SUT Journal of Mathematics
Vol. 47, No. 2 (2011), 217–227

Nonexistence of global solution to some second order quasilinear hyperbolic equation

Yuusuke Sugiyama

(Received May 23, 2011; Revised September 1, 2011)

Abstract. We show the nonexistence of a global solution of the Dirichlet problem of a quasilinear hyperbolic equation.

AMS 2010 *Mathematics Subject Classification.* 35L70.

Key words and phrases. Nonlinear wave equation, blow-up, Dirichlet problem.

§1. Introduction

In this paper, we consider the Dirichlet problem for the nonlinear wave equation:

$$(1.1)\quad \begin{cases} \partial_t^2 u = u\,\mathrm{div}(u\nabla u) + u^p, \quad (t,x) \in (0,T] \times \Omega, & (1.1)_a \\ u(0,x) = u_0(x), \quad x \in \Omega, & (1.1)_b \\ \partial_t u(0,x) = u_1(x), \quad x \in \Omega, & (1.1)_c \\ u(t,x) = A,\ \partial_t u(t,x) = 0, \quad (t,x) \in [0,T] \times \partial\Omega, & (1.1)_d \end{cases}$$

where $u(t,x)$ is an unknown real valued function, A is a nonnegative constant and Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary. We denote Lebesgue space $L^2(\Omega)$ with the norm $\|\cdot\|_{L^2}$, Sobolev space $H^m(\Omega)$ with Sobolev norm $\|\cdot\|_{H^m} = (\sum_{k=0}^{m} \|\partial_x^k \cdot\|_{L^2}^2)^{\frac{1}{2}}$ for $m \in \mathbb{N}$ and the closure of $C_0^\infty(\Omega)$ with the topology of $H^1(\Omega)$ by L^2, H^m and H_0^1 respectively, and set $H_\sharp^m := H^m \cap H_0^1$. The hyperbolic equation:

$$\partial_t^2 u = u\,\mathrm{div}(u\nabla u)$$

describes the wave of temperature in the superfluid, which is called second sound equation. In [9], L. D. Landau and E. M. Lifshitz explain the details of second sound equation and its background.

Theorem 1. *Let* $u_0 - A \in H_\sharp^{[\frac{n}{2}]+2}$, $u_1 \in H_\sharp^{[\frac{n}{2}]+1}$ *satisfying the compatibility condition of oder* $[\frac{n}{2}]+1$. *Suppose that* $A > 0$, $p > 3$, $(u_0 - A, u_1)_{L^2} > 0$ *and either*

$$P'(\|u_0 - A\|_{L^2}^2) \geq 0, \tag{1.2}$$

or

$$P((\frac{C_m + 8E(0)}{C_p})^{\frac{2}{p+1}}) + C' > 0 \text{ for } C_m + 8E(0) \geq 0, \tag{1.3}$$

where

$$P(x) = \frac{2C_p}{p+3} x^{\frac{p+3}{2}} - (C_m + 8E(0))x,$$

$P'(x) = \dfrac{dP}{dx}(x)$, C_p *and* C_m *are some positive constants depending only on* A, p *and* Ω,

$$C' = 2(u_0 - A, u_1)_{L^2}^2 - P(\|u_0 - A\|_{L^2}^2)$$

and

$$E(0) = \frac{1}{2}\|u_1\|_{L^2}^2 + \frac{1}{2}\sum_{j=1}^{n} \|u_0 \partial_{x_j} u_0\|_{L^2}^2 - \frac{1}{p+1}\int_\Omega u_0^{p+1}(x)dx.$$

Then a time global solution u *of* (1.1) *satisfying*

$$u - A \in \bigcap_{k=0}^{[n/2]+2} C^k([0,\infty); H_\sharp^{[n/2]-k+2}) \text{ and } u(t,x) > 0 \quad \text{for } (t,x) \in [0,\infty) \times \Omega, \tag{1.4}$$

does not exist.

