

Some transformations on Kenmotsu manifolds

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Abstract. The present paper deals with a study of certain transformations on a Kenmotsu manifold. We study an infinitesimal CL -transformation on a Kenmotsu manifold. We also study CL -transformation on a Kenmotsu manifold and obtain a new tensor field which is invariant under such a transformation. Finally we study CL -semisymmetric Kenmotsu manifold and prove that it is a manifold of constant curvature -1 , from which we obtain some equivalent conditions as characterization of such a manifold.

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§1. Introduction

In 1963, Tashiro and Tachibana [8] introduced a transformation, called CL -transformation, on a Sasakian manifold under which C -loxodrome remains invariant. We mention that a loxodrome is a curve on the unit sphere that intersects the meridians at a fixed angle, and the loxodrome was mainly used in navigation and usually called rhumb lines. We note that a C -loxodrome is a loxodrome cutting geodesic trajectories of the characteristic vector field ξ of the Sasakian manifold with constant angle. We also note that under conformal transformation angle between two intersecting curves remains invariant and the conformal curvature tensor is the invariant of such a transformation [9]. On the other hand, under CL -transformation the angle between two C -loxodromes remains invariant and hence ‘ CL ’ stands for C -loxodrome. The transformation was mainly defined for Sasakian manifold by Tashiro and Tachibana [8] and hence the invariance of such a transformation depends on specific manifold as the invariant tensor field on Sasakian manifold was determined by Koto and Nagao [3]. Also, an invariant tensor field on LP -Sasakian

manifold was obtained by Matsumoto and Mihai [4]. Recently, the invariant tensor field under such a transformation on Lorentzian concircular structure manifold is investigated by Shaikh and Ahmad [6]. In the present paper the invariant tensor field under such a transformation on Kenmotsu manifold is obtained in Section 4. Again, Takamatsu and Mizusawa [7] studied an infinitesimal CL -transformation in a compact Sasakian manifold. We note that an infinitesimal CL -transformation means the vector field associated with a local one-parameter transformation group consisting of CL -transformations and may be named as a CL -Killing vector field.

In 1972, Kenmotsu [2] introduced a class of almost contact Riemannian manifolds which is called Kenmotsu manifold. We note that the structure of a Kenmotsu manifold is normal but not quasi-Sasakian and hence not Sasakian. We also note that a Kenmotsu manifold is not compact. Again, if F is a Kählerian manifold and c is a non-zero constant such that $g(t) = ce^t$ is a function on a line L , then the warped product $M = L \times_g F$ is a Kenmotsu manifold and the converse also is true [2].

The object of the present paper is to study some transformations on a Kenmotsu manifold. The paper is organized as follows. Section 2 provides the rudimentary facts of Kenmotsu manifolds along with some curvature relations. Section 3 is devoted to the study of an infinitesimal CL -transformation on a Kenmotsu manifold and it is proved that such a transformation is not necessarily a projective Killing vector field. However, on an Einstein Kenmotsu manifold an infinitesimal CL -transformation is necessarily a projective Killing vector field. In Section 4 we study a CL -transformation on a Kenmotsu manifold and obtain a new tensor field which is invariant under the CL -transformation. This tensor field is called a CL -curvature tensor field. In the last section we study CL -semisymmetric Kenmotsu manifolds and prove that such a manifold is of constant curvature -1 and the converse also is true. Hence in a Kenmotsu manifold, the concept of CL -semisymmetry, CL -symmetry, CL -flatness, semisymmetry, local symmetry, conformally flatness and manifold of constant curvature -1 are equivalent (see Corollary 5.1).

§2. Kenmotsu manifolds

A $(2n + 1)$ -dimensional smooth manifold M is said to be an almost contact metric manifold [11] if there exist an $(1,1)$ tensor field ϕ , a vector field ξ , an 1-form η and a Riemannian metric g on M such that

$$(2.1) \quad (a) \ \eta(\xi) = 1, \quad (b) \ \phi \circ \xi = 0, \quad (c) \ \eta \circ \phi = 0,$$

$$(2.2) \quad (a) \ \eta(X) = g(X, \xi), \quad (b) \ \phi^2 X = -X + \eta(X)\xi,$$

$$(2.3) \quad g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y)$$

for any vector fields X and Y on M . An almost contact metric manifold M equipped with an almost contact metric structure (ϕ, ξ, η, g) is said to be a Kenmotsu manifold [2] if

$$(2.4) \quad (\nabla_X \phi)(Y) = -\eta(Y)\phi X - g(X, \phi Y)\xi$$

and

$$(2.5) \quad \nabla_X \xi = X - \eta(X)\xi$$

for any vector fields X and Y on M , where ∇ is the Levi-Civita connection of g .

