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On (Super) Edge-Magic Total Labeling of Subdivision of $K_{1,3}$

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Abstract. Let G be a finite graph, with $V(G)$ and $E(G)$ the vertex-set and edge-set of G, respectively. An *edge-magic total labeling* is a one-to-one mapping f from $V(G) \cup E(G)$ onto $\{1, 2, 3, \cdots, |V(G)| + |E(G)|\}$ such that there exists a constant c satisfying $f(u) + f(uv) + f(v) = c$, for each $uv \in E(G)$. Such a labeling is called a *super edge-magic total labeling* if all vertices of G receive all smallest labels. In this paper, we consider (super) edge-magic total labeling for subdivision of a star $K_{1,3}$.

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§1. Introduction

All graphs considered here are finite and simple. The graph G has the vertex-set $V(G)$ and the edge-set $E(G)$.

Let $p = |V(G)|$ and $q = |E(G)|$. A bijection $f : V(G) \cup E(G) \to \{1, 2, 3, \cdots, p+q\}$ is called an *edge-magic total labeling* of G if $f(x) + f(xy) + f(y)$ is a constant c (called the *magic constant* of f) for every edge xy of G. The graph that admits such a labeling is called an *edge-magic graph.* An edge-magic total labeling f is called a *super edge-magic total labeling* if $f(V(G)) = \{1, 2, 3, \cdots, p\}$. A graph that admits a super edge-magic total labeling is called a *super edge-magic graph.* The edge-magic and super edge-magic concepts were first introduced by Kotzig and Rosa [7] and Enomoto, Lladó, Nakamigawa and Ringel [3], respectively.

Given a total labeling f, the *dual* labeling, which Kotzig and Rosa [7] called the complementary labeling, f' is defined as follows,

$$f'(x) = p + q + 1 - f(x) \quad \text{for} \quad \text{every } x \in E(G) \cup V(G).$$

If f is an edge-magic total labeling with magic-constant c, then f' is an edge-magic total labeling with magic-constant $c' = 3(p+q+1) - c$. Notice that this dual labeling does not preserve super edge-magic total labeling unless $G = \overline{K_n}$.

Another definition of a dual labeling was also introduced in [1]. By this definition the dual labeling preserves the property of super edge-magicness.

Lemma 1. [1] *If g is a super edge-magic total labeling of G with the magic constant c, then the function $g' : V(G) \cup E(G) \to \{1, 2, 3, \cdots, p+q\}$ defined by*

$$g'(x) = \begin{cases} p + 1 - g(x), & \text{if } x \in V(G), \\ 2p + q + 1 - g(x), & \text{if } x \in E(G), \end{cases}$$

is also a super edge-magic total labeling of G with the magic constant $c' = 4p + q + 3 - c$.

The labeling g' defined in Lemma 1 is called a *dual super* labeling of g.

In the original papers about (super) edge-magic total labeling, Kotzig and Rosa [7], and Enomoto et al. [3] conjectured that every tree is edge-magic and every tree is super edge-magic, respectively. These conjectures have become very popular in the area of graph labeling. Some classes of tree have been proved to admit a (super) edge-magic labelings, such as paths, caterpillars [7], stars [4, 11], tree with at most 17 vertices [9], and path-like trees [2]. Additionally, Fukuchi [6] gives recursive formula for constructing super edge-magic trees. However, the conjectures are still remain open.

In this paper, we prove that a particular type of tree, namely a subdivision of a star $K_{1,3}$ is (super) edge-magic. These results provide more examples to support the correctness of the two conjectures on trees.

§2. The Results

For $m, n, k \geq 1$, let $T(m, n, k)$ be a graph obtained by inserting $m-1$, $n-1$, and $k-1$ vertices to the first, second, and third edges, respectively, of a star $K_{1,3}$. Thus, the star $K_{1,3}$ can be written as $T(1, 1, 1)$. We define the the vertex-set and the edge-set of graph $T(m, n, k)$ as follows.

$$V(T(m, n, k)) = \{w\} \cup \{x_i : 1 \leq i \leq m\} \cup \{y_i : 1 \leq i \leq n\} \cup \{z_i : 1 \leq i \leq k\},$$

and

$$\begin{aligned} E(T(m, n, k)) = \; & \{wx_1, wy_1, wz_1\} \cup \{x_i x_{i+1} : 1 \leq i \leq m-1\} \\ & \cup \{y_i y_{i+1} : 1 \leq i \leq n-1\} \cup \{z_i z_{i+1} : 1 \leq i \leq k-1\}. \end{aligned}$$

Clearly, a graph $T(m, n, k)$ has $m + n + k + 1$ vertices and $m + n + k$ edges. Among these vertices, one vertex has degree three, three vertices have degree one, and the remaining vertices have degree two. As an example, Figure 1 shows the graph $T(4, 5, 7)$.