Many authors(e.g. M. Tsutsumi[1], J. M. Ball[2], R. T. Glassey[3], H. A. Levine[4] and B. Straughan[5]) have considered the nonexistence of a global solution of the following semilinear wave equation:

$$\begin{cases} \partial_t^2 u = \Delta u + |u|^{p-1}u, & (t,x) \in (0,T] \times \Omega, \\ u(0,x) = u_0(x), \quad \partial_t u(0,x) = u_1(x), & x \in \Omega, \\ u(t,x) = 0, \quad (t,x) \in [0,T] \times \partial\Omega. \end{cases} \tag{1.5}$$

The strategy of their proofs is based on the argument of the differential inequality which is derived by the conservation law of energy such that

$$\tilde{E}(t) := \frac{1}{2}\sum_{j=1}^{n} \|\partial_{x_j} u(t,\cdot)\|_{L^2}^2 + \frac{1}{2}\|\partial_t u(t,\cdot)\|_{L^2}^2 - \frac{1}{p+1}\int_\Omega |u(t,x)|^{p+1}dx = \tilde{E}(0)\ .$$

Roughly speaking, the theorems of the above papers state that the solution $u(t,x)$ of (1.5) blows up in finite time with the negative energy ($\tilde{E}(0) \le 0$). In [5], B. Straughan introduces a condition for the initial data that the solution $u(t,x)$ of (1.5) blows up in finite time with the positive energy.

Our proof of Theorem 1 is based on the same strategy as above. The solution of (1.1) does not vanish on the boundary, which is the necessary condition for (1.1) being strictly hyperbolic. We introduce the local well-posedness of the strictly hyperbolic equation (1.1) in Section 2. In Section 4, we show the nonexistence of a global solution of (1.1) in the case where $A = 0$.

Remark 2. The constants C_p and C_m introduced in the assumption of Theorem 1 are

$$C_p = \frac{p-3}{p+1}|\Omega|^{-\frac{p-1}{2}},$$
$$C_m = C|\Omega|A^{p+1},$$

where $|\Omega| = \int_\Omega 1dx$ and C depends only on p.

In the case where $E(0) < 0$, $C_m + E(0)$ is negative for sufficiently small A. Hence the assumption (1.2) is satisfied.

One can verify that there exists the initial data of (1.1) satisfying (1.3) for sufficiently small $|E(0)|$, A and large $|\Omega|$.

Remark 3. If (1.1) does not have a global solution u satisfying $(u(t) - A, \partial_t u(t)) \in H^{[\frac{n}{2}]+2} \times H^{[\frac{n}{2}]+1}$ and $u(t,x) > 0$ for $(t,x) \in [0,\infty) \times \Omega$, then Theorem 5 implies that there exists a time $T_m < \infty$ such that the solution satisfies either

$$\lim_{t \nearrow T_m} \|u(t)\|_{H^{[\frac{n}{2}]+2}} + \|\partial_t u(t)\|_{H^{[\frac{n}{2}]+1}} = \infty,$$

or

$$\lim_{t \nearrow T_m} u(t,x_0) = 0 \text{ for some } x_0 \in \Omega.$$

Remark 4. By almost the same proof as the one of Theorem 1, (1.1) does not have a global weak solution u satisfying $(u(t) - A, \partial_t u(t)) \in H^1 \times L^2$ and $u(t,x) > 0$ for $(t,x) \in [0,\infty) \times \Omega$.

§2. Local existence and Uniqueness

In this section, we introduce the result of the well-posedness of (1.1), which is proved by using Theorem 14.3 in [7] and Theorem 5.3 in [6]. In [7], by the abstract theorem, T. Kato shows the well-posedness of some class of second order quasilinear hyperbolic equations including $(1.1)_a$. In [6], by using the energy method, C. M. Dafermos and W. J. Hrusa show almost the same result as [7].

Let $[x]$ denote the largest integer not greater than $x \in \mathbb{R}$. We introduce the compatibility condition, which is derived by (1.1) and the regularity of the solution of (1.1).

Substituting 0 for t in $(1.1)_a$, we have the compatibility condition of oder 2 such that

$$u_2 = u_0^2 \Delta u_0 + u_0 |\nabla u_0|^2 + u_0^p \in H^1_\sharp$$

for $u_0 - A \in H^3_\sharp$ and $u_1 \in H^2_\sharp$.

By differentiating the both side of $(1.1)_a$ with respect to t formally, we have

$$\partial_t^3 u = u^2 \Delta \partial_t u + 2u\partial_t u \Delta u + \partial_t u |\nabla u|^2 + 2u \nabla u \cdot \nabla \partial_t u + p u^{p-1} \partial_t u,$$

from which we have the compatibility condition of oder 3,

$$\begin{cases} u_2 \in H^2_\sharp \\ u_3 = u_0^2 \Delta u_1 + 2u_0 u_1 \Delta u_0 + u_1 |\nabla u_0|^2 + 2u_0 \nabla u_0 \cdot \nabla u_1 + p u_0^{p-1} u_1 \in H^1_\sharp \end{cases}$$

for $u_0 - A \in H^4_\sharp$ and $u_1 \in H^3_\sharp$.

