, v_p$ be the vertices of G with size q and $u_1, u_2, \ldots, u_n$ be the vertices of P_n. Let f be an even vertex odd mean labeling of G.

Then define g on $V(P_n(G))$ as follows:
$g(v_i) = f(v_i), 1 \leq i \leq p$ and
$g(u_j) = 2q + 2j - 2, 1 \leq j \leq n$.
Then, g is an even vertex odd mean labeling of $P_n(G)$. □

Corollary 2.17. *Dragon $P_n(C_m)$ is an even vertex odd mean graph for $n \geq 1, m \equiv 0 \pmod 4$.*

Proof. Since C_m is an even vertex odd mean graph for $m \equiv 0 \pmod 4$, by Theorem 2.16, $P_n(C_m)$ as also an even vertex odd mean graph.

For example, an even vertex odd mean labeling of $P_5(C_8)$ is shown in Figure 16. □

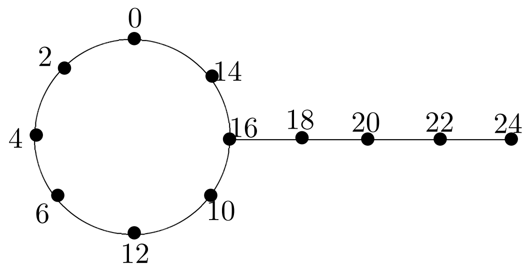


Figure 16: An even vertex odd mean labeling of $P_5(C_8)$

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