In a Kenmotsu manifold, the following relations hold [2]:

$$(2.6) \quad (\nabla_X \eta)(Y) = g(X, Y) - \eta(X)\eta(Y),$$

$$(2.7) \quad \eta(R(X, Y)Z) = \{g(X, Z)\eta(Y) - g(Y, Z)\eta(X)\},$$

$$(2.8) \quad R(\xi, Y)Z = \{\eta(Z)Y - g(Y, Z)\xi\},$$

$$(2.9) \quad S(X, \xi) = -2n\eta(X),$$

$$(2.10) \quad (\nabla_Z R)(X, Y)\xi = -R(X, Y)Z - \{g(Y, Z)X - g(X, Z)Y\}$$

for any vector fields X , Y and Z on M , where R and S are the curvature tensor and the Ricci tensor of g respectively. For various results of Kenmotsu manifolds, we refer the reader to the book of Pitiş [5] and also references therein.

Throughout the paper we will consider a Kenmotsu manifold M of dimension $2n + 1$, $n \geq 1$, endowed with the Levi-Civita connection ∇ . In particular, from Section 4 to the last section we assume that $n > 1$.

§3. Infinitesimal CL -transformation on a Kenmotsu manifold

Definition 3.1. A vector field V on a Kenmotsu manifold M is said to be an infinitesimal CL -transformation [7] if it satisfies

$$(3.1) \quad \mathcal{L}_V \{^h_{ji}\} = \rho_j \delta_i^h + \rho_i \delta_j^h + \alpha(\eta_j \phi_i^h + \eta_i \phi_j^h)$$

for a certain constant α , where ρ_i are the components of an 1-form ρ , \mathcal{L}_V denotes the Lie derivative with respect to V and $\{^h_{ji}\}$ is the Christoffel symbol of the Riemannian metric g .

Proposition 3.1. *If V is an infinitesimal CL-transformation on a Kenmotsu manifold, then the 1-form ρ is closed.*

Proof. Contracting h and j in (3.1), it can be easily seen that ρ_i is a gradient. Hence the 1-form ρ is closed. \square

Theorem 3.1. *If V is an infinitesimal CL-transformation on a Kenmotsu manifold M , then the relation*

$$(3.2) \quad (\mathcal{L}_V g)(Y, Z) = (\nabla_Y \rho)(Z) - \alpha g(Y, \phi Z)$$

holds for any vector fields Y and Z on M .

Proof. It is known from [10] that

$$(3.3) \quad \mathcal{L}_V R_{kji}^h = \nabla_k \mathcal{L}_V \{j_i^h\} - \nabla_j \mathcal{L}_V \{k_i^h\}.$$

Substituting (3.1) into (3.3) and then using (2.4) and (2.6), we obtain

$$(3.4) \quad \begin{aligned} (\mathcal{L}_V R)(X, Y)Z &= (\nabla_X \rho)(Z)Y - (\nabla_Y \rho)(Z)X \\ &\quad + \alpha \{g(X, Z)\phi Y - g(Y, Z)\phi X\} \\ &\quad - \{\eta(Y)\phi X - \eta(X)\phi Y\}\eta(Z) + \{g(Y, \phi Z)\eta(X) \\ &\quad - g(X, \phi Z)\eta(Y) + 2g(Y, \phi X)\eta(Z)\}\xi \end{aligned}$$

for any vector fields X , Y and Z on M . Operating η to (3.4), we get

$$(3.5) \quad \begin{aligned} \eta((\mathcal{L}_V R)(X, Y)Z) &= (\nabla_X \rho)(Z)\eta(Y) - (\nabla_Y \rho)(Z)\eta(X) \\ &\quad + \alpha \{g(Y, \phi Z)\eta(X) - g(X, \phi Z)\eta(Y) \\ &\quad + 2g(Y, \phi X)\eta(Z)\}. \end{aligned}$$