By the same process as the above argument, we get $u_2,\ u_3, \dots, u_k$ inductively, which are introduced in the the compatibility condition of oder $2, 3, \dots, k$ respectively. However it is not easy to give an explicit representation of u_k. The compatibility condition of oder m is $u_{k+2} \in H^{m-k-1}_\sharp$ for $u_0 - A \in H^{m+1}_\sharp$, $u_1 \in H^m_\sharp$ and $k = 0, 1, \dots, m-2$.

Theorem 5. *Let $u_0 - A \in H^{m+1}_\sharp$, $u_1 \in H^m_\sharp$ satisfying the compatibility condition of oder m with $m \geq [\frac{n}{2}] + 1$ and $p \in \mathbb{R}$. Suppose that there exists a positive constant $\delta \leq A$ such that $u_0(x) \geq \delta$ for all $x \in \Omega$. Then there exists $T > 0$ and a unique solution u of* (1.1) *such that*

$$u - A \in \bigcap_{k=0}^{m} C^k([0,T]; H^{m-k+1}_\sharp) \ \ \text{and} \ \ u(t,x) \geq \delta/2 \ \text{for} \ (t,x) \in [0,T] \times \Omega,$$

where T depends only on $\|u_0 - A\|_{H^{m+1}}$, $\|u_1\|_{H^m}$ and A.

Remark 6. For $p \in \mathbb{R}$, the local existence and the uniqueness of the solution of (1.1) obtained by Theorem 1 hold by the positivity of u.

§3. Proof of Theorem 1

In this section, we prove Theorem 1.

First we introduce the conservation law of (1.1). We set

$$E(t) = \frac{1}{2}\|\partial_t u\|_{L^2}^2 + \frac{1}{2}\sum_{j=1}^{n}\|u\partial_{x_j}u\|_{L^2}^2 - \frac{1}{p+1}\int_\Omega u^{p+1}(t,x)dx,$$

which is a conserved quantity. In fact, multiplying the both side of $(1.1)_a$ by $\partial_t u$, integrating over Ω and using the divergence theorem, we have

$$\frac{dE(t)}{dt} = 0.$$

We assume that there exists a global solution $u(t,x)$ of (1.1) satisfying (1.4). We define the functional $F(t)$ as

$$F(t) := \int_\Omega (u(t,x) - A)^2 dx.$$

From (1.1), we have

$$\begin{aligned}\frac{d^2}{dt^2}F(t) &\geq 2\int_\Omega (\partial_t^2 u)(x)(u(x)-A)dx \\ &= 2\int_\Omega (u(x)-A)u(x)\operatorname{div}(u(x)\nabla u(x))dx + 2\int_\Omega u^p(x)(u(x)-A)dx.\end{aligned}$$

The divergence theorem of Gauss and the conservation law yield that

$$\begin{aligned}&2\int_\Omega (u(x)-A)u(x)\operatorname{div}(u(x)\nabla u(x))dx \\ &= -2\int_\Omega (\nabla((u(x)-A)u(x)))\cdot(u(x)\nabla u(x))dx \\ &\geq -4\int_\Omega u^2(x)|\nabla u(x)|^2 dx \\ &\geq -\frac{8}{p+1}\int_\Omega u^{p+1}(x)dx - 8E(0),\end{aligned}$$

from which we have

$$\begin{aligned}\frac{d^2}{dt^2}F(t) &\geq (2-\frac{8}{p+1})\int_\Omega u^{p+1}(x)dx - 2A\int_\Omega u^p(x)dx - 8E(0) \\ &= \int_\Omega u^p(x)((\frac{2(p-3)}{p+1})u(x) - 2A)dx - 8E(0). \qquad (3.1)\end{aligned}$$

We set $v = u - A,\ C_1 = \frac{2(p-3)}{p+1}$. We prove the following elementary inequality.

Proposition 7. *Let u, v, A and C_1be as above. We have*

(3.2) $u^p(C_1u - 2A) \geq \frac{C_1}{2}v^{p+1} - C_2A^{p+1}$ *for* $v \geq \frac{2A}{C_1} - A \geq 0$,

(3.3) $u^p(C_1u - 2A) \geq C_1v^{p+1} + A^{p+1}(C_1 - 2(\frac{2}{C_1})^p)$ *for* $\frac{2A}{C_1} - A \geq v \geq 0$,

(3.4) $u^p(C_1u - 2A) \geq C_1|v|^{p+1} - A^{p+1}(C_1 + 2)$ *for* $0 \geq v \geq -A$,

where $C_2 = (\frac{4p}{C_1(p+1)})^p \frac{2}{p+1} - \frac{C_1}{2}$.

