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Canonical Bases for Subspaces of a Vector Space, and 5-Dimensional Subalgebras of Lie Algebra of Lorentz Group

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Abstract

Canonical bases for subspaces of a vector space are introduced as a new effective method to analyze subalgebras of Lie algebras. This method generalizes well known Gauss-Jordan elimination method.

Keywords: Vector space; Subspaces; Lie algebras; Subalgebras

Introduction

This article has two parts. In Part I, the canonical bases for 5-dimensional subspaces of a 6-dimensional vector spaces are introduced, and all of them are found. Then the nonequivalent canonical bases are classified in Theorem 1. The corresponding evaluation in Part I has a universal character, and it can be called a generalization of the well-known Gauss - Jordan elimination method [1]. Canonical bases for subspaces of 3-, 4-, and 5-dimensional vector spaces are already found too, and they will be demonstrated in the separate manuscripts. The canonical bases for the (n–1)-dimensional subspaces of vector spaces of dimension n>6 can be constructed in the way similar to this in Part I. This new method of canonical bases helps to study all objects associated with subspaces of vector spaces.

In Part II, this method is applied to study subalgebras of Lie algebra of Lorentz group. It's a fact that a classification problem of subalgebras of low dimensional real Lie algebras was discussed during 1970-1980 years. That classification of subalgebras of all real Lie algebras of dimension $n \leq 4$ only was obtained in the form of representatives for equivalent classes of subalgebras considering under their groups of inner automorphisms [2,3]. The subalgebras of real Lie algebras of dimension $n \geq 5$ were not classified before. As a step of the further classification, the 5-dimensional hypothetical subalgebras of 6-dimensional Lie algebra of Lorentz group are investigated in Part II [4]. The corresponding procedure involves nonequivalent canonical bases from Part I. It is proved that Lie algebra of Lorentz group has no subalgebras of the dimension 5. This means also that Lorentz group has no connected 5-dimensional subgroups.

Part I

Canonical bases for 5-dimensional subspaces of a 6-dimensional vector space

Let $\vec{a}=a_1\vec{e_1}+a_2\vec{e_2}+a_3\vec{e_3}+a_4\vec{e_4}+a_5\vec{e_5}+a_6\vec{e_6}$, $\vec{b}=b_1\vec{e_1}+b_2\vec{e_2}+b_3\vec{e_3}+b_4\vec{e_4}+b_5\vec{e_5}+b_6\vec{e_6}$,

$$\vec{c}=c_1\vec{e_1}+c_2\vec{e_2}+c_3\vec{e_3}+c_4\vec{e_4}+c_5\vec{e_5}+c_6\vec{e_6}, \vec{d}=d_1\vec{e_1}+d_2\vec{e_2}+d_3\vec{e_3}+d_4\vec{e_4}+d_5\vec{e_5}+d_6\vec{e_6}, \quad \text{(I)}$$

$\vec{f}=f_1\vec{e_1}+f_2\vec{e_2}+f_3\vec{e_3}+f_4\vec{e_4}+f_5\vec{e_5}+f_6\vec{e_6}$ be a general basis for arbitrary 5-dimensional subspace S of a 6-dimensional vector space V with its standard basis $\{\vec{e_1},\vec{e_2},\vec{e_3},\vec{e_4},\vec{e_5},\vec{e_6}\}$. We associate the next matrix M with the basis (I)

$$M=\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \\ c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \\ d_1 & d_2 & d_3 & d_4 & d_5 & d_6 \\ f_1 & f_2 & f_3 & f_4 & f_5 & f_6 \end{bmatrix}.$$

Definition 1

The basis (I) is called canonical if its associated matrix M is in reduced row echelon form.

Definition 2

Two bases are called equivalent if they generate the same subspace of a given vector space, and two bases are nonequivalent if they generate different subspaces.

We start our transformation procedure for the basis (I) to find all canonical nonequivalent bases for the subspace S.

Suppose that at least one coefficient from a_1, b_1, c_1, d_1, f_1 is not zero. Without any loss in the generality, we can propose that $a_1 \neq 0$. Perform the linear operation $\vec{a}/a_1$ first, and the operations $\vec{b}-b_1\vec{a}$, $\vec{c}-c_1\vec{a}$, $\vec{d}-d_1\vec{a}$, $\vec{f}-f_1\vec{a}$ after the first one. The following basis is obtained $\vec{a}=\vec{e_1}+a_2\vec{e_2}+a_3\vec{e_3}+a_4\vec{e_4}+a_5\vec{e_5}+a_6\vec{e_6}$, $\vec{b}=b_2\vec{e_2}+b_3\vec{e_3}+b_4\vec{e_4}+b_5\vec{e_5}+b_6\vec{e_6}$,

$$\vec{c}=c_2\vec{e_2}+c_3\vec{e_3}+c_4\vec{e_4}+c_5\vec{e_5}+c_6\vec{e_6}, \vec{d}=d_2\vec{e_2}+d_3\vec{e_3}+d_4\vec{e_4}+d_5\vec{e_5}+d_6\vec{e_6}, \quad \text{(a)}$$

$$\vec{f}=f_2\vec{e_2}+f_3\vec{e_3}+f_4\vec{e_4}+f_5\vec{e_5}+f_6\vec{e_6}.$$

Remark

The first components of vectors $\vec{a},\vec{b},\vec{c},\vec{d},\vec{f}$ are changed as the result of the operations performed but all other components of them are saved just for convenience. This idea will be used throughout of Part I.

Suppose now that at least one coefficient from b_2, c_2, d_2, f_2 in the basis (a) is not zero. Without any loss in generality, let $b_2 \neq 0$. Perform the first linear operation $\vec{b}/b_2$, and the operations $\vec{a}-a_2\vec{b}$, $\vec{c}-c_2\vec{b}$, $\vec{f}-f_2\vec{b}$, $\vec{f}-f_2\vec{b}$ after the first one. The following new basis is obtained

$$\vec{a}=\vec{e_1}+a_3\vec{e_3}+a_4\vec{e_4}+a_5\vec{e_5}+a_6\vec{e_6}, \vec{b}=\vec{e_2}+b_3\vec{e_3}+b_4\vec{e_4}+b_5\vec{e_5}+b_6\vec{e_6},$$

$$\vec{c}=c_3\vec{e_3}+c_4\vec{e_4}+c_5\vec{e_5}+c_6\vec{e_6}, \vec{d}=d_3\vec{e_3}+d_4\vec{e_4}+d_5\vec{e_5}+d_6\vec{e_6}, \quad (1)$$

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$$\vec f = f_3\vec e_3 + f_4\vec e_4 + f_5\vec e_5 + f_6\vec e_6 .$$

Suppose that at least one coefficient among c_3, d_3, f_3 in the basis (1) is not zero. Again, without any loss in the generality, let $c_3 \neq 0$. Perform the first operation $\vec c / c_3$, and the operations $\vec a - a_3\vec c$, $\vec b - b_3\vec c$, $\vec d - d_3\vec c$, $\vec f - f_3\vec c$ then. We obtain the following basis

$$\vec b = \vec e_2 + b_4\vec e_4 + b_5\vec e_5 + b_6\vec e_6,\ \vec b = \vec e_2 + b_4\vec e_4 + b_5\vec e_5 + b_6\vec e_6,$$
$$\vec c = \vec e_3 + c_4\vec e_4 + c_5\vec e_5 + c_6\vec e_6,\ \vec d = d_4\vec e_4 + d_5\vec e_5 + d_6\vec e_6, \qquad (2)$$
$$\vec f = f_4\vec e_4 + f_5\vec e_5 + f_6\vec e_6 .$$

Suppose now that at least one coefficient from d_4, f_4 in the basis (2) is not zero. Let $d_4 \neq 0$. Perform the operation $\vec d / d_4$ first, and then the operations $\vec a - a_4\vec d$, $\vec b - b_4\vec d$, $\vec c - c_4\vec d$, $\vec f - f_4\vec d$. The new transformed basis is

$$\vec a = \vec e_1 + a_5\vec e_5 + a_6\vec e_6,\ \vec b = \vec e_2 + b_5\vec e_5 + b_6\vec e_6,$$
$$\vec c = \vec e_3 + c_5\vec e_5 + c_6\vec e_6,\ f = f_5 e_5 + f_6 e_6, \qquad (3)$$
$$\vec f = f_5\vec e_5 + f_6\vec e_6 .$$

At least one coefficient from f_5, f_6 is not zero in the basis (3). If $f_5 \neq 0$, then perform the operation $\vec f / f_5$ first, and the operations $\vec a - a_5\vec f$, $\vec b - b_5\vec f$, $\vec c - c_5\vec f$, $\vec d - d_5\vec f$ after the first one. The following canonical basis is obtained

$$\vec a = \vec e_1 + a_6\vec e_6,\quad \vec b = \vec e_2 + b_6\vec e_6,\quad \vec c = \vec e_3 + c_6\vec e_6,\quad \vec d = \vec e_4 + d_6\vec e_6,\quad \vec f = \vec e_5 + f_6\vec e_6 . \qquad (a_1)$$