Taking Lie derivative of (2.7) with respect to V and using (3.5) and then replacing X and Y to Y and ξ , respectively, we get

$$(3.6) \quad (\mathcal{L}_V g)(Y, Z) = (\nabla_Y \rho)(Z) - \{(\nabla_\xi \rho)(Z) - (\mathcal{L}_V g)(\xi, Z)\}\eta(Y) - \alpha g(Y, \phi Z).$$

Interchanging Y and Z in (3.6) and then subtracting it from (3.6), we get

$$(3.7) \quad \begin{aligned} \{(\nabla_\xi \rho)(Z) - (\mathcal{L}_V g)(\xi, Z)\}\eta(Y) &= \{(\nabla_\xi \rho)(Y) - (\mathcal{L}_V g)(\xi, Y)\}\eta(Z) \\ &\quad + 2\alpha g(\phi Y, Z). \end{aligned}$$

Replacing Y to ξ in (3.7) we obtain

$$(3.8) \quad (\nabla_\xi \rho)(Z) - (\mathcal{L}_V g)(\xi, Z) = \{(\nabla_\xi \rho)(\xi) - (\mathcal{L}_V g)(\xi, \xi)\}\eta(Z).$$

From (3.6) and (3.8), we obtain

$$(3.9) \quad (\mathcal{L}_V g)(Y, Z) = (\nabla_Y \rho)(Z) - \{(\nabla_\xi \rho)(\xi) - (\mathcal{L}_V g)(\xi, \xi)\} \eta(Y) \eta(Z) - \alpha g(Y, \phi Z).$$

Now taking inner product of (3.4) with a vector field W on M and then contracting X and W , we get

$$(3.10) \quad (\mathcal{L}_V S)(Y, Z) = -2n(\nabla_Y \rho)(Z).$$

Replacing Y to ξ in (3.10), we have

$$(3.11) \quad (\mathcal{L}_V S)(\xi, Z) = -2n(\nabla_\xi \rho)(Z).$$

Taking Lie derivative of (2.9) with respect to V and using (3.11) and then replacing Z to ξ , we obtain

$$(3.12) \quad (\nabla_\xi \rho)(\xi) = (\mathcal{L}_V g)(\xi, \xi).$$

Using (3.12) in (3.9), we obtain (3.2). This completes the proof. \square

From (3.2), we can state the following:

Theorem 3.2. *An infinitesimal CL-transformation V on a Kenmotsu manifold M is not a projective Killing vector field unless $\alpha = 0$.*

Corollary 3.1. *Any infinitesimal CL-transformation V on an Einstein Kenmotsu manifold M is necessarily a projective Killing vector field.*

Proof. Let M be an Einstein Kenmotsu manifold. Then

$$(3.13) \quad S(Y, Z) = -2ng(Y, Z).$$

Taking Lie derivative of (3.13) with respect to V and using (3.2) and (3.10), we obtain

$$(3.14) \quad \alpha g(Y, \phi Z) = 0$$

from which we get $\alpha = 0$. This completes the proof. \square

Corollary 3.2. *If V is an infinitesimal CL-transformation on an Einstein Kenmotsu manifold M , then $V - \frac{1}{2}\mu$ is a Killing vector field, where μ is the associated vector field of the 1-form ρ .*

§4. *CL*-transformation on a Kenmotsu manifold

Definition 4.1 ([3]). *A transformation f on a $(2n+1)$ -dimensional Kenmotsu manifold M with structure (ϕ, ξ, η, g) is said to be a *CL*-transformation if the Levi-Civita connection ∇ and a symmetric affine connection ∇^f induced from ∇ by f are related by*

$$(4.1) \quad \nabla_X^f Y = \nabla_X Y + \rho(X)Y + \rho(Y)X + \alpha\{\eta(X)\phi Y + \eta(Y)\phi X\},$$

where ρ is an 1-form and α is a constant.

Throughout the section, the geometric objects with respect to the symmetric affine connection ∇^f are represented as R^f and S^f etc., where R^f and S^f denote the curvature tensor and the Ricci tensor of the connection ∇^f respectively.