Proof. First, we show (3.2). By the elementary computation, we have

$$\frac{1}{2}C_1u^{p+1} - 2Au^p \geq -(\frac{4Ap}{C_1(p+1)})^p \frac{2A}{p+1} \quad \text{for } u \geq \frac{2A}{C_1}.$$

The inequality $u^{p+1} \geq v^{p+1} + A^{p+1}$ for $v \geq 0$ yields that

$$u^p(C_1u - 2A) = \frac{1}{2}C_1u^{p+1} + \frac{1}{2}C_1u^{p+1} - 2Au^p \geq \frac{C_1}{2}v^{p+1} - C_2A^{p+1}.$$

Secondly, we show (3.3). By $u^{p+1} \geq v^{p+1} + A^{p+1}$ for $v \geq 0$ and $u^p \leq (\frac{2A}{C_1})^p$, we have

$$u^p(C_1u - 2A) \geq C_1u^{p+1} - 2Au^p \geq C_1v^{p+1} + A^{p+1}(C_1 - 2(\frac{2}{C_1})^p).$$

Finally, we show (3.4). From $u^p \leq A^p$ for $v \leq 0$ and $C_1u - 2A \leq 0$, it follows that

$$u^p(C_1u - 2A) \geq A^p(C_1u - 2A).$$

By $A \geq |v|$ and $u \geq |v| - A$, we have

$$A^p(C_1u - 2A) \geq C_1|v|^{p+1} - A^{p+1}(C_1 + 2).$$

□

We divide Ω into three parts as

$$\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3,$$

with $\Omega_1 = \{x \in \Omega|\ v(t,x) \geq \frac{2A}{C_1} - A\}$, $\Omega_2 = \{x \in \Omega|\ \frac{2A}{C_1} - A \geq v(t,x) \geq 0\}$ and $\Omega_3 = \{x \in \Omega|\ 0 \geq v(t,x) \geq -A\}$ for $t \geq 0$.

From (3.1) and the above devision of Ω, we have

$$\int_\Omega u^p(x)(C_1u(x) - 2A)dx - 8E(0) = \sum_{j=1}^{3} \int_{\Omega_j} u^p(x)(C_1u(x) - 2A)dx - 8E(0).$$

By Proposition 7, we have

$$\int_\Omega u^p(x)(C_1u(x)-2A)dx \geq \frac{C_1}{2}\int_\Omega |v(x)|^{p+1}dx - C_m,$$

where $C_m = |\Omega|\max\{C_2, 2(\frac{2}{C_1})^p - C_1, C_1+2\} \times A^{p+1}$.

By Hölder's inequality, we estimate the first term of the right hand side of the above inequality as

$$\frac{C_1}{2}\int_\Omega |v(x)|^{p+1}dx \geq C_p F^{\frac{p+1}{2}}(t),$$

where $C_p = \frac{C_1}{2}|\Omega|^{-\frac{p-1}{2}}$.

Therefore, we obtain the following differential inequality,

$$\frac{d^2F}{dt^2}(t) \geq C_p F^{\frac{p+1}{2}}(t) - 8E(0) - C_m.$$

Next, we prove the following lemma.

Lemma 8. *Let $G(t)$ be a solution of the differential equation*

$$\frac{d^2G}{dt^2}(t) = \alpha|G(t)|^q - \beta \quad \textit{for } \alpha > 0,\ \beta \in \mathbb{R} \textit{ and } q > 1. \tag{3.5}$$

If $G(0)$ and $\frac{dG}{dt}(0)$ satisfy $G(0) \geq 0$, $\frac{dG}{dt}(0) > 0$ and either

$$P'(G(0)) \geq 0, \tag{3.6}$$

or

$$P((\frac{\beta}{\alpha})^{\frac{1}{q}}) + C'' > 0 \textit{ for } \beta \geq 0, \tag{3.7}$$

where

$$P(x) = \frac{\alpha}{q+1}x^{q+1} - \beta x,$$

$$P'(x) = \frac{dP}{dx}(x) \textit{ and } C'' = \frac{1}{2}(\frac{dG(0)}{dt})^2 - P(G(0)),$$

then there exists a time $T < \infty$ such that

$$\frac{dG}{dt}(t) > 0 \textit{ on } [0,T) \textit{ and } \lim_{t\nearrow T} G(t) = \infty.$$

Proof. First, we show that $\frac{dG}{dt}(t)$ is a strictly positive function of t.