If $f_6 \neq 0$, then perform operation $\vec f / f_6$ first, and the operations $\vec b - b_6\vec f$, $\vec b - b_6\vec f$,

$\vec c - c_6\vec f$, $\vec d - d_6\vec f$ after the first one. The new basis is obtained

$$\vec a = \vec e_1 + a_5\vec e_5,\quad \vec b = \vec e_2 + b_5\vec e_5,\quad \vec c = \vec e_3 + c_5\vec e_5,\quad \vec d = \vec e_4 + d_5\vec e_5,\quad \vec f = f_5\vec e_5 + \vec e_6 .$$

The last basis is equivalent to the basis (a_1) if $f_5 \neq 0$. So, $f_5 = 0$, and the new canonical basis is obtained

$$\vec a = \vec e_1 + a_5\vec e_5,\ \vec b = \vec e_2 + b_5\vec e_5,\ \vec c = \vec e_3 + c_5\vec e_5,\ \vec d = \vec e_4 + d_5\vec e_5,\ \vec f = \vec e_6 . \qquad (a_2)$$

1. Suppose that both coefficients d_4, f_4 at the basis (2) are zero. We have

$$\vec a = \vec e_1 + a_4\vec e_4 + a_5\vec e_5 + a_6\vec e_6,\ \vec b = \vec e_2 + b_4\vec e_4 + b_5\vec e_5 + b_6\vec e_6,$$
$$\vec c = \vec e_3 + c_4\vec e_4 + c_5\vec e_5 + c_6\vec e_6,\ \vec d = d_5\vec e_5 + d_6\vec e_6,\ \vec f = f_5\vec e_5 + f_6\vec e_6 . \qquad (4)$$

Suppose that at least one coefficient from d_5, f_5 at (4) is not zero. It's easy to see that the alternative case with $d_5 = 0$ and $f_5 = 0$ is impossible because the corresponding vectors $\vec d = d_6\vec e_6$, $\vec f = f_6\vec e_6$ are linearly dependent. Let $d_5 \neq 0$. Perform operation $\vec d / d_5$ first, and the operations $\vec a - a_5\vec d$, $\vec b - b_5\vec d$, $\vec f - f_5\vec d$, $\vec f - f_5\vec d$ next. The following basis is obtained

$$\vec a = \vec e_1 + a_4\vec e_4 + a_6\vec e_6,\ \vec b = \vec e_2 + b_4\vec e_4 + b_6\vec e_6,$$
$$\vec c = \vec e_3 + c_4\vec e_4 + c_6\vec e_6,\ \vec d = \vec e_5 + d_6\vec e_6,$$
$$\vec f = f_6\vec e_6 .$$

It's obvious that $f_6 \neq 0$ for the vector $\vec f$ at the last basis. Perform the operation $\vec f / f_6$ first, and the operations $\vec a - a_6\vec f$, $\vec b - b_6\vec f$, $\vec c - c_6\vec f$, $\vec d - d_6\vec f$ after the first one. We obtain the new canonical basis

$$\vec b = \vec e_2 + b_4\vec e_4,\ \vec b = \vec e_2 + b_4\vec e_4,\ \vec c = \vec e_3 + c_4\vec e_4,\ \vec d = \vec e_5,\ \vec f = \vec e_6 . \qquad (a_3)$$

If $f_5 \neq 0$ at the basis (4), perform the operation $\vec f / f_5$ first, and then the operations $\vec a - a_5\vec f$, $\vec b - b_5\vec f$, $\vec c - c_5\vec f$, $\vec d - d_5\vec f$. The following basis is obtained

$$\vec a = \vec e_1 + a_4\vec e_4 + a_6\vec e_6,\ \vec b = \vec e_2 + b_4\vec e_4 + b_6\vec e_6,$$
$$\vec c = \vec e_3 + c_4\vec e_4 + c_6\vec e_6,\ \vec d = d_6\vec e_6,\ \vec f = \vec e_5 + f_6\vec e_6 .$$

We have $d_6 \neq 0$ in the last basis. Perform the first operation $\vec d / d_6$, and then the operations $\vec a - a_6\vec d$, $\vec b - b_6\vec d$, $\vec c - c_6\vec d$, $\vec f - f_6\vec d$. The following canonical basis is obtained

$$\vec a = \vec e_1 + a_4\vec e_4,\ \vec b = \vec e_2 + b_4\vec e_4,\ \vec c = \vec e_3 + c_4\vec e_4,\ \vec d = \vec e_6,\ \vec f = \vec e_5 .$$

This basis is not new, it's equivalent to the basis (a_3).

2. Suppose, in opposition to the step 2, that all coefficients c_3, d_3, f_3 are zero in the basis (1). We have

$$\vec b = \vec e_2 + b_3\vec e_3 + b_4\vec e_4 + b_5\vec e_5 + b_6\vec e_6,\ \vec b = \vec e_2 + b_3\vec e_3 + b_4\vec e_4 + b_5\vec e_5 + b_6\vec e_6,$$
$$\vec c = c_4\vec e_4 + c_5\vec e_5 + c_6\vec e_6,\ \vec d = d_4\vec e_4 + d_5\vec e_5 + d_6\vec e_6, \qquad (5)$$
$$\vec f = f_4\vec e_4 + f_5\vec e_5 + f_6\vec e_6 .$$

Consider coefficients c_4, d_4, f_4 in the basis (5). Suppose that at least one of them is not zero.

Let $c_4 \neq 0$ (without any loss in generality). Perform the operation $\vec c / c_4$ first, and then the operations $\vec a - a_4\vec c$, $\vec b - b_4\vec c$, $\vec d - d_4\vec c$, $\vec f - f_4\vec c$. The following basis is obtained

$$\vec a = \vec e_1 + a_3\vec e_3 + a_5\vec e_5 + a_6\vec e_6,\ \vec b = \vec e_2 + b_3\vec e_3 + b_5\vec e_5 + b_6\vec e_6,$$
$$\vec c = \vec e_4 + c_5\vec e_5 + c_6\vec e_6,\ \vec d = d_5\vec e_5 + d_6\vec e_6,\ \vec f = f_5\vec e_5 + f_6\vec e_6 . \qquad (5a)$$

At least one coefficient among d_5, f_5 is not zero in the basis (5*a*). If both coefficients d_5, f_5 are zero, then $\vec d = d_6\vec e_6$, $\vec f = f_6\vec e_6$, and vectors $\vec d, \vec f$ are linearly dependent but it's impossible for any basis. Let $d_5 0$. Perform the operation $\vec d / d_5$ first, and then the operations $\vec a - a_5\vec d$, $\vec b - b_5\vec d$, $\vec c - c_5\vec d$, $\vec f - f_5\vec d$. The following basis is obtained

$$\vec a = \vec e_1 + a_3\vec e_3 + a_6\vec e_6,\ c = e_4 + c_6 e_6,$$
$$\vec c = \vec e_4 + c_6\vec e_6,\ \vec d = \vec e_5 + d_6\vec e_6,\ \vec f = f_6\vec e_6 .$$

The coefficient f_6 is not zero at the last basis. Perform the operation $\vec f / f_6$ first, and The operations $\vec a - a_6\vec f$, $\vec b - b_6\vec f$, $\vec c - c_6\vec f$, $\vec d - d_6\vec f$ after the first one. We obtain the new canonical basis

$$\vec a = \vec e_1 + a_3\vec e_3,\ \vec b = \vec e_2 + b_3\vec e_3,\ \vec d = \vec e_5,\ \vec d = \vec e_5,\ \vec f = \vec e_6 . \qquad (a_4)$$

If $f_5 \neq 0$ at the basis (5*a*), then perform the operation $\vec f / f_5$ first, and the operations $\vec a - a_5\vec f$, $\vec b - b_5\vec f$, $\vec c - c_5\vec f$, $\vec d - d_5\vec f$ after the first operation. The following basis is obtained

$$\vec a = \vec e_1 + a_3\vec e_3 + a_6\vec e_6,\ \vec b = \vec e_2 + b_3\vec e_3 + b_6\vec e_6,$$
$$\vec c = \vec e_4 + c_6\vec e_6,\ \vec d = d_6\vec e_6,\ \vec f = \vec e_5 + f_6\vec e_6 .$$

The previous basis generates the next canonical basis

$$\vec a = \vec e_1 + a_3\vec e_3,\ \vec b = \vec e_2 + b_3\vec e_3,\ \vec c = \vec e_4,\ \vec d = \vec e_6,\ \vec f = \vec e_5 .$$

This basis is obviously equivalent to the basis (a_4), so it's not a new one.