If f is a *CL*-transformation on a Kenmotsu manifold M , then by virtue of (4.1), (2.2), (2.4) and (2.6), the curvature tensor $R^f(X, Y)Z$ of the connection ∇^f is given by

$$(4.2) \quad \begin{aligned} R^f(X, Y)Z &= R(X, Y)Z + \{B(X, Y) - B(Y, X)\}Z + B(X, Z)Y \\ &\quad - B(Y, Z)X - \alpha\{\{\eta(Y)\phi X - \eta(X)\phi Y\}\eta(Z) \\ &\quad + \{g(Y, Z)\phi X - g(X, Z)\phi Y\} - \{g(Y, \phi Z)\eta(X) \\ &\quad - g(X, \phi Z)\eta(Y) - 2g(X, \phi Y)\eta(Z)\}\xi], \end{aligned}$$

for any vector fields X , Y and Z on M , where the tensor field $B(X, Y)$ is defined by

$$(4.3) \quad \begin{aligned} B(X, Y) &= (\nabla_X \rho)(Y) - \rho(X)\rho(Y) - \alpha^2\eta(X)\eta(Y) \\ &\quad - \alpha\{\eta(X)\rho(\phi Y) + \eta(Y)\rho(\phi X)\}. \end{aligned}$$

From (4.2), we have

$$(4.4) \quad \begin{aligned} g(R^f(X, Y)Z, U) &= g(R(X, Y)Z, U) + \{B(X, Y) - B(Y, X)\}g(Z, U) \\ &\quad + B(X, Z)g(Y, U) - B(Y, Z)g(X, U) \\ &\quad - \alpha[\{\eta(Y)g(\phi X, U) - \eta(X)g(\phi Y, U)\}\eta(Z) \\ &\quad + \{g(Y, Z)g(\phi X, U) - g(X, Z)g(\phi Y, U)\} \\ &\quad - \{g(Y, \phi Z)\eta(X) - g(X, \phi Z)\eta(Y) \\ &\quad - 2g(X, \phi Y)\eta(Z)\}\eta(U)], \end{aligned}$$

where U is any vector field on M . Taking contraction of (4.4) over X and U and also over Z and U and then adding the results and proceeding same manner as in [3], it is easy to check that $B(X, Y)$ is symmetric and hence from (4.3) we see that the 1-form ρ is closed.

Theorem 4.1. *Let A be the tensor field of type $(1, 3)$ given by*

$$(4.5) \quad A(X, Y)Z := R(X, Y)Z - \frac{1}{2n} [\{S(Y, Z)X - S(X, Z)Y\} \\ - \{g(Y, Z) + \eta(Y)\eta(Z)\}QX + \{g(X, Z) + \eta(X)\eta(Z)\}QY \\ + [\{S(X, Z) + 2ng(X, Z)\}\eta(Y) - \{S(Y, Z) \\ + 2ng(Y, Z)\}\eta(X) + 2\{S(X, Y) + 2ng(X, Y)\}\eta(Z)]\xi] \\ + \{g(Y, Z) + \eta(Y)\eta(Z)\}X - \{g(X, Z) + \eta(X)\eta(Z)\}Y,$$

where Q is the tensor field of type $(1, 1)$ given by $g(QX, Y) = S(X, Y)$, where X and Y are vector fields on M . This tensor field A is invariant under CL -transformations of a Kenmotsu manifold M .

Proof. Assume that f is a CL -transformation on a Kenmotsu manifold M . Then the relations (4.1)-(4.4) hold. Since the tensor $B(X, Y)$ is symmetric, (4.2) can be written as

$$(4.6) \quad R^f(X, Y)Z = R(X, Y)Z + B(X, Z)Y - B(Y, Z)X - \alpha[\{\eta(Y)\phi X \\ - \eta(X)\phi Y\}\eta(Z) + \{g(Y, Z)\phi X - g(X, Z)\phi Y\} \\ - \{g(Y, \phi Z)\eta(X) - g(X, \phi Z)\eta(Y) - 2g(X, \phi Y)\eta(Z)\}\xi],$$

which yields

$$(4.7) \quad 2nB(Y, Z) = S(Y, Z) - S^f(Y, Z).$$

Substituting (4.7) into (4.6), we obtain

$$(4.8) \quad P^f(X, Y)Z = P(X, Y)Z - \alpha[\{\eta(Y)\phi X - \eta(X)\phi Y\}\eta(Z) \\ + \{g(Y, Z)\phi X - g(X, Z)\phi Y\} - \{g(Y, \phi Z)\eta(X) \\ - g(X, \phi Z)\eta(Y) - 2g(X, \phi Y)\eta(Z)\}\xi],$$