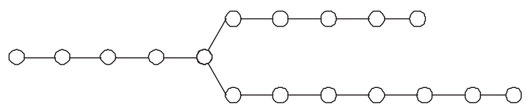

Figure 1: A tree $T(4, 5, 7)$

Lu [12, 13] called the graph $T(m, n, k)$ as a three-path trees and proved that $T(m, n, k)$ is super edge-magic if n and k are odd, or $k = n + 1$, or $k = n + 2$. In this paper, we prove that $T(m, n, k)$ is also super edge-magic if $k = n + 3$, and $k = n + 4$.

In proving the main results, the following lemma will be frequently used.

Lemma 2. [4] *A graph G with p vertices and q edges is super edge-magic if and only if there exists a bijective function $f : V(G) \to \{1, 2, \cdots, p\}$ such that the set $S = \{f(x) + f(y) : xy \in E(G)\}$ consists of q consecutive integers. In such a case, f extends to a super edge-magic total labeling of G with the magic constant $c = p + q + min(S)$.*

Suppose $T(m, n, k)$ has an edge-magic total labeling with the magic constant c. Then tc, where $t = m + n + k$, cannot be smaller than the sum obtained by assigning the smallest labels to the vertex of degree 3, the $t - 3$ next smallest labels to the vertices of degree 2, and three next smallest labels to the vertices of degree 1; in other words

$$tc \geq 3 + 2\sum_{i=2}^{t-2} i + \sum_{i=t-1}^{t+1} i + \sum_{i=t+2}^{2t+1} i.$$

An upper bound for tc is achieved by giving the the largest labels to the vertices of degree 3, and the $t - 3$ next largest labels to the vertices of degree 2, and 3 next largest labels to the rest of vertices; namely

$$tc \leq 3(2t + 1) + 2\sum_{i=t+4}^{2t} i + \sum_{i=t+1}^{t+3} i + \sum_{i=1}^{t} i.$$

Thus, we have the following result.

Lemma 3. *If a $T(m, n, k)$ is an edge-magic graph, then magic constant c is in the following interval:*

$$\tfrac{1}{2t}(5t^2+3t+6) \le c \le \tfrac{1}{2t}(7t^2+9t-6).$$

By a similar argument, it is easy to verify that the following lemma holds.

Lemma 4. *If a $T(m,n,k)$ is a super edge-magic graph, then magic constant c is in the following interval:*

$$\tfrac{1}{2t}(5t^2+3t+6) \le c \le \tfrac{1}{2t}(5t^2+11t-6).$$

In the next two theorems, we will show that $T(m,n,k)$, for $k=n+3$ and $k=n+4$, is super edge-magic. First, we introduce two constants α and β used in the proposed labeling of graph $T(m,n,k)$ as follows:

$$\alpha = \begin{cases} 0, & \text{if } n \equiv 2 \pmod 4, \\ 1, & \text{if } n \equiv 0,1,3 \pmod 4, \end{cases}$$

and

$$\beta = \begin{cases} \lceil \frac{1}{2}(m-4) \rceil, & \text{if } n \equiv 0 \pmod 2, \\ \lceil \frac{1}{2}(m-2) \rceil, & \text{if } n \equiv 1 \pmod 2. \end{cases}$$

Theorem 1. *For all integers $m,n \ge 1$, $T(m,n,n+3)$ is a super edge-magic graph.*

Proof. Consider the vertex labeling $f : V(T(m,n,n+3)) \to \{1,2,3,\cdots,m+2n+4\}$ defined as follows.

$$f(u) = \begin{cases} m+n+3, & \text{if } u=w, \\ \lceil \frac{m}{2} \rceil - \frac{1}{2}(i-1), & \text{if } u=x_i \text{ for } i \equiv 1 \pmod 2, \\ m+n+3-\frac{1}{2}i, & \text{if } u=x_i \text{ for } i \equiv 0 \pmod 2, \\ \lceil \frac{m}{2} \rceil + 1 - \alpha + \frac{1}{2}(i+1), & \text{if } u=y_i \text{ for } i \equiv 1 \pmod 2, \\ m+n+3+\frac{1}{2}i, & \text{if } u=y_i \text{ for } i \equiv 0 \pmod 2. \end{cases}$$

For the remaining vertices, we consider the following four cases.