3. If all coefficients c_4, d_4, f_4 are zero at (5), then the basis has the following structure:

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$$\vec{a}=\vec{e_1}+a_3\vec{e_3}+a_4\vec{e_4}+a_5\vec{e_5}+a_6\vec{e_6}\ ,\ \vec{b}=\vec{e_2}+b_3\vec{e_3}+b_4\vec{e_4}+b_5\vec{e_5}+b_6\vec{e_6}\ ,$$
$$\vec{c}=c_5\vec{e_5}+c_6\vec{e_6}\ ,\ \vec{d}=d_5\vec{e_5}+d_6\vec{e_6}\ ,\ \vec{f}=f_5\vec{e_5}+f_6\vec{e_6}\ . \tag{6}$$

It's obvious that 3 vectors $\vec{c},\vec{d},\vec{f}$ at (6) are located at the same plane determined by vectors $\vec{e_5},\ \vec{e_6}$. So, they are linearly dependent that contradicts the fact that all vectors $\vec{a},\vec{b},\vec{c},\vec{d},\vec{f}$ are linearly independent. So, the case c_4=0, d_4=0, f_4=0 doesn't generate any canonical basis.

4. Suppose, in opposition to the Step 1, that the second coefficients b_2, c_2, d_2, f_2 at the basis **(a)** are zero. We obtain the following basis

$$\vec{a}=\vec{e_1}+a_2\vec{e_2}+a_3\vec{e_3}+a_4\vec{e_4}+a_5\vec{e_5}+a_6\vec{e_6},\ \vec{b}=b_3\vec{e_3}+b_4\vec{e_4}+b_5\vec{e_5}+b_6\vec{e_6}\ ,$$
$$\vec{c}=c_3\vec{e_3}+c_4\vec{e_4}+c_5\vec{e_5}+c_6\vec{e_6}\ ,\ \vec{d}=d_3\vec{e_3}+d_4\vec{e_4}+d_5\vec{e_5}+d_6\vec{e_6}\ , \tag{7}$$
$$\vec{f}=f_3\vec{e_3}+f_4\vec{e_4}+f_5\vec{e_5}+f_6\vec{e_6}\ .$$

Consider coefficients b_3, c_3, d_3, f_3 at the basis (7). Suppose that at least one of them is not zero. Without any loss in the generality, let $b_3 \neq 0$. Perform the operation $\vec{b}/b_3$ first, and then the operations $\vec{a}-a_3\vec{b}$, $\vec{c}-c_3\vec{b}$, $\vec{d}-d_3\vec{b}$, $\vec{f}-f_3\vec{b}$. The following basis is obtained

$$\vec{a}=\vec{e_1}+a_2\vec{e_2}+a_4\vec{e_4}+a_5\vec{e_5}+a_6\vec{e_6}\ ,\ \vec{b}=\vec{e_3}+b_4\vec{e_4}+b_5\vec{e_5}+b_6\vec{e_6}\ ,$$
$$\vec{c}=c_4\vec{e_4}+c_5\vec{e_5}+c_6\vec{e_6}\ ,\ \vec{d}=d_4\vec{e_4}+d_5\vec{e_5}+d_6\vec{e_6}\ ,$$
$$\vec{f}=f_4\vec{e_4}+f_5\vec{e_5}+f_6\vec{e_6}\ .$$

Consider coefficients c_4, d_4, f_4 at the previous basis. Suppose that at least one of them is not zero. Let $c_4 \neq 0$. Perform the operation $\vec{c}/c_4$ first, and the operations $\vec{a}-a_4\vec{c}$, $\vec{b}-b_4\vec{c}$, $\vec{d}-d_4\vec{c}$, $\vec{f}-f_4\vec{c}$ after the first one. The following basis is obtained

$$\vec{a}=\vec{e_1}+a_2\vec{e_2}+a_5\vec{e_5}+a_6\vec{e_6}\ ,\ \vec{b}=\vec{e_3}+b_5\vec{e_5}+b_6\vec{e_6}\ ,$$
$$\vec{c}=\vec{e_4}+c_5\vec{e_5}+c_6\vec{e_6}\ ,\ \vec{d}=d_5\vec{e_5}+d_6\vec{e_6}\ , \tag{8}$$
$$\vec{f}=f_5\vec{e_5}+f_6\vec{e_6}\ .$$

At least one coefficient among d_5, f_5 in the basis (8) is not zero. If both coefficients d_5, f_5 are zero, then vectors $\vec{d}=d_6\vec{e_6}$, $\vec{f}=f_6\vec{e_6}$ are linearly dependent but it's impossible for (8) to be a basis. Let $d_5 \neq 0$. Perform the operation $\vec{d}/d_5$ first, and the operations $\vec{a}-a_5\vec{d}$, $\vec{b}-b_5\vec{d}$, $\vec{c}-c_5\vec{d}$, $\vec{f}-f_5\vec{d}$ after the first one at the basis (8). The following new basis is obtained

$$\vec{a}=\vec{e_1}+a_2\vec{e_2}+a_6\vec{e_6}\ ,\ \vec{b}=\vec{e_3}+b_6\vec{e_6}\ ,$$
$$\vec{c}=\vec{e_4}+c_6\vec{e_6}\ ,\ \vec{d}=\vec{e_5}+d_6\vec{e_6}\ ,\ \vec{f}=f_6\vec{e_6}\ .$$

It's obvious that $f_6 \neq 0$ at the last basis, and it generates the new canonical basis

$$\vec{a}=\vec{e_1}+a_2\vec{e_2}\ ,\ \vec{b}=\vec{e_3}\ ,\ \vec{c}=\vec{e_4}\ ,\ \vec{d}=\vec{e_5}\ ,\ \vec{f}=\vec{e_6}\ . \tag{a_5}$$

If $f_5 \neq 0$ at the basis (8), perform the operation $\vec{f}/f_5$ first, and then the operations $\vec{a}-a_5\vec{f}$,

$\vec{b}-b_5\vec{f}$, $\vec{c}-c_5\vec{f}$, $\vec{d}-d_5\vec{f}$ at the basis (8). The following basis obtained

$$\vec{a}=\vec{e_1}+a_2\vec{e_2}+a_6\vec{e_6}\ ,\ \vec{b}=\vec{e_3}+b_6\vec{e_6}\ ,$$
$$\vec{c}=\vec{e_4}+c_6\vec{e_6}\ ,\ \vec{d}=d_6\vec{e_6}\ ,\ \vec{f}=\vec{e_5}+f_6\vec{e_6}\ .$$

It's obvious that $d_6 \neq 0$ in the last basis. So, the following canonical basis is generated

$$\vec{a}=\vec{e_1}+a_2\vec{e_2}\ ,\ \vec{b}=\vec{e_3}\ ,\ \vec{c}=\vec{e_4}\ ,\ \vec{d}=\vec{e_6}\ ,\ \vec{f}=\vec{e_5}\ .$$

The last basis is equivalent to the basis (a_5), so it's not a new one.

5. Suppose that coefficients b_3, c_3, d_3, f_3 at (8) are zero. We receive the basis

$$\vec{a}=\vec{e_1}+a_2\vec{e_2}+a_4\vec{e_4}+a_5\vec{e_5}+a_6\vec{e_6}\ ,\ \vec{b}=b_4\vec{e_4}+b_5\vec{e_5}+b_6\vec{e_6}\ ,$$
$$\vec{c}=c_3\vec{e_3}+c_5\vec{e_5}+c_6\vec{e_6}\ ,\ \vec{d}=d_4\vec{e_4}+d_5\vec{e_5}+d_6\vec{e_6}\ , \tag{9}$$
$$\vec{f}=f_4\vec{e_4}+f_5\vec{e_5}+f_6\vec{e_6}\ .$$

The four vectors $\vec{b}$, $\vec{c}$, $\vec{d}$, $\vec{f}$ in the basis (9) are linearly dependent because they are located at the same 3-dimensional subspace $Span\{\vec{e_4},\vec{e_5},\vec{e_6}\}$ but it contradicts to the fact that vectors $\vec{a},\vec{b},\vec{c},\vec{d},\vec{f}$ form a basis. So, this case doesn't generate any canonical basis.