where P is the projective curvature tensor [9] given by

$$(4.9) \quad P(X, Y)Z = R(X, Y)Z - \frac{1}{2n}\{S(Y, Z)X - S(X, Z)Y\}.$$

Replacing X to ξ in (4.8) and operating η , we obtain

$$(4.10) \quad \eta(P^f(\xi, Y)Z) - \eta(P(\xi, Y)Z) = \alpha g(Y, \phi Z).$$

From (4.8) and (4.10), we have

$$(4.11) \quad H^f(X, Y)Z = H(X, Y)Z - \alpha[\{g(Y, Z)\phi X - g(X, Z)\phi Y\} \\ + \{\eta(Y)\phi X - \eta(X)\phi Y\}\eta(Z)],$$

where we put

$$(4.12) \quad H(X, Y)Z = P(X, Y)Z - \{\eta(P(\xi, Y)Z)\eta(X) - \eta(P(\xi, X)Z)\eta(Y) - 2\eta(P(\xi, X)Y)\eta(Z)\}\xi.$$

Taking the inner product of (4.11) with W and then using (4.10), we obtain

$$(4.13) \quad g(L^f(X, Y)Z, W) = g(L(X, Y)Z, W)$$

where the tensor field L is defined by

$$(4.14) \quad g(L(X, Y)Z, W) = g(H(X, Y)Z, W) + \{g(X, Z) + \eta(X)\eta(Z)\}\eta(P(\xi, Y)W) - \{g(Y, Z) + \eta(Y)\eta(Z)\}\eta(P(\xi, X)W)$$

and L^f also is defined similarly. Using (4.9), (4.12), (2.7), (2.9) and (4.14), we obtain

$$g(L(X, Y)Z, W) = g(A(X, Y)Z, W),$$

that is, $L = A$. Similarly we obtain $L^f = A^f$. Hence we obtain

$$g(A^f(X, Y)Z, W) = g(A(X, Y)Z, W).$$

This completes the proof. \square

This tensor field A on a Kenmotsu manifold M invariant under a CL -transformation is said to be the CL -curvature tensor field on M .

§5. CL -semisymmetric Kenmotsu manifolds

Definition 5.1. A Kenmotsu manifold M is said to be CL -flat if the CL -curvature tensor field A of type (1,3) vanishes identically on M .

We mention that CL -flat manifold was introduced by Koto and Nagao in [3] for a Sasakian manifold.

Definition 5.2. A Kenmotsu manifold M is said to be CL -symmetric if $\nabla A = 0$.

A Riemannian manifold M is said to be locally symmetric due to Cartan [1] if it satisfies $\nabla R = 0$.

Definition 5.3. A Kenmotsu manifold M is said to be CL -semisymmetric if $R(U, W) \cdot A = 0$.

From (4.5), we have

$$\begin{aligned}
(5.1) \quad & 2n(R(U, W) \cdot A)(X, Y)Z \\
& = 2n(R(U, W) \cdot R)(X, Y)Z \\
& \quad - \{(R(U, W) \cdot S)(Y, Z)X - (R(U, W) \cdot S)(X, Z)Y\} \\
& \quad + \{g(Y, Z) + \eta(Y)\eta(Z)\}(R(U, W) \cdot Q)(X) \\
& \quad - \{g(X, Z) + \eta(X)\eta(Z)\}(R(U, W) \cdot Q)(Y) \\
& \quad - \{(R(U, W) \cdot S)(X, Z)\eta(Y) - (R(U, W) \cdot S)(Y, Z)\eta(X) \\
& \quad + 2(R(U, W) \cdot S)(X, Y)\eta(Z)\}\xi - \{E(X, Z)\eta(Y) \\
& \quad - E(Y, Z)\eta(X) + 2E(X, Y)\eta(Z)\}\{\eta(U)W - \eta(W)U\} \\
& \quad + \{g(U, Y)\eta(W) - g(W, Y)\eta(U)\}\{E(X, Z)\xi - \eta(Z)\mathcal{E}X\} \\
& \quad - \{g(U, X)\eta(W) - g(W, X)\eta(U)\}\{E(Y, Z)\xi - \eta(Z)\mathcal{E}Y\} \\
& \quad + \{g(U, Z)\eta(W) - g(W, Z)\eta(U)\}[2E(X, Y)\xi \\
& \quad - \eta(Z)\{\eta(Y)\mathcal{E}X - \eta(X)\mathcal{E}Y\}],
\end{aligned}$$

where we put

$$(5.2) \quad E(X, Y) = S(X, Y) + 2ng(X, Y)$$

and \mathcal{E} is given by $g(\mathcal{E}X, Y) = E(X, Y)$.