Case 1. $n \equiv 0 \bmod 4$,

$$f(z_i) = \begin{cases} \lceil \frac{m}{2} \rceil + n + 1 - \frac{1}{2}(i-1), & \text{for } i \equiv 1 \pmod 4, \\ \lceil \frac{m}{2} \rceil + n + 3 - \frac{1}{2}(i-1), & \text{for } i \equiv 3 \pmod 4, \\ m+2n+4-\frac{1}{2}i, & \text{for } i \equiv 2 \pmod 4,\ i \ne n+2, \\ m+2n+6-\frac{1}{2}i, & \text{for } i \equiv 0 \pmod 4, \\ m+\frac{3}{2}n+4, & \text{for } i = n+2. \end{cases}$$

Case 2. $n \equiv 1 \bmod 4$,

$$f(z_i) = \begin{cases} \lceil \frac{m}{2} \rceil + n + 1 - \frac{1}{2}(i-1), & \text{for } i \equiv 1 \pmod 4, \\ \lceil \frac{m}{2} \rceil + n + 3 - \frac{1}{2}(i-1), & \text{for } i \equiv 3 \pmod 4, \\ m+2n+4-\frac{1}{2}i, & \text{for } i \equiv 2 \pmod 4, \\ m+2n+6-\frac{1}{2}i, & \text{for } i \equiv 0 \pmod 4. \end{cases}$$

Case 3. $n \equiv 2 \bmod 4$,

$$f(z_i) = \begin{cases} \lceil \frac{m}{2} \rceil + 1, & \text{for } i = 1, \\ \lceil \frac{m}{2} \rceil + n + 3 - \frac{1}{2}(i-1), & \text{for } i \equiv 1 \pmod 2),\ i \neq 1, \\ m + 2n + 5 - \frac{1}{2}i, & \text{for } i \equiv 0 \pmod 2). \end{cases}$$

Case 4. $n \equiv 3 \bmod 4$,

$$f(z_i) = \begin{cases} \lceil \frac{m}{2} \rceil + n + 1 - \frac{1}{2}(i-1), & \text{for } i \equiv 1 \pmod 4),\ i \neq n+2, \\ \lceil \frac{m}{2} \rceil + n + 3 - \frac{1}{2}(i-1), & \text{for } i \equiv 3 \pmod 4), \\ m + 2n + 4 - \frac{1}{2}i, & \text{for } i \equiv 2 \pmod 4),\ i \neq n-1, \\ m + 2n + 6 - \frac{1}{2}i, & \text{for } i \equiv 0 \pmod 4),\ i \neq n+1, \\ m + 5 + \frac{1}{2}(3n+1), & \text{for } i = n-1, \\ m + 3 + \frac{1}{2}(3n+1), & \text{for } i = n+1, \\ \lceil \frac{m}{2} \rceil + \lceil \frac{n}{2} \rceil + 1, & \text{for } i = n+2, \\ m + 4 + \frac{1}{2}(3n+1), & \text{for } i = n+3. \end{cases}$$

Under the vertex labeling f, we have the following sums of labels of two adjacent vertices.

$$f(w) + f(x_1) = m + \left\lceil \frac{m}{2} \right\rceil + n + 3,$$

$$f(w) + f(y_1) = m + \left\lceil \frac{m}{2} \right\rceil + n + 5 - \alpha,$$

$$f(w) + f(z_1) = \begin{cases} m + \lceil \frac{m}{2} \rceil + n + 4, & \text{if } n \equiv 2 \pmod 4), \\ m + \lceil \frac{m}{2} \rceil + 2n + 4, & \text{if } n \equiv 0, 1, 3 \pmod 4), \end{cases}$$