A. Suppose that $a_1=b_1=c_1=d_1=f_1=0$ in the basis (I). The following basis (b) is obtained

$$\vec{a}=a_2\vec{e_2}+a_3\vec{e_3}+a_4\vec{e_4}+a_5\vec{e_5}+a_6\vec{e_6}\ ,\ \vec{c}=c_2\vec{e_2}+c_3\vec{e_3}+c_4\vec{e_4}+c_5\vec{e_5}+c_6\vec{e_6}\ ,$$
$$\vec{c}=c_2\vec{e_2}+c_3\vec{e_3}+c_4\vec{e_4}+c_5\vec{e_5}+c_6\vec{e_6}\ ,\ \vec{d}=d_2\vec{e_2}+d_3\vec{e_3}+d_4\vec{e_4}+d_5\vec{e_5}+d_6\vec{e_6}\ , \tag{b}$$
$$\vec{f}=f_2\vec{e_2}+f_3\vec{e_3}+f_4\vec{e_4}+f_5\vec{e_5}+f_6\vec{e_6}\ .$$

Suppose that at least one coefficient among a_2, b_2, c_2, d_2, f_2 is not zero in (b). Like before, we can suppose that $a_2 \neq 0$. Perform the operation $\vec{a}/a_2$ first, and the operations $\vec{b}-b_2\vec{a}$, $\vec{c}-c_2\vec{a}$, $\vec{d}-d_2\vec{a}$, $\vec{f}-f_2\vec{a}$ after the first one. The following new basis appears

$$\vec{a}=\vec{e_2}+a_3\vec{e_3}+a_4\vec{e_4}+a_5\vec{e_5}+a_6\vec{e_6}\ ,\ \vec{c}=c_3\vec{e_3}+c_4\vec{e_4}+c_5\vec{e_5}+c_6\vec{e_6}\ ,$$
$$\vec{c}=c_3\vec{e_3}+c_4\vec{e_4}+c_5\vec{e_5}+c_6\vec{e_6}\ ,\ \vec{d}=d_3\vec{e_3}+d_4\vec{e_4}+d_5\vec{e_5}+d_6\vec{e_6}\ , \tag{1}$$
$$\vec{f}=f_3\vec{e_3}+f_4\vec{e_4}+f_5\vec{e_5}+f_6\vec{e_6}\ .$$

Consider coefficients b_3, c_3, d_3, f_3 in the basis (1). Suppose that at least one of them is not zero. We can suggest that $b_3 \neq 0$ (without any loss in the generality). Perform the operation $\vec{b}/b_3$ first, and the operations $\vec{a}-a_3\vec{b}$, $\vec{c}-c_3\vec{b}$, $\vec{d}-d_3\vec{b}$, $\vec{f}-f_3\vec{b}$ after the first one. The following basis is obtained

$$\vec{a}=\vec{e_2}+a_4\vec{e_4}+a_5\vec{e_5}+a_6\vec{e_6}\ ,\ \vec{b}=\vec{e_3}+b_4\vec{e_4}+b_5\vec{e_5}+b_6\vec{e_6}\ ,$$
$$\vec{c}=c_4\vec{e_4}+c_5\vec{e_5}+c_6\vec{e_6}\ ,\ \vec{d}=d_4\vec{e_4}+d_5\vec{e_5}+d_6\vec{e_6}\ , \tag{2}$$
$$\vec{f}=f_4\vec{e_4}+f_5\vec{e_5}+f_6\vec{e_6}\ .$$

If all coefficients b_3, c_3, d_3, f_3 in (1) are zero, then vectors $\vec{b},\vec{c},\vec{d},\vec{f}$ are linearly dependent but it's impossible because these vectors form a basis.

1. Consider coefficients c_4, d_4, f_4 in the basis (2). At least one of them is not zero. If all of them are zero, then vectors

$$\vec{c}=c_5\vec{e_5}+c_6\vec{e_6}\ ,\ \vec{d}=d_5\vec{e_5}+d_6\vec{e_6}\ ,\ \vec{f}=f_5\vec{e_5}+f_6\vec{e_6}$$

are linearly dependent because they are located at the same 2-dimensional subspace generated by vectors $\vec{e_5},\vec{e_6}$. This means that the case $c_4=d_4=f_4=0$ is impossible.

Suppose that $c_4 \neq 0$. Perform the operation $\vec{c}/c_4$ in the basis (2) first, and the operations $\vec{a}-a_4\vec{c}$, $\vec{d}-d_4\vec{c}$, $\vec{d}-d_4\vec{c}$, $\vec{f}-f_4\vec{c}$ after the first operation. The following basis is obtained

$$\vec{a}=\vec{e_2}+a_5\vec{e_5}+a_6\vec{e_6}\ ,\ \vec{b}=\vec{e_3}+b_5\vec{e_5}+b_6\vec{e_6}\ ,$$
$$\vec{c}=\vec{e_4}+c_5\vec{e_5}+c_6\vec{e_6}\ ,\ \vec{d}=d_5\vec{e_5}+d_6\vec{e_6}\ , \tag{3}$$
$$\vec{f}=f_5\vec{e_5}+f_6\vec{e_6}\ .$$

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Consider coefficients d_5, f_5 in the basis (3). At least one of them is not zero. Otherwise, the vectors $\vec{d} = d_6\vec{e_6}$ and $\vec{f} = f_6\vec{e_6}$ are linearly dependent but it's impossible. Suppose that $d_5 \neq 0$. Perform the operation $\vec{d}/d_5$ first, and the operations $\vec{b} - b_5\vec{d}$, $\vec{b} - b_5\vec{d}$, $\vec{c} - c_5\vec{d}$, $\vec{f} - f_5\vec{d}$ after the first one. The following basis is obtained

$$\vec{a} = \vec{e_2} + a_6\vec{e_6}, \ \vec{b} = \vec{e_3} + b_6\vec{e_6},$$
$$\vec{c} = \vec{e_4} + c_6\vec{e_6}, \ \vec{d} = \vec{e_5} + d_6\vec{e_6}, \tag{4}$$
$$\vec{f} = f_6\vec{e_6}.$$

The coefficient f_6 is not zero in the basis (4). Perform the operation $\vec{f}/f_6$ in the basis (4) first, and the linear operations $\vec{a} - a_6\vec{f}$, $\vec{b} - b_6\vec{f}$, $\vec{c} - c_6\vec{f}$, $\vec{d} - d_6\vec{f}$ after the first one. We obtain the following new canonical basis

$$\vec{a} = \vec{e_2}, \ \vec{b} = \vec{e_3}, \ \vec{c} = \vec{e_4}, \ \vec{d} = \vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{b_1}$$

If $f_5 \neq 0$ in the basis (4), we obtain the same canonical basis (b_1).

2. Suppose that all coefficients a_2, b_2, c_2, d_2, f_2 are zero in the basis (b). The following possible basis is obtained

$$\vec{a} = a_3\vec{e_3} + a_4\vec{e_4} + a_5\vec{e_5} + a_6\vec{e_6}, \ \vec{b} = b_3\vec{e_3} + b_4\vec{e_4} + b_5\vec{e_5} + b_6\vec{e_6},$$
$$\vec{c} = c_3\vec{e_3} + c_4\vec{e_4} + c_5\vec{e_5} + c_6\vec{e_6}, \ \vec{d} = d_3\vec{e_3} + d_4\vec{e_4} + d_5\vec{e_5} + d_6\vec{e_6}, \tag{5}$$
$$\vec{f} = f_3\vec{e_3} + f_4\vec{e_4} + f_5\vec{e_5} + f_6\vec{e_6}.$$

These 5 vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{f}$ in (5) are linearly dependent because they are located at the same 4-dimensional subspace generated by vectors $\vec{e_3}, \vec{e_4}, \vec{e_5}, \vec{e_6}$. So, the system (5) of the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{f}$ doesn't produce any canonical basis.

The research performed for the set of coefficients a_1, b_1, c_1, d_1, f_1 in the cases **A** and **B** produces 6 canonical bases (a_1) – (a_5) and (b_1). To find other canonical bases, we should repeat the similar evaluations considering the following five sets of coefficients $\{a_2, b_2, c_2, d_2, f_2\}$, $\{a_3, b_3, c_3, d_3, f_3\}$, $\{a_4, b_4, c_4, d_4, f_4\}$, $\{a_5, b_5, c_5, d_5, f_5\}$, and $\{a_6, b_6, c_6, d_6, f_6\}$ in the basis (I). According the equity principle, we will obtain 6 similar canonical bases for each set of coefficients. Details are very close to those in the cases **A, B**, and we omit them. The total list of canonical bases is

$$\vec{a} = \vec{e_1} + a_6\vec{e_6}, \quad \vec{b} = \vec{e_2} + b_6\vec{e_6}, \quad \vec{c} = \vec{e_3} + c_6\vec{e_6}, \quad \vec{d} = \vec{e_4} + d_6\vec{e_6}, \quad \vec{f} = \vec{e_5} + f_6\vec{e_6}. \tag{a_1}$$

$$\vec{a} = \vec{e_1} + a_5\vec{e_5}, \ \vec{b} = \vec{e_2} + b_5\vec{e_5}, \ \vec{c} = \vec{e_3} + c_5\vec{e_5}, \ \vec{d} = \vec{e_4} + d_5\vec{e_5}, \ \vec{f} = \vec{e6}. \tag{a_2}$$

$$\vec{a} = \vec{e_1} + a_4\vec{e_4}, \ \vec{b} = \vec{e_2} + b_4\vec{e_4}, \ \vec{c} = \vec{e_3} + c_4\vec{e_4}, \ \vec{d} = \vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{a_3}$$

$$\vec{a} = \vec{e_1} + a_3\vec{e_3}, \ \vec{b} = \vec{e_2} + b_3\vec{e_3}, \ \vec{c} = \vec{e_4}, \ \vec{d} = \vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{a_4}$$

$$\vec{a} = \vec{e_1} + a_2\vec{e_2}, \ \vec{b} = \vec{e_3}, \ \vec{c} = \vec{e_4}, \ \vec{d} = \vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{a_5}$$