Lemma 5.1. *A Kenmotsu manifold M is CL-semisymmetric if and only if*

$$\begin{aligned}
(5.3) \quad & 2n(R(U, W) \cdot R)(X, Y)Z \\
& = \{(R(U, W) \cdot S)(Y, Z)X - (R(U, W) \cdot S)(X, Z)Y\} \\
& \quad - \{g(Y, Z) + \eta(Y)\eta(Z)\}(R(U, W) \cdot Q)(X) \\
& \quad - \{g(X, Z) + \eta(X)\eta(Z)\}(R(U, W) \cdot Q)(Y) \\
& \quad + \{(R(U, W) \cdot S)(X, Z)\eta(Y) - (R(U, W) \cdot S)(Y, Z)\eta(X) \\
& \quad + 2(R(U, W) \cdot S)(X, Y)\eta(Z)\}\xi + \{E(X, Z)\eta(Y) \\
& \quad - E(Y, Z)\eta(X) + 2E(X, Y)\eta(Z)\}\{\eta(U)W - \eta(W)U\} \\
& \quad - \{g(U, Y)\eta(W) - g(W, Y)\eta(U)\}\{E(X, Z)\xi - \eta(Z)\mathcal{E}X\} \\
& \quad + \{g(U, X)\eta(W) - g(W, X)\eta(U)\}\{E(Y, Z)\xi - \eta(Z)\mathcal{E}Y\} \\
& \quad - \{g(U, Z)\eta(W) - g(W, Z)\eta(U)\}[2E(X, Y)\xi \\
& \quad - \eta(Z)\{\eta(Y)\mathcal{E}X - \eta(X)\mathcal{E}Y\}]
\end{aligned}$$

for any vector fields X, Y, Z, U and W on M .

Proof. The result follows from (5.1). □

Lemma 5.2. *A CL-semisymmetric Kenmotsu manifold M is Einstein and Ricci semisymmetric.*

Proof. Let M be a CL -semisymmetric Kenmotsu manifold. Then according to Lemma 5.1 we have the relation (5.3). Replacing X and Z to ξ in (5.3), we obtain

$$(5.4) \quad (R(U, W) \cdot Q)(Y) = \{E(W, Y)\eta(U) - E(U, Y)\eta(W)\}\xi.$$

Again, replacing U to ξ in (5.4) and operating η , we get

$$(5.5) \quad S(W, Y) = -2ng(W, Y),$$

i.e., M is an Einstein manifold. Now in view of (5.5), (5.4) yields

$$(5.6) \quad (R(U, W) \cdot Q)(Y) = 0.$$

Thus the manifold M is Ricci semisymmetric. \square

Remark 5.1. *A CL -symmetric (resp. CL -flat) Kenmotsu manifold M is Einstein and Ricci symmetric.*

Theorem 5.1. *A Kenmotsu manifold M is CL -semisymmetric if and only if it is a manifold of constant curvature -1 .*

Proof. Assume that M is CL -semisymmetric. Then replacing X and W to ξ in (5.3) and using (5.5), we obtain

$$(5.7) \quad R(U, Y)Z = -\{g(Y, Z)U - g(U, Z)Y\},$$

i.e. M is of constant curvature -1 .

Conversely if M is of constant curvature -1 , then (5.7) implies (5.5) and hence it follows from (5.1) that M is CL -semisymmetric. \square

From Theorem 5.1, we can state the following:

Corollary 5.1. *In a Kenmotsu manifold M , the following assertions are equivalent:*

- (a) M is CL -semisymmetric;
- (b) M is CL -symmetric;
- (c) M is CL -flat;
- (d) M is semisymmetric;
- (e) M is locally symmetric;
- (f) M is conformally flat;
- (g) M is a manifold of constant curvature -1 .

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