$\{f(x_i) + f(x_{i+1}) : 1 \leq i \leq m-1\}$
$= \{\lceil \frac{m}{2} \rceil + n + 4, \lceil \frac{m}{2} \rceil + n + 5, \cdots, m + \lceil \frac{m}{2} \rceil + n + 2\}$,
$\{f(y_i) + f(y_{i+1}) : 1 \leq i \leq n-1\}$
$= \{m + \lceil \frac{m}{2} \rceil + n + 6 - \alpha, m + \lceil \frac{m}{2} \rceil + n + 7 - \alpha, \cdots, m + \lceil \frac{m}{2} \rceil + 2n + 4 - \alpha\}$,
$\{f(z_i) + f(z_{i+1}) : 1 \leq i \leq n+2\}$
$= \{m + \lceil \frac{m}{2} \rceil + 2n + 5, m + \lceil \frac{m}{2} \rceil + 2n + 6, \cdots, m + \lceil \frac{m}{2} \rceil + 3n + 6\}$.

Thus, the set $S = \{f(v) + f(w) : vw \in E\big(T(m, n, n+3)\big)\}$ consists of consecutive integers with $\max(S) = m + \lceil \frac{m}{2} \rceil + 3n + 6$. By Lemma 2, f extends to a super edge-magic total labeling of $T(m, n, n+3)$ with magic constant $c = 2m + \lceil \frac{m}{2} \rceil + 5n + 11$. □

Figure 2 shows the vertex labeling of a super edge-magic tree $T(4, 6, 9)$.

Theorem 2. *For all integers $m, n \geq 1$, $T(m, n, n+4)$ is a super edge-magic graph.*

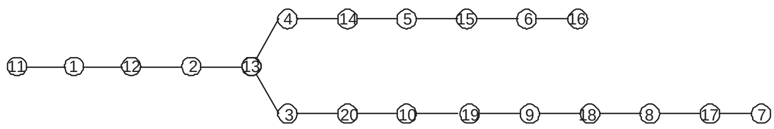

Figure 2: A super edge-magic tree $T(4,6,9)$

Proof. Label the vertices of $T(m,n,n+4)$ in the following way. We consider 2 cases, where n is even and odd.

Case 1. $n \equiv 0 \bmod 2$,

$$g(u) = f(u), \quad \text{for } u = w, x_i\text{'s}, \text{ and } y_i\text{'s},$$

where f is the vertex labeling in the proof of Theorem 1 with $\alpha = 1$.

Subcase 1.1. $n \equiv 0 \bmod 4$,

$$g(z_i) = \begin{cases} \lceil \frac{m}{2} \rceil + n + 1 - \frac{1}{2}(i-1), & \text{for } i \equiv 1 \pmod 4, \\ \lceil \frac{m}{2} \rceil + n + 3 - \frac{1}{2}(i-1), & \text{for } i \equiv 3 \pmod 4, \\ m + 2n + 5 - \frac{1}{2}i, & \text{for } i \equiv 2 \pmod 4, \\ m + 2n + 7 - \frac{1}{2}i, & \text{for } i \equiv 0 \pmod 4. \end{cases}$$

Subcase 1.2. $n \equiv 2 \bmod 4$,

$$g(z_i) = \begin{cases} \lceil \frac{m}{2} \rceil + n + 1 - \frac{1}{2}(i-1), & \text{for } i \equiv 1 \pmod 4, \ i \neq n+3, \\ \lceil \frac{m}{2} \rceil + n + 3 - \frac{1}{2}(i-1), & \text{for } i \equiv 3 \pmod 4, \\ m + 2n + 5 - \frac{1}{2}i, & \text{for } i \equiv 2 \pmod 4, \ i \neq n, n+4, \\ m + 2n + 7 - \frac{1}{2}i, & \text{for } i \equiv 0 \pmod 4, \ i \neq n+2, \\ m + 6 + \frac{3}{2}n, & \text{for } i = n, \\ m + 4 + \frac{3}{2}n, & \text{for } i = n+2, \\ \lceil \frac{m+n}{2} \rceil + 1, & \text{for } i = n+3, \\ m + 5 + \frac{3}{2}n, & \text{for } i = n+4. \end{cases}$$