$$\vec{a} = \vec{e_2}, \ \vec{b} = \vec{e_3}, \ \vec{c} = \vec{e_4}, \ \vec{d} = \vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{b_1}$$

$$\vec{a} = \vec{e_2} + a_6\vec{e_6}, \quad \vec{b} = \vec{e_1} + b_6\vec{e_6}, \quad \vec{c} = \vec{e_3} + c_6\vec{e_6}, \quad \vec{d} = \vec{e_4} + d_6\vec{e_6}, \quad \vec{f} = \vec{e_5} + f_6\vec{e_6}. \tag{c_1}$$

$$\vec{a} = \vec{e_2} + a_5\vec{e_5}, \ \vec{b} = \vec{e_1} + b_5\vec{e_5}, \ \vec{d} = \vec{e_4} + d_5\vec{e_5}, \ \vec{d} = \vec{e_4} + d_5\vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{c_2}$$

$$\vec{a} = \vec{e_2} + a_4\vec{e_4}, \ \vec{b} = \vec{e_1} + b_4\vec{e_4}, \ \vec{c} = \vec{e_3} + c_4\vec{e_4}, \ \vec{d} = \vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{c_3}$$

$$\vec{a} = \vec{e_2} + a_3\vec{e_3}, \ \vec{b} = \vec{e_1} + b_3\vec{e_3}, \ \vec{c} = \vec{e_4}, \ \vec{f} = \vec{e_6}, \ \vec{f} = \vec{e_6}. \tag{c_4}$$

$$\vec{a} = a_1\vec{e_1} + \vec{e_2}, \ \vec{b} = \vec{e_3}, \ \vec{c} = \vec{e_4}, \ \vec{d} = \vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{c_5}$$

$$\vec{a} = \vec{e_1}, \ \vec{b} = \vec{e_3}, \ \vec{c} = \vec{e_4}, \ \vec{d} = \vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{d_1}$$

$$\vec{a} = \vec{e_3} + a_6\vec{e_6}, \quad \vec{b} = \vec{e_1} + b_6\vec{e_6}, \quad \vec{c} = \vec{e_2} + c_6\vec{e_6}, \quad \vec{d} = \vec{e_4} + d_6\vec{e_6}, \quad \vec{f} = \vec{e_5} + f_6\vec{e_6}. \tag{e_1}$$

$$\vec{a} = \vec{e_3} + a_5\vec{e_5}, \ \vec{b} = \vec{e_1} + b_5\vec{e_5}, \ \vec{c} = \vec{e_2} + c_5\vec{e_5}, \ \vec{d} = \vec{e_4} + d_5\vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{e_2}$$

$$\vec{a} = \vec{e_3} + a_4\vec{e_4}, \ \vec{b} = \vec{e_1} + b_4\vec{e_4}, \ \vec{c} = \vec{e_2} + c_4\vec{e_4}, \ \vec{d} = \vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{e_3}$$

$$\vec{a} = a_2\vec{e_2} + \vec{e_3}, \ \vec{b} = \vec{e_1} + b_2\vec{e_2}, \ \vec{c} = \vec{e_4}, \ \vec{d} = \vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{e_4}$$

$$\vec{a} = a_1\vec{e_1} + \vec{e_3}, \ \vec{b} = \vec{e_2}, \ \vec{c} = \vec{e_4}, \ \vec{d} = \vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{e_5}$$

$$\vec{a} = \vec{e_1}, \ \vec{b} = \vec{e_2}, \ \vec{c} = \vec{e_4}, \ \vec{d} = \vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{f_1}$$

$$\vec{a} = \vec{e_4} + a_6\vec{e_6}, \quad \vec{b} = \vec{e_1} + b_6\vec{e_6}, \quad \vec{d} = \vec{e_3} + d_6\vec{e_6}, \quad \vec{d} = \vec{e_3} + d_6\vec{e_6}, \quad \vec{f} = \vec{e_5} + f_6\vec{e_6}. \tag{g_1}$$

$$\vec{a} = \vec{e_4} + a_5\vec{e_5}, \ \vec{b} = \vec{e_1} + b_5\vec{e_5}, \ \vec{c} = \vec{e_2} + c_5\vec{e_5}, \ \vec{d} = \vec{e_3} + d_5\vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{g_2}$$

$$\vec{a} = a_3\vec{e_3} + \vec{e_4}, \ \vec{b} = \vec{e_1} + b_3\vec{e_3}, \ \vec{c} = \vec{e_2} + c_3\vec{e_3}, \ \vec{f} = \vec{e_6}, \ \vec{f} = \vec{e_6}. \tag{g_3}$$

$$\vec{a} = a_2\vec{e_2} + \vec{e_4}, \ \vec{b} = \vec{e_1} + b_2\vec{e_2}, \ \vec{c} = \vec{e_3}, \ \vec{d} = \vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{g_4}$$

$$\vec{a} = a_1\vec{e_1} + \vec{e_4}, \ \vec{b} = \vec{e_2}, \ \vec{c} = \vec{e_3}, \ \vec{d} = \vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{g_5}$$

$$\vec{a} = \vec{e_1}, \ \vec{b} = \vec{e_2}, \ \vec{c} = \vec{e_3}, \ \vec{d} = \vec{e_5}, \ \vec{f} = \vec{e_6}. \tag{h_1}$$

$$\vec{a} = \vec{e_5} + a_6\vec{e_6}, \quad \vec{b} = \vec{e_1} + b_6\vec{e_6}, \quad \vec{c} = \vec{e_2} + c_6\vec{e_6}, \quad \vec{d} = \vec{e_3} + d_6\vec{e_6}, \quad \vec{a} = a_4\vec{e_4} + \vec{e_5}. \tag{i_1}$$

$$\vec{a} = a_4\vec{e_4} + \vec{e_5}, \ \vec{b} = \vec{e_1} + b_4\vec{e_4}, \ \vec{c} = \vec{e_2} + c_4\vec{e_4}, \ \vec{d} = \vec{e_3} + d_4\vec{e_4}, \ \vec{f} = \vec{e_6}. \tag{i_2}$$

$$\vec{a} = a_3\vec{e_3} + \vec{e_5}, \ \vec{b} = \vec{e_1} + b_3\vec{e_3}, \ \vec{c} = \vec{e_2} + c_3\vec{e_3}, \ \vec{d} = \vec{e_4}, \ \vec{f} = \vec{e_6}. \tag{i_3}$$

$$\vec{b} = \vec{e_1} + b_2\vec{e_2}, \ \vec{b} = \vec{e_1} + b_2\vec{e_2}, \ \vec{c} = \vec{e_3}, \ \vec{d} = \vec{e_4}, \ \vec{f} = \vec{e_6}. \tag{i_4}$$

$$\vec{a} = a_1\vec{e_1} + \vec{e_5}, \ \vec{b} = \vec{e_2}, \ \vec{c} = \vec{e_3}, \ \vec{d} = \vec{e_4}, \ \vec{f} = \vec{e_6}. \tag{i_5}$$

$$\vec{a} = \vec{e_1}, \ \vec{b} = \vec{e_2}, \ \vec{c} = \vec{e_3}, \ \vec{d} = \vec{e_4}, \ \vec{f} = \vec{e_6}. \tag{j_1}$$

$$\vec{a} = a_5\vec{e_5} + \vec{e_6}, \quad \vec{b} = \vec{e_1} + b_5\vec{e_5}, \quad \vec{c} = \vec{e_2} + c_5\vec{e_5}, \quad \vec{d} = \vec{e_3} + d_5\vec{e_5}, \quad \vec{f} = \vec{e_4} + f_5\vec{e_5}. \tag{k_1}$$

$$\vec{a} = a_4\vec{e_4} + \vec{e_6}, \ \vec{b} = \vec{e_1} + b_4\vec{e_4}, \ \vec{c} = \vec{e_2} + c_4\vec{e_4}, \ \vec{d} = \vec{e_3} + d_4\vec{e_4}, \ \vec{f} = \vec{e_5}. \tag{k_2}$$

$$\vec{a} = a_3\vec{e_3} + \vec{e_6}, \ \vec{b} = \vec{e_1} + b_3\vec{e_3}, \ \vec{c} = \vec{e_2} + c_3\vec{e_3}, \ \vec{d} = \vec{e_4}, \ \vec{f} = \vec{e_5}. \tag{k_3}$$

$$\vec{a} = a_2\vec{e_2} + \vec{e_6}, \ \vec{b} = \vec{e_1} + b_2\vec{e_2}, \ \vec{c} = \vec{e_3}, \ \vec{d} = \vec{e_4}, \ \vec{f} = \vec{e_5}. \tag{k_4}$$

$$\vec{a} = a_1\vec{e_1} + \vec{e_6}, \ \vec{b} = \vec{e_2}, \ \vec{c} = \vec{e_3}, \ \vec{d} = \vec{e_4}, \ \vec{f} = \vec{e_5}. \tag{k_5}$$

$$\vec{a} = \vec{e_1}, \ \vec{b} = \vec{e_2}, \ \vec{c} = \vec{e_3}, \ \vec{d} = \vec{e_4}, \ \vec{f} = \vec{e_5}. \tag{l_1}$$

