TOKYO J. MATH.
VOL. 34, NO. 1, 2011

Erratum to "New Techniques for Classifying Williams Solenoids" (Tokyo Journal of Mathematics, Vol. 30, No. 1, pp. 139–157, June 2007)

Marcy BARGE and Richard SWANSON

Montana State University

(Communicated by S. Miyoshi)

We regret that we have found a mistake in our paper [1]. The error lay not in our independent theorems, but in our statement of a result in [2]. In the following discussion, we describe the setting for the original paper, the incorrect theorem, and we give an explicit counterexample, which we feel is new and of independent interest. We note that the error did not affect any of our conclusions, and was not actually used in our paper.

A **presentation** is a mapping pair (K, f) such that $f : K \to K$ is a continuous endomorphism of a graph K, with inverse limit space $\varprojlim(f, K)$ whose shift map $\overline{f}$ is conjugate to the given homeomorphism on $\mathcal{S}$, a generalized solenoid. Williams showed (in [2, Theorem 3.3]) that two shift maps $\overline{f}_1$ and $\overline{f}_2$, on presentations (K_1, f_1) and (K_2, f_2), are topologically conjugate if and only if the maps f_1 and f_2 are shift equivalent. He was able to show further that shift equivalence is equivalent to "strong shift equivalence" in the category of maps on branched 1-manifolds satisfying the Williams axioms (see [1] or Axioms 2.1 in [2]). This reduces checking shift equivalence to seeking a sequence of "elementary" (or lag 1) shift equivalences.

This erratum treats Williams' efforts to link the shift equivalence of pointed presentations (corresponding to pointed conjugacy classes of shifts $\overline{f} : (\varprojlim(K, f), \overline{x}) \to (\varprojlim(K, f), \overline{x})$, $\overline{x} = (x, x, \dots)$ to the shift equivalence of π_1 representations. Williams defines the *shift class* $S(\overline{f}, x)$ of $\overline{f}$ to be the shift equivalence class of $\pi_1(f, x) : \pi_1(K, x) \to \pi_1(K, x)$. A presentation (K, f) is **elementary** if K is a wedge of circles and f fixes the branch point y of K.

Here is the *incorrect* theorem that we had in our paper.

THEOREM. *Suppose the elementary presentations* (K_i, f_i), $i = 1, 2$, *satisfy the Williams axioms, and* $f_i(y_i) = y_i$, $i = 1, 2$. *There is a pointed conjugacy*

$$\overline{r} : (\varprojlim(K_1, f_1), \overline{y}_1) \to (\varprojlim(K_2, f_2), \overline{y}_2)$$

Received July 9, 2010

of $\overline{f}_1$ *with* $\overline{f}_2$ *if and only if the fundamental group homomorphisms* $\pi_1(f_1, y_1)$ *and* $\pi_1(f_2, y_2)$ *are shift equivalent.*

PROPOSITION. *This theorem is false as stated.*

PROOF. The wrapping rules

$$\psi : \begin{cases} \mathsf{a} \mapsto \mathsf{abbbbc} \\ \mathsf{b} \mapsto \mathsf{abc} \\ \mathsf{c} \mapsto \mathsf{ac} \end{cases} \qquad \varphi : \begin{cases} \mathsf{a} \mapsto \mathsf{aabbbb} \\ \mathsf{b} \mapsto \mathsf{ab} \end{cases}$$

generate pointed graph maps $f = f_\psi : (K_3, p) \to (K_3, p)$ and $g = g_\varphi : (K_2, q) \to (K_2, q)$, where K_3 is a wedge of 3 circles, K_2 of 2 circles and p, q are the branch points. In the proof of Proposition 4.7 of [1], we prove that there is no shift equivalence (r,s) between these maps such that $r(p) = q$. That is, there is no conjugacy of $\overline{f}$ with $\overline{g}$ that takes $\overline{p}$ to $\overline{q}$.

However, here is an explicit shift equivalence of lag 3 between the group endomorphisms $\pi_1(f, p)$ and $\pi_1(g, q)$:

$$\rho : \begin{cases} \mathsf{a} \mapsto \mathsf{b}^{-1}\mathsf{abbbbababababab} \\ \mathsf{b} \mapsto \mathsf{b}^{-1}\mathsf{abbbbabab} \\ \mathsf{c} \mapsto \mathsf{b}^{-1}\mathsf{abbbbab} \end{cases} \qquad \sigma : \begin{cases} \mathsf{a} \mapsto \mathsf{c}^{-1}\mathsf{bcacabbbc} \\ \mathsf{b} \mapsto \mathsf{c}^{-1}\mathsf{bcac} \end{cases}$$

The reader may wish to check that $\varphi^3 = \rho \circ \sigma$ and $\psi^3 = \sigma \circ \rho$. In fact, this is a strong shift equivalence which passes through a series of 3 elementary (lag 1) shift equivalences. □

In the above example, there is a conjugacy between $\overline{f}$ and $\overline{g}$ — it just doesn't take $\overline{p}$ to $\overline{q}$. This leaves the following question open: Is is possible, in the context of elementary presentations satisfying the Williams axioms, to have $\pi_1(f_1, y_1)$ shift equivalent to $\pi(f_2, y_2)$, while $\overline{f}_1$ is not conjugate to $\overline{f}_2$?

References

[1] M. BARGE and R. SWANSON, New techniques for classifying Williams solenoids, Tokyo J. Math., **30** (2007), 139–157.

[2] R. F. WILLIAMS, Classification of 1-dimensional attractors, Proc. Symp. Pure Math. (AMS) **14** (1970), 341–361.