Under the assumption of (3.6), since $\frac{dG(t)}{dt} - \frac{dG(0)}{dt} = \int_0^t (\alpha|G(s)|^q - \beta) ds$, it follows that $\frac{dG}{dt}(t)$ is a nondecreasing function of t.

We assume (3.7). If there exists a time T_0 such that

$$\frac{dG(T_0)}{dt} = 0 \text{ and } \frac{dG(t)}{dt} > 0 \text{ for } t \in [0, T_0),$$

then multiplying the both side of the (3.5) by $\frac{dG}{dt}(t)$ and integrating on $[0, T_0]$, we have

$$P(G(T_0)) + C'' = 0,$$

which is contradictory to (3.7).

Therefore, $\frac{dG}{dt}(t)$ is a positive function.

From the positivity of $\frac{dG(t)}{dt}$ and the same argument as above, we have

$$\frac{1}{2}(\frac{dG(t)}{dt})^2 = P(G(t)) + C''. \tag{3.8}$$

(3.7) and (3.8) yield that $\frac{dG}{dt}(t) \geq \sqrt{2(P((\frac{\beta}{\alpha})^{\frac{1}{q}}) + C'')}$ for all $t \in [0, \infty)$.

Hence we obtain $\lim_{t \to T} G(t) = \infty$ for some $T \in (0, \infty]$ under the assumption either (3.6) or (3.7).

We prove $T < \infty$.

By the inverse function theorem, we can construct the inverse function $G^{-1} : [G(0), \infty) \longrightarrow [0, T)$, which satisfies

$$\frac{dG^{-1}(r)}{dr} = \frac{1}{\frac{dG}{dt}(G^{-1}(r))}.$$

By (3.8), we have

$$\frac{dG^{-1}(r)}{dr} = \sqrt{\frac{1}{2(P(r) + C'')}}.$$

Integrating both side over $[G(0),\ \infty)$, we have

$$T = \int_{G(0)}^{\infty} \sqrt{\frac{1}{2(P(r) + C'')}} dr < \infty.$$

Therefore, G(t) blows up in finite time. $\square$

Let G(t) be a solution of the differential equation

$$\frac{d^2G}{dt^2}(t) = C_p|G(t)|^{\frac{p+1}{2}} - 8E(0) - C_m,$$

satisfying $G(0) = F(0)$, $0 < \dfrac{dG(0)}{dt} < 2(u_0 - A, u_1)_{L^2}$ and either

$$P'(G(0)) \geq 0,$$

or

$$P((\frac{C_m + 8E(0)}{C_p})^{\frac{2}{p+1}}) + \tilde{C} > 0 \text{ for } C_m + 8E(0) \geq 0,$$

where $P(x)$ is the function introduced in the assumption of Theorem 1 and $\tilde{C} = \dfrac{1}{2}(\dfrac{dG(0)}{dt})^2 - P(G(0))$.

The standard comparison argument yields $F(t) \geq G(t)$ for $t \geq 0$, from which, Lemma 8 yields that $F(t)$ becomes infinite in finite time. We complete the proof of Theorem 1.

Remark 9. Under the assumption (3.6), the solution of (3.5) blows up in finite time with the nonnegative initial data, which is proved in [1], [2], [3], [4] and [5].

Remark 10. By the above computation, we have

$$T_m \leq \int_{F(0)}^{\infty} \sqrt{\frac{1}{2(\frac{2C_p}{p+3}r^{\frac{p+3}{2}} - (C_m + 8E(0))r + C')}} dr,$$

where T_m is the positive constant introduced in Remark 3.