Analyze all these bases comparing them step by step. The bases (b_1), (d_1), (f_1), (h_1), (j_1), (l_1) are particular cases of (c_5), (a_5), (c_4), (a_3), (a_2), (a_1) respectively. The bases (c_1) – (c_4) are obviously equivalent to the bases (a_1) – (a_4). The basis (c_5) is equivalent to the basis (a_5) if $a_1 \neq 0$, and (c_5) is equivalent to the basis (b_1) if $a_1 = 0$. The bases

(e_1) – (e_3) are equivalent to the bases (a_1) – (a_3). If $a_2 \neq 0$ then the basis (e_4) is equivalent to the basis (a_4); if $a_2 = 0$ then (e_4) is equivalent to (a_5). The basis (e_3) is a particular case of the basis (a_4) if $a_1 \neq 0$, and (e_5) is a particular case of (g_5) if $a_1 = 0$. The bases (g_1), (g_2) are obviously equivalent to the bases (a_1), (a_2). The basis (g_3) is equivalent to the basis (a_3) if $a_3 \neq 0$, and (g_3) is equivalent to (a_5) if $a_3 = 0$. The basis (g_4) is equivalent to the basis (a_3) if $a_2 \neq 0$, and (g_4) is equivalent to (a_5) if $a_2 = 0$. The basis (g_5) is a particular case of the basis (a_3) if $a_1 \neq 0$, and (g_5) is a particular case of (i_5) if $a_1 = 0$. The basis (i_1) is equivalent to the basis (a_1). The basis (i_2) is equivalent to the basis (a_2) if $a_4 \neq 0$, so consider the basis (i_2) if $a_4 = 0$. The new basis (i_2) is a particular case of the basis (a_1) if $f_4 = 0$, and (i_2) is equivalent to (a_3) if f_4=0. The basis (i_3) is a particular case of the basis (a_2) if $a_3 \neq 0$, and the basis (i_3) is equivalent to the basis (a_4) if a_3=0. The basis (i_4) is a particular case of the basis (a_2) if $a_2 \neq 0$, and (i_4) is equivalent to (a_5) if a_2=0. The basis (i_5) is a particular case of the basis (a_2) if $a_1 \neq 0$, and (i_5) is a particular case of the basis (k_5) if a_1=0. The basis (k_1) is equivalent to the basis (a_2). The basis (k_2) is equivalent to the basis (i_2). The basis (k_3) is a particular case of the basis (a_1) if $a_3 \neq 0$, and (k_3) is equivalent to the basis (a_4) if a_3=0. The basis (k_4) is a particular case of the basis (a_1) if $a_2 \neq 0$, and (k_4) is equivalent to the basis (a_5) if a_2=0. The basis (k_5) is a particular case of the basis (a_1) if $a_1 \neq 0$, and (k_5) is equivalent to the basis (b_1) if a_1=0.

The analysis performed above implies the following statement.

Theorem 1

Each basis of any 5-dimensional subspace in a 6-dimensional vector space is equivalent to one and only one of the following 6 canonical bases

$$\vec a = \vec{e_1} + a_6\vec{e_6}, \quad \vec b = \vec{e_2} + b_6\vec{e_6}, \quad \vec c = \vec{e_3} + c_6\vec{e_6}, \quad \vec d = \vec{e_4} + d_6\vec{e_6}, \quad \vec f = \vec{e_5} + f_6\vec{e_6}; \qquad (a_1)$$

$$\vec a = \vec{e_1} + a_5\vec{e_5}, \ \vec b = \vec{e_2} + b_5\vec{e_5}, \ \vec c = \vec{e_3} + c_5\vec{e_5}, \ \vec d = \vec{e_4} + d_5\vec{e_5}, \ \vec f = \vec{e_6}; \qquad (a_2)$$

$$\vec b = \vec{e_2} + b_4\vec{e_4}, \ \vec b = \vec{e_2} + b_4\vec{e_4}, \ \vec c = \vec{e_3} + c_4\vec{e_4}, \ \vec d = \vec{e_5}, \ \vec f = \vec{e_6}; \qquad (a_3)$$

$$\vec a = \vec{e_1} + a_3\vec{e_3}, \ \vec b = \vec{e_2} + b_3\vec{e_3}, \ \vec c = \vec{e_4}, \ \vec d = \vec{e_5}, \ \vec f = \vec{e_6}; \qquad (a_4)$$

$$\vec a = \vec{e_1} + a_2\vec{e_2}, \ \vec b = \vec{e_3}, \ \vec c = \vec{e_4}, \ \vec d = \vec{e_5}, \ \vec f = \vec{e_6}; \qquad (a_5)$$

$$\vec a = \vec{e_2}, \ \vec b = \vec{e_3}, \ \vec c = \vec{e_4}, \ \vec d = \vec{e_5}, \ \vec f = \vec{e_6}. \qquad (b_1)$$

Part II

5-dimensional subalgebras of Lie algebra of Lorentz group

Introduction: Lorentz group is the group of transformations of Minkowski space-time R^4. This group is not compact, not abelian, and not connected 6-dimensional real Lie group. The identity component of Lorentz group is the group $SO^+(3, 1)$. This component contains the generators for boots along *x*-, *y*- and *z*-axis, and it contains the generators for rotations in Minkowski space-time [4]. Lie algebra of the group $SO^+(3, 1)$ is 6-dimensional real Lie algebra denoted below by *L* that has the following standard basis:

$$\vec{e_1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \vec{e_2} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \vec{e_3} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

$$\vec{e_4} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \vec{e_5} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \quad \vec{e_6} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}.$$

The Lie product of any two square matrices *A*, *B* is defined by [*A*, *B*]=*AB* – *BA*. For the standard basis of Lie algebra of Lorentz group, the non-zero products are

$$\left[\vec{e_1},\vec{e_2}\right] = \vec{e_4}, \ \left[\vec{e_1},\vec{e_3}\right] = \vec{e_5}, \ \left[\vec{e_1},\vec{e_4}\right] = \vec{e_2}, \ \left[\vec{e_1},\vec{e_5}\right] = \vec{e_3}, \ \left[\vec{e_2},\vec{e_3}\right] = \vec{e_6}, \ \left[\vec{e_2},\vec{e_4}\right] = -\vec{e_1},$$

$$\left[\vec{e_2},\vec{e_6}\right] = \vec{e_3}, \ \left[\vec{e_3},\vec{e_5}\right] = -\vec{e_1}, \ \left[\vec{e_3},\vec{e_6}\right] = -\vec{e_2}, \ \left[\vec{e_4},\vec{e_5}\right] = -\vec{e_6}, \ \left[\vec{e_4},\vec{e_6}\right] = \vec{e_5}, \ \left[\vec{e_5},\vec{e_6}\right] = -\vec{e_4}. \quad (*)$$

To determine which 5-dimensional subspace *h* of the given Lie algebra *L* is a subalgebra of *L*, we will check the condition [*h, h*]⊂ *h* applying to the nonequivalent canonical bases that are described in the Theorem 1.

Let the subspace h_1 be generated by the canonical basis (a_1). Compute all products between vectors $\vec a, \vec b, \vec c, \vec d, \vec f$ in this basis. Utilizing the table of products **(*),** we have

$$\left[\vec a,\vec b\right] = \left[\vec{e_1} + a_6\vec{e_6}, \vec{e_2} + b_6\vec{e_6}\right] = \vec{e_4} - a_6\vec{e_3} = x_1\vec a + x_2\vec b + x_3\vec c + x_4\vec d + x_5\vec f \cdot$$

So, x_1=0, x_2=0, x_3=$-a_6$, x_4=1, x_5=0, and $-a_6c_6+d_6$=0.

$$\left[\vec a,\vec c\right] = \left[\vec{e_1} + a_6\vec{e_6}, \vec{e_3} + c_6\vec{e_6}\right] = \vec{e_5} + a_6\vec{e_2} = y_1\vec a + y_2\vec b + y_3\vec c + y_4\vec d + y_5\vec f \cdot$$

So, y_1=0, y_2=a_6, y_1=0, y_3=0, y_4=0, y_5=1, and $a_6b_6+f_6$=0.

$$\left[\vec a,\vec d\right] = \left[\vec{e_1} + a_6\vec{e_6}, \vec{e_4} + d_6\vec{e_6}\right] = \vec{e_2} - a_6\vec{e_5} = z_1\vec a + z_2\vec b + z_3\vec c + z_4\vec d + z_5\vec f \cdot$$

So, z_1=0, z_2=1, z_3=0, z_4=0, z_5=$-a_6$ and $b_6 - a_6f_6$=0.

$$\left[\vec a,\vec f\right] = \left[\vec{e_1} + a_6\vec{e_6}, \vec{e_5} + f_6\vec{e_6}\right] = \vec{e_3} + a_6\vec{e_4} = s_1\vec a + s_2\vec b + s_3\vec c + s_4\vec d + s_5\vec f \cdot$$