Case 2. $n \equiv 1 \bmod 2$,

$$g(u) = \begin{cases} m + n + 4, & \text{if } u = w, \\ \lceil \frac{m}{2} \rceil - \frac{1}{2}(i-1), & \text{if } u = x_i \text{ for } i \equiv 1 \pmod 2, \\ m + n + 4 - \frac{1}{2}i, & \text{if } u = x_i \text{ for } i \equiv 0 \pmod 2, \\ \lceil \frac{m}{2} \rceil + 2 + \frac{1}{2}(i-1), & \text{if } u = y_i \text{ for } i \equiv 1 \pmod 2, \\ m + n + 4 + \frac{1}{2}i, & \text{if } u = y_i \text{ for } i \equiv 0 \pmod 2. \end{cases}$$

$$g(z_i) = \begin{cases} \lceil \frac{m}{2} \rceil + 1, & \text{for } i = 1, \\ \lceil \frac{m}{2} \rceil + n + 4 - \frac{1}{2}(i-1), & \text{for } i \equiv 1 \pmod 2, \ i \neq 1, \\ m + 2n + 6 - \frac{1}{2}i, & \text{for } i \equiv 0 \pmod 2. \end{cases}$$

It is a routine procedure to verify that g is a vertex labeling of $T(m,n,n+4)$. Under the vertex labeling g, for each case of n, we can count the sums of labels of two adjacent vertices as follows.

Case 1. $n \equiv 0 \bmod 2$,

$$g(w)+g(x_1)=m+\left\lceil\frac{m}{2}\right\rceil+n+3,$$

$$g(w)+g(y_1)=m+\left\lceil\frac{m}{2}\right\rceil+n+4,$$

$$g(w)+g(z_1)=m+\left\lceil\frac{m}{2}\right\rceil+2n+4,$$

$\{g(x_i)+g(x_{i+1}) : 1 \le i \le m-1\}$
$= \{\lceil\frac{m}{2}\rceil+n+4, \lceil\frac{m}{2}\rceil+n+5, \cdots, m+\lceil\frac{m}{2}\rceil+n+2\}$,
$\{g(y_i)+g(y_{i+1}) : 1 \le i \le n-1\}$
$= \{m+\lceil\frac{m}{2}\rceil+n+5, m+\lceil\frac{m}{2}\rceil+n+6, \cdots, m+\lceil\frac{m}{2}\rceil+2n+3\}$,
$\{g(z_i)+g(z_{i+1}) : 1 \le i \le n+3\}$
$= \{m+\lceil\frac{m}{2}\rceil+2n+5, m+\lceil\frac{m}{2}\rceil+2n+6, \cdots, m+\lceil\frac{m}{2}\rceil+3n+7\}$.

Case 2. $n \equiv 1 \bmod 2$,

$$g(w)+g(x_1)=m+\left\lceil\frac{m}{2}\right\rceil+n+4,$$

$$g(w)+g(y_1)=m+\left\lceil\frac{m}{2}\right\rceil+n+6,$$

$$g(w)+g(z_1)=m+\left\lceil\frac{m}{2}\right\rceil+n+5,$$

$\{g(x_i)+g(x_{i+1}) : 1 \le i \le m-1\}$
$= \{\lceil\frac{m}{2}\rceil+n+5, \lceil\frac{m}{2}\rceil+n+6, \cdots, m+\lceil\frac{m}{2}\rceil+n+3\}$,
$\{g(y_i)+g(y_{i+1}) : 1 \le i \le n-1\}$
$= \{m+\lceil\frac{m}{2}\rceil+n+7, m+\lceil\frac{m}{2}\rceil+n+8, \cdots, m+\lceil\frac{m}{2}\rceil+2n+5\}$,
$\{g(z_i)+g(z_{i+1}) : 1 \le i \le n+3\}$
$= \{m+\lceil\frac{m}{2}\rceil+2n+6, m+\lceil\frac{m}{2}\rceil+2n+7, \cdots, m+\lceil\frac{m}{2}\rceil+3n+8\}$.

Hence, the set $S=\{g(v)+g(w) : vw \in E(T(m,n,n+4))\}$ is a set of consecutive integers with $\max(S)=m+3n+9+\beta$. By Lemma 2, g extends to a super edge-magic total labeling of $T(m,n,n+4)$ with magic constant $c=2m+5n+15+\beta$. □

Figure 3 shows the vertex labeling of a super edge-magic tree $T(3,6,10)$.