§4. Case where $A = 0$

In Sections 1, 2 and 3, we assumed that A is a positive constant. In this section, we treat (1.1) in the case where $A = 0$, that is, we consider the following Dirichlet problem:

$$(4.1) \quad \begin{cases} \partial_t^2 u = u \operatorname{div}(u\nabla u) + |u|^{p-1}u, \quad (t,x) \in (0,T] \times \Omega, & (4.1)_a \\ u(0,x) = u_0(x), \quad x \in \Omega, & (4.1)_b \\ \partial_t u(0,x) = u_1(x), \quad x \in \Omega, & (4.1)_c \\ u(t,x) = \partial_t u(t,x) = 0, \quad (t,x) \in [0,T] \times \partial\Omega. & (4.1)_d \end{cases}$$

Theorem 11. *Suppose that* $p > 3$, $(u_0, u_1)_{L^2} > 0$ *and either*

$$P'(\|u_0\|_{L^2}^2) \geq 0,$$

or

$$P((\frac{8E(0)}{C_p})^{\frac{2}{p+1}}) + C' > 0 \text{ for } E(0) \geq 0,$$

where

$$P(x) = \frac{2C_p}{p+3} x^{\frac{p+3}{2}} - 8E(0)x,$$

$P'(x) = \frac{dP}{dx}(x)$, C_p *is the same constant as the one of Theorem* 1,

$$C' = 2(u_0, u_1)_{L^2}^2 - P(\|u_0\|_{L^2}^2),$$

and

$$E(0) = \frac{1}{2}\|u_1\|_{L^2}^2 + \frac{1}{2}\sum_{j=1}^{n} \|u_0 \partial_{xj} u_0\|_{L^2}^2 - \frac{1}{p+1}\int_\Omega u_0^{p+1}(x)dx.$$

Then a time global solution u *of* (4.1) *satisfying* $u \in C^2([0,\infty) \times \Omega)$ *does not exist.*

Proof. We give the outline of the proof which is similar to the argument in [1], [2], [3], [4] and [5].

We assume that there exists a global solution $u(t,x)$ of (4.1) and set the functional $F(t) := \int_\Omega u(t,x)^2 dx$.

By the same argument as Theorem 1, we have the following differential inequality as

$$\frac{d^2F}{dt^2}(t) \geq C_p F^{\frac{p+1}{2}}(t) - 8E(0).$$

By Lemma 8, $F(t)$ does not exist on $[0,\infty)$. □

Remark 12. If p is an odd number greater than 3 and the initial data of (4.1) is analytic, by Cauchy-Kowalewsky Theorem(e.g. [8]), we can construct the unique analytic solution of $(4.1)_a$. The solution of $(4.1)_a$, which is constructed by Cauchy-Kowalewsky Theorem, vanishes at the boundary of Ω. In fact, By the uniqueness of the solution of the ordinary differential equation, $u(0,x_0) = 0$ and $\partial_t u(0,x_0) = 0$ for $x_0 \in \partial\Omega$ are equivalent to $u(t,x_0) = 0$ and $\partial_t u(t,x_0) = 0$ for $t \in [0,T]$.

Acknowledgments

I would like to express my gratitude to Professor Keiichi Kato for many valuable comments and discussions.

References

[1] M. Tsutsumi, On solutions of semilinear differential equations in a Hilbert space, Math. Japon. **17** (1972) 173-193.

[2] J. M. Ball, Remarks on blow-up and nonexistence theorems for nonlinear evolution equations, Quart. J. Math. Oxford Ser(2). **28** (1977) 473-486.

[3] R. T. Glassey, Blow-up theorems for nonlinear wave equations, Math. Z. **132** (1973) 183-203.

[4] H. A. Levine, Instability and nonexistence of global solutions to nonlinear wave equations of the form $Pu_{tt} = -Au + \mathcal{F}(u)$, Trans. Amer. Math. Soc. **192** (1974) 1-21.

[5] B. Straughan, Further global nonexistence theorems for abstract nonlinear wave equations, Proc. Amer. Math. Soc. **48** (1975) 381-390.

[6] C. M. Dafermos, W. J. Hrusa, Energy methods for quasilinear hyperbolic initial-boundary value problems. Applications to elastodynamics, Arch. Rat. Mech. Anal. **87** (1985) 267-292.

[7] T. Kato, Abstract differential equations and nonlinear mixed problems, Lezioni Fermiane [Fermi Lectures]. Scuola Normale Superiore, Pisa; Accademia Nazionale dei Lincei, Rome, 1985.

[8] M. E. Taylor, Partial Differential Equations Ⅲ Nonlinear Equations, New York Springer-Verlag, (1996).

[9] L. D. Landau and E. M. Lifshitz. Fluid mechanics, volume 6 of course of theoretical physics. Pergamon, (1959).

Yuusuke Sugiyama
Department of Mathematics, Tokyo University of Science
Kagurazaka 1-3, Shinjuku-ku, Tokyo 162-8601, Japan
E-mail: `yoursmine1986@c3-net.ne.jp`