So, s_1=0, s_2=0, s_3=1, s_4=a_6, s_5=0, and $c_6+a_6d_6$=0.

$$\left[\vec b,\vec c\right] = \left[\vec{e_2} + b_6\vec{e_6}, \vec{e_3} + c_6\vec{e_6}\right] = \vec{e_6} + b_6\vec{e_2} = p_1\vec a + p_2\vec b + p_3\vec c + p_4\vec d + p_5\vec f \cdot$$

So, p_1=0, p_2=b_6, p_3=0, p_4=0, p_5=0, and b_6b_6=1.

$$\left[\vec b,\vec d\right] = \left[\vec{e_2} + b_6\vec{e_6}, \vec{e_4} + d_6\vec{e_6}\right] = -\vec{e_1} + d_6\vec{e_3} - b_6\vec{e_5} = q_1\vec a + q_2\vec b + q_3\vec c + q_4\vec d + q_5\vec f \cdot$$

So, q_1=-1, q_2=0, q_3=d_6, q_4=0, q_5=$-b_6$, and $-a_6 + d_6c_6 - b_6f_6$

$$\left[\vec b,\vec f\right] = \left[\vec{e_2} + b_6\vec{e_6}, \vec{e_5} + f_6\vec{e_6}\right] = f_6\vec{e_3} + b_6\vec{e_4} = r_1\vec a + r_2\vec b + r_3\vec c + r_4\vec d + r_5\vec f \cdot$$

So, r_1=0, r_2=0, r_3=f_6, r_4=b_6, r_5=0, and $f_6c_6+b_6d_6$=0.

$$\left[\vec c,\vec d\right] = \left[\vec{e_3} + c_6\vec{e_6}, \vec{e_4} + d_6\vec{e_6}\right] = -d_6\vec{e_2} - c_6\vec{e_5} = t_1\vec a + t_2\vec b + t_3\vec c + t_4\vec d + t_5\vec f \cdot$$

So, t_1=0, t_2=$-d_6$, t_3=0, t_4=0, t_5=$-c_6$, and $-d_6b_6-c_6f_6$=0.

$$\left[\vec c,\vec f\right] = \left[\vec{e_3} + c_6\vec{e_6}, \vec{e_5} + f_6\vec{e_6}\right] = -\vec{e_1} - f_6\vec{e_2} + c_6\vec{e_4} = v_1\vec a + v_2\vec b + v_3\vec c + v_4\vec d + v_5\vec f \cdot$$

So, v_1=-1, v_2=$-f_6$, v_3=0, v_4=c_6, v_5=0, and $-a_6-f_6b_6+c_6d_6$=0.

$$\left[\vec d,\vec f\right] = \left[\vec{e_4} + d_6\vec{e_6}, \vec{e_5} + f_6\vec{e_6}\right] = -\vec{e_6} + f_6\vec{e_5} + d_6\vec{e_4} = n_1\vec a + n_2\vec b + n_3\vec c + n_4\vec d + n_5\vec f \cdot$$

So, n_1=0, n_2=0, n_3=0, n_4=d_6, n_5=f_6, and $d_6d_6 + f_6f_6$=-1.

The equation $d_6^{\ 2} + f_6^{\ 2} = -1$ has no solution in the set of all real numbers. This means that no 5-dimensional subalgebra of Lie algebra *L* with the basis (a_1) exists.

Let the subspace h_2 be generated by the canonical basis (a_2). Compute all products between vectors $\vec a, \vec b, \vec c, \vec d, \vec f$ in this basis. Utilizing the table of products (*), we have

$$\left[\vec a,\vec b\right] = \left[\vec{e_1} + a_5\vec{e_5}, \vec{e_2} + b_5\vec{e_5}\right] = \vec{e_4} + b_5\vec{e_3} = x_1\vec a + x_2\vec b + x_3\vec c + x_4\vec d + x_5\vec f \cdot$$

So, x_1=0, x_2=0, x_3=b_5, x_4=1, x_5=0, and $b_5c_5+d_5$=0.

$$\left[\vec a,\vec c\right] = \left[\vec{e_1} + a_5\vec{e_5}, \vec{e_3} + c_5\vec{e_5}\right] = \vec{e_5} + c_5\vec{e_3} + a_5\vec{e_1} = y_1\vec a + y_2\vec b + y_3\vec c + y_4\vec d + y_5\vec f \cdot$$

So, y_1=a_5, y_2=0, y_3=c_5, y_4=0, y_5=0, and $a_5a_5 + c_5c_5$.

$$\left[\vec a,\vec d\right] = \left[\vec{e_1} + a_5\vec{e_5}, \vec{e_4} + d_5\vec{e_5}\right] = \vec{e_2} + d_5\vec{e_3} + a_5\vec{e_6} = z_1\vec a + z_2\vec b + z_3\vec c + z_4\vec d + z_5\vec f \cdot$$

So, z_1=0, z_2=1, z_3=d_5, z_4=0, z_5=a_5, and $b_5 + d_5c_5$=0.

$$\left[\vec a,\vec f\right] = \left[\vec{e_1} + a_5\vec{e_5}, \vec{e_6}\right] = -a_5\vec{e_4} = s_1\vec a + s_2\vec b + s_3\vec c + s_4\vec d + s_5\vec f \ .$$

So, s_1=0, s_2=0, s_3=0, s_4=$-a_5$, s_5=0, and a_5d_5=0.

$$\left[\vec b,\vec c\right] = \left[\vec{e_2} + b_5\vec{e_5}, \vec{e_3} + c_5\vec{e_5}\right] = \vec{e_6} + b_5\vec{e_1} = p_1\vec a + p_2\vec b + p_3\vec c + p_4\vec d + p_5\vec f \cdot$$

So, p_1=b_5, p_2=0, p_3=0, p_4=0, p_5=1, and b_5a_5=0.

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$$\left[\vec{b},\vec{d}\right]=\left[\vec{e_2}+b_5\vec{e_5},\vec{e_4}+d_5\vec{e_5}\right]=-\vec{e_1}+b_5\vec{e_6}=q_1\vec{a}+q_2\vec{b}+q_3\vec{c}+q_4\vec{d}+q_5\vec{f}\cdot$$

So, $q_1=-1$, $q_2=0$, $q_3=0$, $q_4=0$, $q_5=b_5$, and $-a_5=0$.

$$\left[\vec{b},\vec{f}\right]=\left[\vec{e_2}+b_5\vec{e_5},\vec{e_6}\right]=\vec{e_3}-b_5\vec{e_4}=r_1\vec{a}+r_2\vec{b}+r_3\vec{c}+r_4\vec{d}+r_5\vec{f}\cdot$$

So, $r_1=0$, $r_2=0$, $r_3=1$, $r_4=-b_5$, $r_5=0$, and $c_5-b_5d_5=0$.

$$\left[\vec{c},\vec{d}\right]=\left[\vec{e_3}+c_5\vec{e_5},\vec{e_4}+d_5\vec{e_5}\right]=-d_5\vec{e_1}+c_5\vec{e_6}=t_1\vec{a}+t_2\vec{b}+t_3\vec{c}+t_4\vec{d}+t_5\vec{f}\cdot$$

So, $t_1=-d_5$, $t_2=0$, $t_4=0$, $t_5=c_5$, and $-d_5a_5=0$.

$$\left[\vec{c},\vec{f}\right]=\left[\vec{e_3}+c_5\vec{e_5},\vec{e_6}\right]=-\vec{e_2}-c_5\vec{e_4}=v_1\vec{a}+v_2\vec{b}+v_3\vec{c}+v_4\vec{d}+v_5\vec{f}\cdot$$

So, $v_1=0$, $v_2=-1$, $v_3=0$, $v_4=-c_5$, $v_{5=}0$, and $-b_5-c_5d_5=0$.

$$\left[\vec{d},\vec{f}\right]=\left[\vec{e_4}+d_5\vec{e_5},\vec{e_6}\right]=\vec{e_5}-d_5\vec{e_4}=n_1\vec{a}+n_2\vec{b}+n_3\vec{c}+n_4\vec{d}+n_5\vec{f}\cdot$$

So, $n_1=0$, $n_2=0$, $n_3=0$, $n_4=-d_5$, $n_5=0$, and $-d_5d_5=1$.

The last equation $d_5^2=-1$ has no solution in the set of all real numbers. This means that no 5-dimensional subalgebra of Lie algebra L with the basis (a_2) exists.