By the dual super property (Lemma 1), $T(m,n,n+3)$ and $T(m,n,n+4)$ also have a super edge-magic total labeling with magic constant as in the following corollaries.

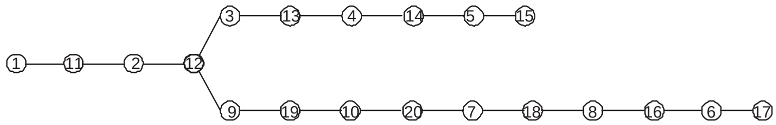

Figure 3: A super edge-magic tree $T(3, 6, 10)$

Corollary 1. *For all integers* $m, n \geq 1$, $T(m, n, n+3)$ *has a super edge-magic total labeling with magic constants* $c = 3m - \lceil \frac{m}{2} \rceil + 5n + 11$. $\square$

Corollary 2. *For all integers* $m, n \geq 1$, $T(m, n, n+4)$ *has a super edge-magic total labeling with magic constants* $c = 3m + 5n + 12 - \beta$. $\square$

Additionally, by applying the duality property to Theorems 1 and 2, and Corollaries 1 and 2, we have the following results.

Corollary 3. *For all integers* $m, n \geq 1$, $T(m, n, n + 3)$ *has edge-magic total labelings with magic constants* $c = 4m - \lceil \frac{m}{2} \rceil + 7n + 13$ *and* $c = 3m + \lceil \frac{m}{2} \rceil + 7n + 13$. $\square$

Corollary 4. *For all integers* $m, n \geq 1$, $T(m, n, n + 4)$ *has edge-magic total labelings with magic constants* $c = 4m + 7n + 15 - \beta$ *and* $c = 3m + 7n + 18 + \beta$. $\square$

We can also construct an edge-magic total labeling of $T(m, n, n + 3)$ and $T(m, n, n + 4)$ for which all the odd labels are on the vertices, as follows.

Theorem 3. *For all integers* $m, n \geq 1$, $T(m, n, n+3)$ *has an edge-magic total labeling with all vertices receive odd labels. This labeling has magic constant* $c = 2m + 2\lceil \frac{m}{2} \rceil + 6n + 12$ *and the dual has magic constant* $c = 4m - 2\lceil \frac{m}{2} \rceil + 6n + 12$.

Proof. Define a labeling h of $T(m, n, n + 3)$ as follows.

$$h(v) = 2f(v) - 1, \text{ for all } v \in V\big(T(m, n, n + 3)\big),$$

where f is the vertex labeling in the proof of Theorem 1. It is not difficult to verify that all vertices receive odd labels, and

$$S = \{h(u) + h(v) : uv \in E\big(T(m, n, n + 3)\big)\}$$

forms an arithmetic progression with initial term $2n+2\lceil \frac{m}{2} \rceil+6$ having common difference 2. If we define

$$h(uv) = 2m + 2\left\lceil \frac{m}{2} \right\rceil + 6n + 12 - h(u) - h(v),$$

then h is an edge-magic total labeling of $T(m,n,n+3)$ with magic constant $c = 2m + 2\lceil \frac{m}{2} \rceil + 6n + 12$. By the duality property, it also has an edge-magic total labeling with magic constant $c = 4m - 2\lceil \frac{m}{2} \rceil + 6n + 12$. □

A similar result for $T(m,n,n+4)$ can be stated in the next theorem.

Theorem 4. *For all integers $m,n \geq 1$, $T(m,n,n+4)$ has an edge-magic total labeling with all vertices receive odd labels. This labeling has magic constant $c = 2m+6n+18+2\beta$ and the dual has magic constant $c = 4m+6n+12-2\beta$, where β is a constant as defined before.*

We have proved the super edge-magicness of $T(m,n,k)$ only for $k = n+3$ and $k = n+4$ (not for any value of k). Additionally, we proved that $T(m,n,n+3)$ and $T(m,n,n+4)$ are (super) edge-magic for several values of magic constants c but not for all possible values of c. So, we have the following open problems.

Open problem 1. *Find a (super) edge-magic total labeling of $T(m,n,k)$ for any remaining values of m,n and k.*

Open problem 2. *Find a (super) edge-magic total labeling of $T(m,n,n+3)$ and $T(m,n,n+4)$ for other values of magic constants c.*

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