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A CORRECTION ON "SOME NONDIFFEOMORPHIC HOMEOMORPHIC HOMOGENEOUS 7-MANIFOLDS WITH POSITIVE SECTIONAL CURVATURE"

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As was pointed out to us by L. Astey, E. Micha & G. Pastor the homeomorphism result in [3] Theorem 3.1 is not correct. They have examples [1] of two non-homeomorphic smooth spin manifolds as in Theorem 3.1 with fourth cohomology group of the same order where the invariants $\bar{s}_i$ for $1 \leq i \leq 3$ agree. In these examples the invariant s_2 is different suggesting that the statement has to be modified by requesting instead of the equality of $\bar{s}_2$ the equality of s_2 (recall that $\bar{s}_2$ was simply defined as $2s_2$). Theorem 3.1 is based on Proposition 3.2 which in [2] was originally only proved in the smooth category. We assumed that the proof also works in the topological category; this is not true and Proposition 3.2 only holds in the smooth category. At the moment we do not have a classification for topological manifolds. As we will explain below, at least for smooth manifolds the invariant s_2 is a homeomorphism invariant and one can obtain a homeomorphism classification of smooth manifolds from their diffeomorphism classification. A correct formulation of Theorem 3.1 is:

Theorem 1. *Let M and M' be smooth manifolds of type (2.1) such that $|H^4(M;\mathbb{Z})| = |H^4(M';\mathbb{Z})|$ which are both spin or both nonspin. Then M is diffeomorphic (homeomorphic) to M' if and only if $s_i(M) = s_i(M')$ for $i = 1, 2, 3$ (resp. $28s_1(M) = 28s_1(M')$ and $s_i(M) = s_i(M')$ for $i = 2, 3$).*

Note that the applications to the homeomorphism and diffeomorphism classification of the Wallach spaces is not affected by the mistake

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since by Lemmas 4.4 and 5.1 the condition $\bar{s}_i(M) = \bar{s}_i(M')$ for $i = 1, 2, 3$ is equivalent to the condition $28s_1(M) = 28s_1(M')$ and $s_i(M) = s_i(M')$ for $i = 2, 3$. This follows since the order of the fourth cohomology group of a Wallach space (denoted N in 4.4 and 5.1) is always odd.

Now we want to explain why s_2 is a homeomorphism invariant for smooth spin manifolds M of the type under consideration. Let W be compact topological spin manifold whose boundary is M such that the class $u \in H^2(M;\mathbb{Z})$ extends to a class $z \in H^2(W;\mathbb{Z})$. Since M is smooth, the first obstruction for a lift of the topological normal bundle of W to a linear bundle is an element $KS(W) \in H^4(W, M; \mathbb{Z}/2)$. We say that W is admissible for the computation of s_i if $z^2 \cup KS(W) = 0 \in \mathbb{Z}/2$.

This follows from the fact that both invariants vanish on smooth manifolds and depend only on the class of (W, z) in the reduced bordism group $\tilde{\Omega}_8^{\mathrm{TopSpin}}(\mathbb{CP}^\infty)$ of topological spin manifolds. Moreover, both invariants are non-trivial (cf. [3], Lemma 6.2) and the cokernel of the forgetful homeomorphism $\tilde{\Omega}_8^{\mathrm{Spin}}(\mathbb{CP}^\infty) \to \tilde{\Omega}_8^{\mathrm{TopSpin}}(\mathbb{CP}^\infty)$ is $Z/2$; this follows by comparing the Atiyah-Hirzebruch spectral sequences as in [3], Section 6.

As a consequence one can assume, after perhaps adding a closed topological manifold, that for given M the bounding manifold W is admissible. Using only admissible W's one obtains that $s_2(M)$ mod $\mathbb{Z}$ is also in the spin case a well defined homeomorphism invariant.

The proof of the theorem above for the homeomorphism classification is reduced to the smooth case by noting that if $\bar{s}_1(M) = \bar{s}_1(M')$ then after adding an appropriate homotopy sphere one can assume that $s_1(M) = s_2(M')$.

References

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[3] M. Kreck & S. Stolz, *Some nondiffeomorphic homeomorphic homogeneous 7-manifolds with positive sectional curvature*, Differential Geom. **33** (1991) 465–486.

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