Let the subspace h_3 be generated by the canonical basis (a_3). Consider all products between vectors $\vec{a},\vec{b},\vec{c},\vec{d},\vec{f}$ of this basis. Utilizing the table of products (*), we have

$$\left[\vec{a},\vec{b}\right]=\left[\vec{e_1}+a_4\vec{e_4},\vec{e_2}+b_4\vec{e_4}\right]=\vec{e_4}+b_4\vec{e_2}+a_4\vec{e_1}=x_1\vec{a}+x_2\vec{b}+x_3\vec{c}+x_4\vec{d}+x_5\vec{f}\cdot$$

So, $x_{1=}a_4$, $x_2=b_4$, $x_3=0$, $x_4=0$, $x_4=0$, $x_5=0$ and $a_4a_4+b_4b_4=1$.

$$\left[\vec{a},\vec{c}\right]=\left[\vec{e_1}+a_4\vec{e_4},\vec{e_3}+c_4\vec{e_4}\right]=\vec{e_5}+c_4\vec{e_2}=y_1\vec{a}+y_2\vec{b}+y_3\vec{c}+y_4\vec{d}+y_5\vec{f}\cdot$$

So, $y_1=0$, $y_2=c_4$, $y_3=0$, $y_4=1$, $y_5=0$, and $c_4b_4=0$.

$$\left[\vec{a},\vec{d}\right]=\left[\vec{e_1}+a_4\vec{e_4},\vec{e_5}\right]=\vec{e_3}-a_4\vec{e_6}=z_1\vec{a}+z_2\vec{b}+z_3\vec{c}+z_4\vec{d}+z_5\vec{f}\cdot$$

So, $z_1=0$, $z_2=0$, $z_3=1$, $z_4=0$, $z_5=-a_4$, and $c_4=0$.

$$\left[\vec{a},\vec{f}\right]=\left[\vec{e_1}+a_4\vec{e_4},\vec{e_6}\right]=a_4\vec{e_5}=s_1\vec{a}+s_2\vec{b}+s_3\vec{c}+s_4\vec{d}+s_5\vec{f}\cdot$$

So, $s_1=0$, $s_2==0$, $s_3=0$, $s_4=a_4$, $s_5=0$, and $0=0$.

$$\left[\vec{b},\vec{c}\right]=\left[\vec{e_2}+b_4\vec{e_4},\vec{e_3}+c_4\vec{e_4}\right]=\vec{e_6}-c_4\vec{e_1}=p_1\vec{a}+p_2\vec{b}+p_3\vec{c}+p_4\vec{d}+p_5\vec{f}\cdot$$

So, $p_1=-c_4$, $p_2=0$, $p_3=0$, $p_4=0$, $p_5=1$, and $-c_4a_4=0$.

$$\left[\vec{b},\vec{d}\right]=\left[\vec{e_2}+b_4\vec{e_4},\vec{e_5}\right]=-b_4\vec{e_6}=q_1\vec{a}+q_2\vec{b}+q_3\vec{c}+q_4\vec{d}+q_5\vec{f}\cdot$$

So, $q_1=0$, $q_2=0$, $q_3=0$, $q_4=0$, $q_5=-b_4$, and $0=0$.

$$\left[\vec{b},\vec{f}\right]=\left[\vec{e_2}+b_4\vec{e_4},\vec{e_6}\right]=\vec{e_3}+b_4\vec{e_5}=r_1\vec{a}+r_2\vec{b}+r_3\vec{c}+r_4\vec{d}+r_5\vec{f}\cdot$$

So, $r_1=0$, $r_2=0$, $r_3=1$, $r_4=b_4$, $r_5=0$, and $c_4=0$.

$$\left[\vec{c},\vec{d}\right]=\left[\vec{e_3}+c_4\vec{e_4},\vec{e_5}\right]=-\vec{e_1}-c_4\vec{e_6}=t_1\vec{a}+t_2\vec{b}+t_3\vec{c}+t_4\vec{d}+t_5\vec{f}\cdot$$

So, $t_1=-1$, $t_2=0$, $t_3=0$, $t_4=0$, $t_5=-c_4$, and $a_4=0$.

$$\left[\vec{c},\vec{f}\right]=\left[\vec{e_3}+c_4\vec{e_4},\vec{e_6}\right]=-\vec{e_2}+c_4\vec{e_5}=v_1\vec{a}+v_2\vec{b}+v_3\vec{c}+v_4\vec{d}+v_5\vec{f}\cdot$$

So, $v_1=0$, $v_2=-1$, $v_3=0$, $v_4=c_4$, $v_5=0$, and $b_4=0$.

$$\left[\vec{d},\vec{f}\right]=\left[\vec{e_5},\vec{e_6}\right]=-\vec{e_4}=n_1\vec{a}+n_2\vec{b}+n_3\vec{c}+n_4\vec{d}+n_5\vec{f}\cdot$$

So, n1=0, n2=0, n3=0, n4=0, n5=0, and 0=−1.

The last contradiction 0=−1 proves that Lie algebra L has no 5-dimensional subalgebra generated by the basis (a_3).

Let the subspace h_4 be generated by the canonical basis (a_4). Consider all products between vectors $\vec{a},\vec{b},\vec{c},\vec{d},\vec{f}$ in this basis. Utilizing the table of products (*), we have

$$\left[\vec{a},\vec{b}\right]=\left[\vec{e_1}+a_3\vec{e_3},\vec{e_2}+b_3\vec{e_3}\right]=\vec{e_4}+b_3\vec{e_5}-a_3\vec{e_6}=x_1\vec{a}+x_2\vec{b}+x_3\vec{c}+x_4\vec{d}+x_5\vec{f}\cdot$$

So, $x_1=0$, $x_2=0$, $x_3=1$, $x_4=b_3$, $x_5=-a_3$, and $0=0$.

$$\left[\vec{a},\vec{c}\right]=\left[\vec{e_1}+a_3\vec{e_3},\vec{e_4}\right]=\vec{e_2}=y_1\vec{a}+y_2\vec{b}+y_3\vec{c}+y_4\vec{d}+y_5\vec{f}\cdot$$

So, $y_1=0$, $y_2=1$, $y_3=0$, $y_4=0$, $y_5=0$, and $b_3=0$.

$$\left[\vec{a},\vec{d}\right]=\left[\vec{e_1}+a_3\vec{e_3},\vec{e_5}\right]=\vec{e_3}-a_3\vec{e_1}=z_1\vec{a}+z_2\vec{b}+z_3\vec{c}+z_4\vec{d}+z_5\vec{f}\cdot$$

So, $z_1=-a_3$, $z_2=0$, $z_4=0$, $z_5=0$, and $-a_3^2=1$.

The last condition $a_3^2=-1$ is impossible in the set of all real numbers. This means that the product $\left[\vec{a},\vec{d}\right]$ doesn't belong to the 5-dimensional subspace generated by the basis (a_4). Thus, this subspace is not subalgebra of Lie algebra L.

Let the subspace h_5 be generated by the canonical basis (a_5). Consider all products between vectors $\vec{a},\vec{b},\vec{c},\vec{d},\vec{f}$ in this basis. Utilizing the table of products (*), we have

$$\left[\vec{a},\vec{b}\right]=\left[\vec{e_1}+a_2\vec{e_2},\vec{e_3}\right]=\vec{e_5}+a_2\vec{e_6}=x_1\vec{a}+x_2\vec{b}+x_3\vec{c}+x_4\vec{d}+x_5\vec{f}\cdot$$

So, $x_1=0$, $x_2=0$, $x_3=0$, $x_4=1$, $x_5=a_2$, and $0=0$.

$$\left[\vec{a},\vec{c}\right]=\left[\vec{e_1}+a_2\vec{e_2},\vec{e_4}\right]=\vec{e_2}-a_2\vec{e_1}=y_1\vec{a}+y_2\vec{b}+y_3\vec{c}+y_4\vec{d}+y_5\vec{f}\cdot$$

So, $y_1=-a_2$, $y_2=0$, $y_4=0$, $y_5=0$, and $-a_2^2=1$.

The last condition $a_2^2=-1$ is not satisfied in the set of all real numbers. This means that no 5-dimensional subalgebra with the basis (a_5) exists in Lie algebra L.

Let the subspace h_6 be generated by the canonical basis (b_1). Consider all products between vectors $\vec{a},\vec{b},\vec{c},\vec{d},\vec{f}$ in this basis. Utilizing the table of products (*), we have

$$\left[\vec{a},\vec{b}\right]=\left[\vec{e_2},\vec{e_3}\right]=\vec{e_6}=x_1\vec{a}+x_2\vec{b}+x_3\vec{c}+x_4\vec{d}+x_5\vec{f}\cdot$$

So, $x_1=0$, $x_2=0$, $x_3=0$, $x_5=1$, and $1=1$.

$$\left[\vec{a},\vec{c}\right]=\left[\vec{e_2},\vec{e_4}\right]=-\vec{e_1}=y_1\vec{a}+y_2\vec{b}+y_3\vec{c}+y_4\vec{d}+y_5\vec{f}\cdot$$

So, $y_1=0$, $y_2=0$, $y_3=0$, $y_4=0$, $y_5=0$, and $0=-1$.

The last contradiction 0=− 1 shows that Lie algebra L has no 5-dimensional subalgebra generated by the basis (b_1).

The evaluations performed in Part II prove the following statements.

Theorem 2

Lie algebra of the Lorentz group doesn't contain any 5-dimendional subalgebra.

Corollary

Lorentz group doesn't contain any connected 5-dimensional subgroup.

References

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